Socio-Economic Planning Sciences 95 (2024) 102056

Contents lists available at ScienceDirect

Socio-Economic Planning Sciences

journal homepage: www.elsevier.com/locate/seps

Stage stochastic incremental data envelopment analysis models and applications

Bo-wen Wei [a,1], Yi-yi Ma [b,1], Ai-bing Ji [b,*]

[a] Hebei University, College of Management, Baoding, 071002, Hebei, China
[b] Hebei University, College of Mathematics and Information Science, Baoding, 071002, Hebei, China

ARTICLE INFO

Keywords:
Stage stochastic incremental DEA model
Stochastic inputs–outputs
Semi-stochastic inputs–outputs
Investment and financing efficiency

ABSTRACT

Data envelopment analysis (DEA) is a mathematical programming method that can evaluate the relative efficiency of multiple inputs and multiple outputs of a decision-making unit (DMU). The classical DEA model assumes that inputs and outputs are determined. However, there are some applications where the inputs–outputs are stochastic. In practice, it is important to evaluate stage performance. It is essential to eliminate the effect of preceding stage inputs (outputs) on stage performance in order to accurately assess stage performance. In this paper, we propose stage stochastic incremental DEA models that integrate two different kinds of inputs and outputs. The first kind of model takes into account the assessment of stage efficiency when determinate incremental inputs and stochastic incremental outputs are applied at the beginning and end of the stage. The second kind of model uses stochastic incremental inputs–outputs to evaluate stage efficiency. To verify the efficacy of the suggested models, the first kind of model is applied to assess the stage financing efficiency of 15 energy-saving and environmental protection clean enterprises (ESEPCEs). The second kind of model is applied in assessing the stage investment efficiency of 15 ESEPCEs. The empirical results show that the proposed models not only eliminate the effect of prior performance but also more accurately assess stage efficiency in a stochastic environment.

1. Introduction

DEA is a powerful and versatile method used in measuring the efficiency of multi-input and multi-output production systems [1,2]. The CCR model, which is the first DEA model, was originally introduced by Charnes et al. [3]. Then Banker et al. [4] suggested a variable proportional returns version of the DEA-CCR model, which has since become widely known as the DEA-BCC model. Based on the above two models, the DEA model has a series of extensions based on practical applications [5,6]. The method is able to accomplish two goals of the assessment: analyzing historical efficiency and strategizing for future improvements [7–9].

The classical DEA models assume that inputs–outputs are deterministic. But in practical management, inputs–outputs may be stochastic. For example, in the investment and financing process, enterprises are confronted with various uncertainties, such as fluctuations in interest rates, project failure risks, and the potential for asset impairment. These factors contribute to the randomness of inputs and outputs, rendering the financial landscape complex and challenging. In addition, the data collected are often snapshot observations, the values of which may vary if collected a little earlier or later. Therefore, the inputs–outputs of a business over time are often stochastic variables. Another situation is when it is necessary to forecast the performance of a business for the following year, in which operations have not yet begun. In this case, the data of inputs–outputs is unknown and must be predicted, so it is more suitable to represent them by stochastic variables. The stochastic DEA model has attracted substantial attention from scholars and has been extensively investigated, reflecting its prominence in the field.

Sengupta [10] and Land et al. [11] proposed stochastic decision models to explain the randomness of inputs–outputs. Banker et al. [4] considered statistical factors in DEA and proposed a statistical non-parametric DEA method. After, Sengupta [12], Huang and Li [13], Khodabakhshi and Asgharian [14], and Lotfi et al. [15] presented stochastic DEA models and obeyed the normal distribution around probabilistic stochastic problems. For example, Nazari and Behzadi [16] gave the asymptotic distribution function of a stochastic variable with a skewed normal distribution and applied it to DEA. Izadikhah and Saen [17] proposed a two-stage stochastic DEA model

* Corresponding author.
E-mail addresses: 20219012006@stumail.hbu.edu.cn (B.-w. Wei), mayiyi@stumail.hbu.edu.cn (Y.-y. Ma), jab@hbu.edu.cn (A.-b. Ji).
[1] Contributing authors.

https://doi.org/10.1016/j.seps.2024.102056
Received 7 February 2024; Received in revised form 10 August 2024; Accepted 2 September 2024
Available online 3 September 2024
0038-0121/

for intermediate products that can produce both desired and undesired outputs. By convolution of normal and half-normal distributions, Jradi and Ruggiero [18] solves the stochastic frontier model programming problem with incorrect structural constraints. In addition, Tavassoli et al. [19] proposed a stochastic fuzzy DEA model using the α-cut algorithm. Kao and Liu [20] developed a network stochastic model considering the correlation between inputs and outputs and applied the model to 22 commercial banks in Taiwan. Wanke et al. [21] proposed a stochastic ratio-DEA model and utilized a generalized auto-regressive moving average model to examine the temporal dependence within the input/output set, thereby anticipating efficiency. Therefore, the stochastic DEA model enables researchers to quantify the variability in efficiency scores and better understand the impact of stochastic factors on DMUs.

The stochastic DEA models presented above analyze the overall performance of the DMUs. However, stage performance evaluation is more commonly practiced. For instance, the assessment of academic disciplines conducted by Chinese universities every 2–3 years and the development of a strategic plan by an enterprise to be implemented for 3–4 years are all stage performance evaluations. Stage performance evaluation encompasses a thorough assessment of performance at a particular stage, offering a valuable reference for leaders in their strategic planning and decision-making for the subsequent stage. Therefore, many scholars have paid attention to stage performance evaluations that take into account the randomness of inputs and outputs under some practical problems.

The dynamic DEA model, window DEA model, and Malmquist index model are all methods of evaluating the stage performance of a DMU, responding to performance over time. In view of the common randomness in practical applications, scholars have also attempted to incorporate the randomness of inputs–outputs for DMUs into the aforementioned methods to more accurately assess stage performance. For example, Yaghoubi and Fazli [22] proposed a dynamic stochastic DEA model based on a fuzzy environment and combined it with modern banking industry indicators to predict bank efficiency. Gan et al. [23] presented a dynamic network DEA model and used it to analyze the efficiency of industrial metabolism in 18 first-tier cities in China from 2016–2020. Kumar et al. [24] proposed a non-stochastic frontier window DEA model with a window period of three years and analyzed the efficiency of private sector banks in India from 2005–2017. Liu et al. [25] proposed a stochastic semi-parametric frontier three-stage window DEA model for evaluating the green economic efficiency of Chinese industry. Molinos-Senante et al. [26] proposed a stochastic meta-frontier approach to evaluate and compare productivity changes in water and sewerage enterprises in England and Wales. This approach was used to assess the changes over the period 1991–2016 for water utilities in the same regions. However, the aforementioned methods assess changes in stage performance. It is critical to acknowledge that the evaluation results can be influenced by the preceding stage's performance, which consequently may compromise the accuracy of the evaluation under current stage. However, the existing stage performance models cannot solve this stage evaluation problem. This paper also studies stage efficiency over some time. The task of this paper is to solve the limitations of previous methods. Specifically, it aims to eliminate the influence of input and output performance prior to a particular stage by building a stage performance evaluation model. Furthermore, it considers the stochastic nature of inputs and outputs to establish a stage stochastic incremental DEA model.

This paper is to further consider the problem of input–output randomness based on Ji et al. [27], and propose a stage stochastic incremental performance evaluation model. It solves the problem of the possible randomness of inputs–outputs in stage performance evaluation. Therefore, this paper first uses the stage incremental data of inputs (outputs) as the inputs (outputs) data for stage efficiency evaluation. Meanwhile, considering the randomness of inputs–outputs in practical management, this paper proposes two kinds of stage stochastic incremental DEA models. One is the stage stochastic incremental DEA model, where inputs are determined and outputs are stochastic. The other is the stage stochastic incremental DEA model, where inputs and outputs are stochastic. The proposed two kinds of models are applied to evaluate the stage investment and financing efficiency of ESEPCEs, respectively. The practical applications not only validate the validity of the proposed models but also evaluate the stage investment and financing of the enterprise from the perspective of efficiency, which is helpful for enterprises to optimize their growth strategies and achieve more sustainable development. The specific work of this paper is as follows:

- Using the incremental data of inputs and outputs from the assessed stage can eliminate the influence of the previous stage's efficiency.
- Taking into account the randomness of the practical problems, this paper proposes a stage stochastic incremental DEA model under deterministic inputs and stochastic outputs and gives its definitions.
- Then, this paper further expands on the first model, proposes a stage stochastic incremental DEA model under stochastic inputs and outputs, and gives its definitions.
- To validate the proposed two kinds of stage stochastic incremental DEA models, the first form of the proposed models evaluates the stage financing efficiency of 15 ESEPCEs in China after green transformation. The second form evaluates the stage investment efficiency of 15 ESEPCEs in China. The evaluation results provide a reference for promoting the sustainable development of ESEPCEs.

The organization of this paper is outlined below. Section 2 describes the basics. Section 3 presents stage stochastic incremental DEA models under two forms of inputs–outputs and provides relevant definitions and theorems. Section 4 evaluates the stage financing efficiency and stage investment efficiency of ESEPCEs using the two kinds of models, respectively, which not only validates the effectiveness of the models but also evaluates the stage of investment and financing. Finally, Section 5 summarizes the paper.

2. Preliminaries

2.1. DEA-CCR model

Assume that we have n DMUs ($DMU_j, j = 1, \dots, n$), each associates with m inputs $X_j = \left(x_{1j}, x_{2j}, \dots, x_{mj}\right)^T$ and s outputs $Y_j = \left(y_{1j}, y_{2j}, \dots, y_{sj}\right)^T, j = 1, \dots, n$. The production possibility set T can be expressed as follows:

$$T = \left\{ (X,Y) | \, X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j > 0, j = 1, \dots, n \right\} \tag{1}$$

The DEA-CCR model measures the relative efficiency of DMUs is as follows:

$$\max \sum_{r=1}^{s} \mu_r y_{r0}$$
$$s.t. \tag{2}$$
$$\begin{cases} \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \omega_i x_{ij} \leq 0, j = 1, \dots, n, \\ \sum_{i=1}^{m} \omega_i x_{i0} = 1, \\ \mu_r, \omega_i \geq 0, r = 1, \dots, s; i = 1, \dots, m. \end{cases}$$

where the subscript 0 indicates the evaluated DMU, $\omega_i (i = 1, \dots, m)$ and $\mu_r (r = 1, \dots, s)$ express the weights of m inputs and s outputs, respectively.

The dual linear programming model for model (2) is:

$$\min \theta$$
$$s.t. \tag{3}$$
$$\begin{cases} \sum_{j=1}^{n} \lambda_j X_j \le \theta X_{j_0}, \\ \sum_{j=1}^{n} \lambda_j Y_{j_0} \ge Y_j, \\ \lambda_j \ge 0, j = 1, \dots, n. \end{cases}$$

Definition 2.1. If the model (2) has optimal solutions μ_r^*, ω_i^* satisfying the conditions $\sum_{r=1}^{s} \mu_r^* y_{r0} = 1$, and $\mu_r^* > 0, \omega_i^* > 0$, then DMU_{j_0} is defined as DEA efficient.

2.2. Stage incremental DEA model

The incremental DEA model proposed by Ji et al. [27] is the basis for constructing this paper. Assuming that there are a total of n DMUs that require efficiency measurements. Each DMU_j at time t consumes m inputs x_{ij}^t to produce s outputs y_{rj}^t. From the time t_0 to $t_0 + l$ ($t_0 > 1$), that is, stage t_0, the increments of inputs–outputs are defined by $\Delta X_j^{t_0} = X_j^{t_0+l} - X_j^{t_0}$, $\Delta Y_j^{t_0} = Y_j^{t_0+l} - Y_j^{t_0}$. The stage incremental DEA model is as follows:

$$\max \frac{u^T \Delta Y_0^{t_0}}{v^T \Delta X_0^{t_0}}$$
$$s.t. \tag{4}$$
$$\begin{cases} \frac{u^T \Delta Y_j^{t_0}}{v^T \Delta X_j^{t_0}} \le 1, j = 1, \dots, n, \\ u^T \Delta Y_j^{t_0} \ge 0, j = 1, \dots, n, \\ v^T \Delta X_j^{t_0} > 0, j = 1, \dots, n, \\ u \ge 0, \ v \ge 0. \end{cases}$$

where $\frac{u^T \Delta Y_0^{t_0}}{v^T \Delta X_0^{t_0}}$ denotes the stage efficiency of DMU_{j_0}. $v = (v_1, v_2, \dots, v_m)^T > 0$, v_i is the weight of the ith incremental input. $u = (u_1, u_2, \dots, u_s)^T > 0$, u_r is the weight of the rth incremental output. The incremental inputs $\Delta x_{ij}^{t_0} (1 \le i \le m)$ and incremental outputs $\Delta y_{rj}^{t_0} (1 \le r \le s)$ may be positive, negative or zero. The aggregate incremental input $v^T \Delta X_j^{t_0} > 0$ is non-negative. The aggregate incremental output $u^T \Delta Y_j^{t_0} \ge 0$ is positive. The stage incremental DEA model simultaneously minimizes total weighted incremental inputs and maximizes total weighted incremental outputs. By C^2-transformation [3]: $t = \frac{1}{v^T \Delta X_0^{t_0}}$, $\omega = tv$, $\mu = tu$, the fractional programming model (4) can be converted into the following equivalent linear programming model:

$$\max \ \theta = \mu^T \Delta Y_0^{t_0}$$
$$s.t. \tag{5}$$
$$\begin{cases} \omega^T \Delta X_j^{t_0} - \mu^T \Delta Y_j^{t_0} \ge 0, j = 1, \dots, n, \\ \omega^T \Delta X_0^{t_0} = 1, \\ \mu^T \Delta Y_j^{t_0} \ge 0, j = 1, \dots, n, \\ \mu \ge 0, \\ \omega \ge 0. \end{cases}$$

Using Lagrange multiplier method, the dual model of model (5) is presented as follows:

$$\min \ \theta$$
$$s.t. \tag{6}$$
$$\begin{cases} \sum_{j=1}^{n} \lambda_j \Delta X_j^{t_0} \le \theta \Delta X_0^{t_0}, \\ \sum_{j=1}^{n} (\lambda_j - \eta_j) \Delta Y_j^{t_0} \ge \Delta Y_0^{t_0}, \\ \lambda_j \ge 0, \eta_j \ge 0, j = 1, \dots, n. \end{cases}$$

The stage efficiency value θ^* ranges between 0 and 1.

2.3. Stochastic DEA model

2.3.1. Stochastic DEA model with determinate inputs and stochastic outputs

Assume that we have n DMUs ($DMU_j, j = 1, \dots, n$), each associates with m determinate inputs $X_j = (x_{1j}, x_{2j}, \dots, x_{mj})^T$ and s stochastic outputs $\tilde{Y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, \dots, \tilde{y}_{sj})^T, j = 1, \dots, n$.

When the inputs are determinate and the outputs are stochastic, the stochastic DEA model with a normal distribution is the following:

$$\max_{u,v,f} f$$
$$s.t. \tag{7}$$
$$\begin{cases} \Pr\left[\sum_{r=1}^{s} u_r \tilde{y}_{r0} \ge f\right] \ge \alpha, & \text{(a)} \\ \sum_{r=1}^{s} v_i x_{i0} = 1, & \text{(b)} \\ \Pr\left[\sum_{r=1}^{s} u_r \tilde{y}_{rj} - \sum_{r=1}^{s} v_i x_{ij} \le 0\right] \ge \alpha, j = 1, \dots, n, & \text{(c)} \\ v_i, u_r \ge 0, i = 1, \dots, m; r = 1, \dots, s. \end{cases}$$

where Pr represents probability; α is significance level.

In model (7), the objective function is maximized such that the significance level of satisfying the constraints is at least α. Constraint (7a) states that the probability that the sum of weighted outputs is higher than the stochastic efficiency score is at least the significance level α. Constraint (7c) represents the probability that all n constraints (linear inequalities) have a given significance level α.

Assume that $\tilde{Y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, \dots, \tilde{y}_{sj})^T, j = 1, \dots, n$, are the normal output vectors for DMU_j, and stochastic outputs $\tilde{Y}_j$ are independently distributed normal stochastic variables with

$$E\left[\tilde{y}_{rj}\right] = y_{rj}, Var\left[\tilde{y}_{rj}\right] = \left(\sigma_{rj}^O\right)^2 \tag{8}$$

According to Eqs. (8) and Cheng and Lisser [28], model (7) can be transformed into a determinate form of optimization problem, as follows

$$\max_{u,v} \sum_{r=1}^{s} u_r y_{r0} - \phi^{-1}(\alpha) \sqrt{\sum_{r=1}^{s} u_r^2 \operatorname{Var}(\tilde{y}_{r0})}$$
$$s.t. \tag{9}$$
$$\begin{cases} \sum_{i=1}^{m} v_i x_{i0} = 1, \\ \sum_{r=1}^{s} u_r y_{rj} + \phi^{-1}(\alpha^{\lambda_j}) \sqrt{\sum_{r=1}^{s} u_r^2 \operatorname{Var}(\tilde{y}_{r0})} \le \sum_{i=1}^{m} v_i x_{ij}, j = 1, \dots, n, \\ u_r \ge 0, r = 1, \dots, s; v_i \ge 0; i = 1, \dots, m; \lambda_j \ge 0, j = 1, \dots, n. \end{cases}$$

2.3.2. Stochastic DEA model with stochastic inputs–outputs

Assume that we have n DMUs ($DMU_j, j = 1, \dots, n$), each associates with m stochastic inputs $\tilde{X}_j = (\tilde{x}_{1j}, \tilde{x}_{2j}, \dots, \tilde{x}_{mj})^T$ and s stochastic outputs $\tilde{Y}_j = (\tilde{y}_{1j}, \tilde{y}_{2j}, \dots, \tilde{y}_{sj})^T, j = 1, \dots, n$. These components are considered to follow a normally distributed distribution, i.e. $\tilde{x}_{ij} \sim \left(x_{ij}, \delta_{ij}^2\right)$, $\tilde{y}_{rj} \sim N\left(y_{rj}, v_{rj}^2\right)$. Stochastic DEA models are divided into two types: input-oriented and output-oriented. Here we introduce the input-oriented stochastic DEA model. The input-oriented stochastic DEA model is as follows:

$$\min \theta$$
$$s.t. \quad (10)$$
$$\begin{cases} Pr\left(\sum_{j=1}^{n} \lambda_j \tilde{X}_{ij} \le \theta \tilde{X}_{i0}\right) \ge 1-\alpha, i=1,2,\ldots,m, \\ Pr\left(\sum_{j=1}^{n} \lambda_j \tilde{Y}_{rj} \ge \tilde{Y}_{r0}\right) \ge 1-\alpha, r=1,\ldots,s, \\ \lambda_j \ge 0, j=1,\ldots,n. \end{cases}$$

where $1-\alpha$ is confidence level, and $\alpha(0 \le \alpha \le 1)$ represents significance level. θ^* denotes the optimal objective function value for model (10).

Definition 2.2. For the given significance level α, when $\theta^* = 1$, the evaluated DMU_0 is defined as stochastic DEA efficient ; and when $\theta^* < 1$, the evaluated DMU_0 is defined as stochastic DEA inefficient.

Inequality constraints can be replaced with equality constraints by adding slack variables. The equality constraints for stochastic DEA model (10) are as follows:

$$\min \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$
$$s.t. \quad (11)$$
$$\begin{cases} Pr\left(\sum_{j=1}^{n} \lambda_j \tilde{X}_{ij} - \theta \tilde{X}_{i0} \le -s_i^-\right) = 1-\alpha, i=1,\ldots,m, \\ Pr\left(\sum_{j=1}^{n} \lambda_j \tilde{Y}_{rj} - \tilde{Y}_{r0} \ge s_r^+\right) = 1-\alpha, r=1,\ldots,s, \\ \lambda_j, s_i^-, s_r^+ \ge 0, j=1,\ldots,n; i=1,\ldots,m; r=1,\ldots,s. \end{cases}$$

DMU_0 is stochastic efficient if $\theta^* = 1, s_i^- = s_r^+ = 0 (i=1,\ldots,m; r=1,\ldots,s)$.

The equivalent form of model (11) is expressed as follows:

$$\min \theta - \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$
$$s.t. \quad (12)$$
$$\begin{cases} \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- - \phi^{-1}(\alpha)\delta_i(\lambda,\theta) = \theta x_{i0}, i=1,\ldots,m, \\ \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ + \phi^{-1}(\alpha)\upsilon_r(\lambda) = y_{r0}, r=1,\ldots,s, \\ \lambda_j, s_i^-, s_r^+ \ge 0, j=1,\ldots,n; i=1,\ldots,m; r=1,\ldots,s. \end{cases}$$

where ϕ is the cumulative distribution function of the normal distribution. $\delta_i(\lambda,\theta)$ is the standard deviation of the stochastic variable $\sum_{j=1}^{n} \lambda_j \tilde{X}_{ij} - \theta \tilde{X}_{i0}$; $\upsilon_r(\lambda)$ is the standard deviation of the stochastic variable $\sum_{j=1}^{n} \lambda_j \tilde{Y}_{rj} - \tilde{Y}_{r0}$.

3. Model formulation

The model in this section is further considering the randomness of inputs–outputs on the basis of Ji et al. [27]. We also define the period from period t_0 to period t_0+l as stage t_0.

3.1. Stage stochastic incremental DEA model with deterministic inputs and stochastic outputs

Assume that $S = \left\{\left(X_j^t, \tilde{Y}_j^t\right) | j=1,\ldots,n; t=1,\ldots,T\right\}$ is a panel stochastic production possibility set. Each DMU_j at period t consumes m different real-valued inputs $X_j^t = \left(x_{1j}^t, x_{2j}^t, \ldots, x_{mj}^t\right)^T$ to produce s different stochastic outputs $\tilde{Y}_j^t = \left(\tilde{y}_{1j}^t, \tilde{y}_{2j}^t, \ldots, \tilde{y}_{sj}^t\right)^T$.

From the stage t_0, the increments of inputs–outputs are defined by $\Delta X_j^{t_0} = X_j^{t_0+l} - X_j^{t_0}$, $\Delta\tilde{Y}_j^{t_0} = \tilde{Y}_j^{t_0+l} - \tilde{Y}_j^{t_0}$.

Based on the stage incremental DEA model [27] and stochastic DEA model, the stage stochastic incremental DEA model with determinate inputs and stochastic outputs is given as follows:

$$\max f$$
$$s.t.$$
$$\begin{cases} \Pr\left(\mu^T \Delta\tilde{Y}_0^{t_0} \ge f\right) \ge 1-\alpha, & \text{(a)} \\ \omega^T \Delta X_0^{t_0} = 1, & \text{(b)} \\ \Pr\left(\mu^T \Delta\tilde{Y}_j^{t_0} - \omega^T \Delta X_j^{t_0} \le 0\right) \ge 1-\beta, j=1,\ldots,n, & \text{(c)} \\ \Pr\left(\mu^T \Delta\tilde{Y}_j^{t_0} \ge 0\right) \ge 1-\gamma, j=1,\ldots,n, & \text{(d)} \\ \mu \ge 0, \omega \ge 0. \end{cases} \quad (13)$$

where α, β, γ are significance level between 0 and 1.

Then add a positive slack variable to each constraint in optimization problem (13), as follows:

$$(13a) \Rightarrow \Pr\left(\mu^T \Delta\tilde{Y}_0^{t_0} \ge f\right) = 1-\alpha+\xi_0, \quad (14)$$

$$(13c) \Rightarrow \Pr\left(\mu^T \Delta\tilde{Y}_j^{t_0} - \omega^T \Delta X_j^{t_0} \le 0\right) = 1-\beta+\xi_j, \quad (15)$$

$$(13d) \Rightarrow \Pr\left(\mu^T \Delta\tilde{Y}_j^{t_0} \ge 0\right) = 1-\gamma+\eta_j. \quad (16)$$

There must exist positive slack variables s_0^+, s_j^+, s_j^-, such that the Eqs. (14), (15), (16) are converted as follows:

$$\Pr\left(\mu^T \Delta\tilde{Y}_0^{t_0} \ge f + s_0^+\right) = 1-\alpha, \quad (17)$$

$$\Pr\left(\mu^T \Delta\tilde{Y}_j^{t_0} - \omega^T \Delta X_j^{t_0} + s_j^- \le 0\right) = 1-\beta, \quad (18)$$

$$\Pr\left(\mu^T \Delta\tilde{Y}_j^{t_0} \ge s_j^+\right) = 1-\gamma. \quad (19)$$

It is obviously that $\xi_0 = 0$ if and only if $s_0^+ = 0$. Similarly, $\xi_j = 0$ if and only if $s_j^- = 0$, and $\eta_j = 0$ if and only if $s_j^+ = 0$.

Then we introduce the non-Archimedean infinitesimal $\varepsilon > 0$, the stage stochastic incremental DEA model can be characterized by the following chance constraints model:

$$\max f + \varepsilon\left(s_0^+ + \sum_{j=1}^{n} s_j^+ + \sum_{j=1}^{n} s_j^-\right)$$
$$s.t. \quad (20)$$
$$\begin{cases} \Pr\left(\mu^T \Delta Y_0^{t_0} \ge f + s_0^+\right) = 1-\alpha, \\ \omega^T \Delta X_0^{t_0} = 1, \\ \Pr\left(\mu^T \Delta\tilde{Y}_j^{t_0} - \omega^T \Delta X_j^{t_0} + s_j^- \le 0\right) = 1-\beta, j=1,\ldots,n, \\ \Pr\left(\mu^T \Delta\tilde{Y}_j^{t_0} \ge s_j^+\right) = 1-\gamma, \\ \mu \ge 0, \omega \ge 0, s_0^+ \ge 0, s_j^+ \ge 0, s_j^- \ge 0, j=1,\ldots,n. \end{cases}$$

Definition 3.1 (*Stage Stochastic Efficient*)**.** In model (20), for the given significance levels α, β, γ of $[0,1]$, if there exists at least one optimal solution (μ^*, ω^*), such that the following conditions are satisfied:

(1) f = 1,
(2) $s_0^+ = s_j^+ = s_j^- = 0, \forall j=1,\ldots,n$.

then the evaluated decision-making unit DMU_{j_0} is stage stochastic efficient for the given significance levels α, β, γ of $[0,1]$.

For the given significance level α, β, γ, the stage stochastic incremental DEA model (20) can be transformed into the following deterministic equivalent:

$$\max f + \varepsilon\left(s_0^+ + \sum_{j=1}^{n} s_j^+ + \sum_{j=1}^{n} s_j^-\right)$$

$$s.t. \qquad (21)$$
$$\begin{cases} \mu^T \Delta Y_0^{t_0} - f - s_0^+ + \sigma_0(f,u)\phi^{-1}(\alpha) = 0, \\ \omega^T \Delta X_0^{t_0} - 1 = 0, \\ \mu^T \Delta Y_j^{t_0} - \omega^T \Delta X_j^{t_0} + s_j^- + \sigma_j(\mu)\phi^{-1}(1-\beta) = 0, j = 1,\dots,n, \\ \mu^T \Delta Y_j^{t_0} - s_j^+ + \phi^{-1}(\gamma)\sigma_j(\mu) = 0, j = 1,\dots,n, \\ \mu \ge 0, \omega \ge 0, s_0^+ \ge 0, s_j^+ \ge 0, s_j^- \ge 0. \end{cases}$$

where $\left[\sigma_0(f,\mu)\right]^2 = \mu^T \sum \mu$, $\sum$ is the covariance matrix of $\Delta\tilde{Y}_0^{t_0}$, $\left[\sigma_j(\mu)\right]^2 = \left[\sigma_j(\mu)\right]^2 = \mu^T \sum_j \mu$, and $\sum_i$ is the covariance matrix of $\Delta\tilde{Y}_j^{t_0}$. Especially, if the components of $\Delta\tilde{Y}_j^{t_0} = \left(\Delta\tilde{y}_{1j}^{t_0}, \Delta\tilde{y}_{2j}^{t_0}, \dots, \Delta\tilde{y}_{sj}^{t_0}\right)^T$ are independent of each other, the variance of $\mu^T \Delta\tilde{Y}_j^{t_0}$ is $\sum_{i=1}^{s} \mu_i^2 Var\left(\Delta\tilde{y}_{ij}^{t_0}\right)$. Generally, the significance levels α, β, γ can take values of 0.05, 0.01, etc. When the significance levels $\alpha = 0, \beta = 1, \gamma = 0$, model (21) degenerates into a stage incremental DEA model [27].

3.2. Stage stochastic incremental DEA model with stochastic inputs–outputs

Assume that $S^* = \left\{\left(\tilde{X}_j^t, \tilde{Y}_j^t\right) | j = 1,\dots,n; t = 1,\dots,T\right\}$ is a panel stochastic production possibility set. Each DMU_j at period t consumes m different stochastic inputs $\tilde{X}_j^t = \left(\tilde{x}_{1j}^t, \tilde{x}_{2j}^t, \dots, \tilde{x}_{mj}^t\right)^T$ to produce s different stochastic outputs $\tilde{Y}_j^t = \left(\tilde{y}_{1j}^t, \tilde{y}_{2j}^t, \dots, \tilde{y}_{sj}^t\right)^T$.

$\Delta\tilde{X}_j^{t_0} = \tilde{X}_j^{t_0+l} - \tilde{X}_j^{t_0}$ and $\Delta\tilde{Y}_j^{t_0} = \tilde{Y}_j^{t_0+l} - \tilde{Y}_j^{t_0}$ are defined as the stochastic incremental inputs and outputs at stage t_0, respectively. The stage stochastic incremental DEA model with stochastic inputs–outputs is formulated as:

$$\min \theta$$
$$s.t. \qquad (22)$$
$$\begin{cases} \Pr\left[\sum_{j=1}^{n} \lambda_j \Delta\tilde{X}_{ij}^{t_0} \le \theta \Delta\tilde{X}_{i0}^{t_0}\right] \ge 1-\alpha, i = 1,\dots,m, \\ \Pr\left[\sum_{j=1}^{n} \left(\lambda_j - \eta_j\right) \Delta\tilde{Y}_{rj}^{t_0} \ge \Delta\tilde{Y}_{r0}^{t_0}\right] \ge 1-\beta, r = 1,\dots,s, \\ \lambda_j \ge 0, \eta_j \ge 0, j = 1,\dots,n. \end{cases}$$

where $0 \le \alpha \le 1, 0 \le \beta \le 1$ are significance levels. Similar to Section 3.1, the stage stochastic incremental DEA model (22) can transformed into the following chance constraints optimization model with slack variables:

$$\min \theta + \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)$$
$$s.t.$$
$$\begin{cases} \Pr\left[\sum_{j=1}^{n} \lambda_j \Delta\tilde{X}_{ij}^{t_0} + s_i^- \le \theta \Delta\tilde{X}_{i0}^{t_0}\right] = 1-\alpha, i = 1,\dots,m, & \text{(a)} \\ \Pr\left[\sum_{j=1}^{n} \left(\lambda_j - \eta_j\right) \Delta\tilde{Y}_{rj}^{t_0} \ge \Delta\tilde{Y}_{r0}^{t_0} + s_r^+\right] = 1-\beta, r = 1,\dots,s, & \text{(b)} \\ \lambda_j \ge 0, \eta_j \ge 0, j = 1,\dots,n. \end{cases} \qquad (23)$$

where $\varepsilon > 0$ is the non-Archimedean infinitesimal.

Definition 3.2 (*Stage Stochastic Efficient*). In model (23), for the given significance levels α, β of [0, 1], if there exists at least one optimal solution (λ^*, η^*), such that the following conditions are satisfied:

(1) $\theta = 1$,

(2) $s_0^+ = s_j^+ = s_j^- = 0, \forall j = 1,\dots,n$.

then the evaluated decision-making unit DMU_{j_0} is stage stochastic efficient for the given significance levels α, β of [0, 1].

In order to facilitate application and calculation, it is necessary to transform model (23) into a determinate form. The first constraint is equivalent to the following form:

$$\Pr\left[\frac{\sum_{j=1}^{n} \lambda_j \Delta\tilde{X}_{ij}^{t_0} + s_i^- - \theta\Delta\tilde{X}_{i0}^{t_0} - \sum_{j=1}^{n} \lambda_j \Delta X_{ij}^{t_0} - s_i^- + \theta \Delta X_{i0}^{t_0}}{\sigma_i(\lambda,\theta)} \le \frac{0 - \sum_{j=1}^{n} \lambda_j \Delta X_{ij}^{t_0} - s_i^- + \theta \Delta X_{i0}^{t_0}}{\sigma_i(\lambda,\theta)}\right] = 1-\alpha$$

where $\frac{\sum_{j=1}^{n} \lambda_j \Delta\tilde{X}_{ij}^{t_0} + s_i^- - \theta\Delta\tilde{X}_{i0}^{t_0} - \sum_{j=1}^{n} \lambda_j \Delta X_{ij}^{t_0} - s_i^- + \theta \Delta X_{i0}^{t_0}}{\sigma_i(\lambda,\theta)} \sim N(0,1)$,

$$\left[\sigma_i(\lambda,\theta)\right]^2 = \sum_{j\ne0}\sum_{k\ne0} \eta_j\eta_k \operatorname{cov}\left(\Delta\tilde{X}_{ij}, \Delta\tilde{X}_{ik}\right) + \left(\eta_0 - \theta\right)^2 \operatorname{Var}\left(\Delta\tilde{X}_{i0}\right) + 2\left(\eta_0 - \theta\right)\sum_{j\ne0} \lambda_j \operatorname{cov}\left(\Delta\tilde{X}_{ij}, \Delta\tilde{X}_{i0}\right).$$

Then

$$0 - \sum_{j=1}^{n} \lambda_j \Delta X_{ij}^{t_0} - s_i^- + \theta \Delta X_{i0}^{t_0} = \phi^{-1}(1-\alpha)\sigma_i(\lambda,\theta), i = 1,\dots,m.$$

Similar to the constraint (23b), it can be converted to the following form with slack variable s_r^+ :

$$\Pr\left[\sum_{j=1}^{n} \left(\lambda_j - \eta_j\right) \Delta\tilde{Y}_{rj}^{t_0} - \Delta\tilde{Y}_{r0}^{t_0} - s_r^+ \le 0\right] = \beta$$

Then

$$\Pr\left[\frac{\sum_{j=1}^{n} \left(\lambda_j - \eta_j\right) \Delta\tilde{Y}_{rj}^{t_0} - \Delta\tilde{Y}_{r0}^{t_0} - s_r^+ - \left[\sum_{j=1}^{n} \left(\lambda_j - \eta_j\right) \Delta Y_{rj}^{t_0} - \Delta Y_{r0}^{t_0} - s_r^+\right]}{\sigma_r(\lambda,\eta)} \le \frac{0 - \sum_{j=1}^{n} \left(\lambda_j - \eta_j\right) \Delta Y_{rj}^{t_0} + \Delta Y_{r0}^{t_0} + s_r^+}{\sigma_r(\lambda,\eta)}\right] = \beta$$

where

$$\frac{\sum_{j=1}^{n} \left(\lambda_j - \eta_j\right) \Delta\tilde{Y}_{rj}^{t_0} - \Delta\tilde{Y}_{r0}^{t_0} - s_r^+ - \left[\sum_{j=1}^{n} \left(\lambda_j - \eta_j\right) \Delta Y_{rj}^{t_0} - \Delta Y_{r0}^{t_0} - s_r^+\right]}{\sigma_r(\lambda,\eta)} \sim N(0,1),$$

$$\left[\sigma_r(\lambda,\eta)\right]^2 = \sum_{i\ne0}\sum_{j\ne0} (\lambda_i - \eta_i)(\lambda_j - \eta_j) \operatorname{cov}(\Delta\tilde{Y}_{ri}^{t_0}, \Delta\tilde{Y}_{rj}^{t_0}) + (\lambda_0 - \eta_0 - 1)\operatorname{Var}(\Delta\tilde{Y}_{r0}^{t_0}) + 2(\lambda_0 - \eta_0 - 1) \times \sum_{j\ne0} (\lambda_j - \eta_j) \operatorname{cov}(\Delta\tilde{Y}_{rj}^{t_0}, \Delta\tilde{Y}_{r0}^{t_0}).$$

Then

$$\frac{0 - \sum_{j=1}^{n} \left(\lambda_j - \eta_j\right) \Delta Y_{rj}^{t_0} + \Delta Y_{r0}^{t_0} + s_r^+}{\sigma_r(\lambda,\eta)} = \phi^{-1}(\beta), r = 1,\dots,s.$$

Therefore, for the given significance level α, β, the stage stochastic incremental DEA model with stochastic inputs–outputs (23) can be transformed into the following deterministic equivalent:

$$\min\left[\theta + \varepsilon\left(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\right)\right]$$
$$s.t. \qquad (24)$$
$$\begin{cases} \sum_{j=1}^{n} \lambda_j \Delta X_{ij}^{t_0} + s_i^- - \theta \Delta X_{i0}^{t_0} + \phi^{-1}(1-\alpha)\sigma_i(\lambda,\theta) = 0, i = 1,\dots,m, \\ \sum_{j=1}^{n} \left(\lambda_j - \eta_j\right) \Delta Y_{rj}^{t_0} - \Delta Y_{r0}^{t_0} - s_r^+ + \phi^{-1}(\beta)\sigma_r(\lambda,\eta) = 0, r = 1,\dots,s, \\ \lambda_j \ge 0, \eta_j \ge 0, j = 1,\dots,n. \end{cases}$$

where ϕ is the cumulative distribution function of the normal distribution. When the significance levels $\alpha = 1, \beta = 0$, model (24) degenerates into a stage incremental DEA model [27].

Table 1
Inputs–outputs table.

	Index name	Descriptive index
Inputs	Non-current liabilities	Debt financing
	Paid-in capital	Equity financing
	Capital reserve	Equity financing
	Cash outflows from financing activities	Financing cost
	Equity ratio	Finance risk
Outputs	Total operating income	The income from the main business
	Government subsidies	Fiscal financing
	Cash inflows from fund-raising activities	Cash inflows generated from financing activities during operations
	Intangible asset	Enterprise assets
	Earnings per share	Reflects the profitability of the enterprises

4. Empirical research

This section shows that the two stage stochastic incremental DEA models are applied to evaluate the stage investment and financing situations of ESEPCEs, respectively. The proposed stage stochastic incremental DEA models are compared with the stage incremental DEA model [27] to validate their effectiveness and provide a more accurate assessment of the investment and financing situation in stochastic environments.

The first stage stochastic incremental DEA model is applied to assess the 15 ESEPCEs' stage financing efficiency and gain insight into the financial situation of these enterprises at stage. The inputs–outputs of 15 ESEPCEs from 2018–2022 are selected to evaluate their stage financing efficiency. The second form selects 15 ESEPCEs different from those mentioned above and evaluates their stage investment efficiency to understand the stage investment situation. The inputs–outputs of 15 ESEPCEs from 2018–2022 are selected to assess their stage investment situation. The evaluation period for both applications is selected from 2018–2022. The reason is that ESEPCEs are in a period of rising development at this stage. Exploring the investment and financing situations of this kind of enterprise will help to achieve the long-term development of the enterprise.

4.1. Stage financing efficiency evaluation

4.1.1. Selection of indicators and data

A reasonable construction of evaluation indicators is the premise and basis for using DEA to effectively calculate the financing efficiency of ESEPCEs. Considering the applicability, availability, and importance principles of indicators, from a financing perspective, select non-current liabilities representing debt financing, paid-in capital and capital reserves representing equity financing, cash outflows from financing activities representing financing costs, and financing risks. The property rights ratio is used as an input indicator for measuring the financing efficiency of ESEPCEs. Government subsidies, cash inflows from financing activities, total revenue, intangible assets, and earnings per share are selected as the output indicators for measuring the stage financing efficiency of ESEPCEs. Specific indicators are shown in Table 1. The data for stochastic variables is calculated on the basis of the data over the past years.

4.1.2. Analysis of the results

Table 2 shows the calculated stage financing efficiencies of 15 ESEPCEs applying the stage stochastic incremental DEA model and the stage incremental DEA model [27]. The stage financing efficiencies calculated using the stage stochastic incremental DEA model are calculated at 90% and 95% confidence levels, respectively. The comparison results show that when the stage financing efficiency calculated by using the stage incremental DEA model is efficient, the stage stochastic incremental DEA model is also efficient at different confidence levels. When the stage financing efficiency calculated using the stage incremental DEA model is inefficient, the stage stochastic incremental DEA model is also inefficient at different confidence levels. At a 90% confidence level, the inefficiency of stage financing is notably superior compared to that at a 95% confidence level. This further verifies the validity of the proposed models.

Based on the stage financing efficiency determined by the stage stochastic incremental DEA model, it has been found that 10 out of 15 ESEPCEs under the two confidence levels are efficient, which indicates that these 10 enterprises are in a better financing situation at the evaluated stage. This means that these 10 ESEPCEs have excellent performance in terms of capital management, investment decisions, and risk control. They are likely to have a sound financial structure and a good internal and external financing environment, and they are able to effectively utilize internal resources to finance and meet their financing needs.

Among the inefficient 5 enterprises, Grammy's stage financing efficiency is much lower than that of the other enterprises, both at the 90% and 95% confidence levels. Its stage financing efficiency is 0.3768 and 0.3360, respectively, indicating a high likelihood of encountering financing difficulties. After a thorough examination of the underlying factors, the Grammy continues expanding the scale of investment during the stage 2018–2022, resulting in a large number of loans and poor liquidity. In addition, there is a serious backlog of inventory, which also leads to poor asset liquidity. Over-reliance on outside funding, particularly expensive short-term loans, can put the enterprise in danger of having too much financial leverage, which may impair its capacity to raise capital. The stage financing efficiency of Radio and Television Metering at 90% and 95% confidence levels is also very poor, which is 0.4106 and 0.3811 respectively. The reason is that the enterprise does not increase its income. During this stage, the change of main business and the lack of layout of main areas forced enterprises to increase financing, and the stage financing efficiency is relatively poor. The enterprises with low stage financing efficiency in the remaining three stages are Runbang Stock, Huahong Technology, and Tongxing Environmental Protection. The lower stage financing efficiency of these three enterprises is mainly due to the high costs of enterprise financing, inefficient loan time, and the enterprise's management problems. Due to the high business risk of middle and small-sized enterprises (SMEs), financial institutions will increase the higher financing cost to reduce the risk. In addition, the financing needs of SMEs are characterized by frequency, while the long approval process of bank loans may lead to the inability of enterprises to obtain financing in time. At the same time, the opacity of management information and the low level of managers can affect stage financing efficiency.

As can be seen in Table 2, in comparison to the stage incremental DEA model, the stage stochastic incremental DEA model yields similar results for both the efficient and inefficient stage financing efficiencies at the 90% and 95% confidence levels. Among the 15 stage financing efficiencies calculated using the stage incremental DEA model, 10 enterprises have better stage financing efficiency, and 5 enterprises have worse stage financing efficiency. This case indicates that the two kinds of models have comparable prediction accuracy for assessing the enterprises' stage financing efficiency in stage financing efficiency evaluation. This not only proves the validity of the proposed stage stochastic incremental DEA model but also solves the problem of evaluating stage financing efficiency when the outputs are stochastic.

In summary, the stage financing situation of the 15 enterprises in the stage 2018–2022 is generally good. Among them, 10 enterprises have efficient stage financing efficiency, while the remaining 5 enterprises have inefficient efficiency. 2 of the 5 inefficient enterprises have extremely poor financing efficiency. The enterprise's internal factors

Table 2
Stage financing efficiency of stage stochastic incremental DEA model and stage incremental DEA model.

Enterprises	Stage stochastic incremental DEA model		Stage incremental DEA model [27]
	$\alpha = \beta = \gamma = 0.10$	$\alpha = \beta = \gamma = 0.05$	
Grammy	0.3768	0.3360	0.4341
Fuchun Environmental Protection	1.0000	1.0000	1.0000
Runbang Stock	0.5851	0.5646	0.5919
Xizi Clean Energy	1.0000	1.0000	1.0000
Kemet Gas	1.0000	1.0000	1.0000
Clean Environment	1.0000	1.0000	1.0000
Huahong Technology	0.6865	0.6681	0.9803
Cedilone	1.0000	1.0000	1.0000
Invicta	1.0000	1.0000	1.0000
Radio and Television Metering	0.4106	0.3811	0.4233
Overseas Chinese Bank Shares	1.0000	1.0000	1.0000
Youcai Resources	1.0000	1.0000	1.0000
Tongxing Environmental Protection	0.8466	0.8463	0.9307
Sequential Control Development	1.0000	1.0000	1.0000
Centre Testing	1.0000	1.0000	1.0000

Table 3
Inputs–outputs description table.

Classification	Indicator name	Indicator description
Inputs	Fixed asset	Long-term capital investment
	Intangible asset	Long-term capital investment
	Long-term equity investment	Equity investment
	Payroll payable	Labor input
Outputs	Sales revenue	The income from the main business
	Mass of profit	Total profit
	Cash inflows from investment activities	Cash inflows generated from investment activities during operations
	Total assets turnover	The ability of total assets to generate income

account for the majority of the detrimental financing effect, but some of it is also affected by the external environment.

4.2. Stage investment efficiency evaluation

4.2.1. Selection of indicators and data

Considering the principles of comprehensiveness, importance, and comparability in the selection of input–output indicators, this paper constructs the input indicator system from capital inputs and labor inputs. Capital inputs include fixed assets, intangible assets, and long-term investments, which are important assets that can create value for the enterprise. Labor inputs mainly come from the employees of the enterprise.

As for the selection of output indicators, the purpose of enterprise investment is to maximize economic benefits. This paper selects output indicators that can measure the income generated by the daily production and operation activities of the enterprise and the financial performance of the enterprise. Specific indicators are shown in Table 3. The data for stochastic variables is calculated on the basis of the data over the past years.

4.2.2. Evaluation results

Table 4 indicates the calculated stage investment efficiencies of 15 ESEPCEs applying the stage stochastic incremental DEA model and the stage incremental DEA model. The stage stochastic incremental DEA model is employed to calculate the stage investment efficiencies, which are subsequently determined at confidence levels of 90% and 95%, respectively. When the stage investment efficiency is efficient, the efficiency under two confidence levels is also efficient; when the stage investment efficiency is inefficient, there is a difference in the efficiency under two confidence levels. The stage investment efficiency at the 90% confidence level is significantly higher than that at the 95% confidence level.

From the calculation of the stage investment efficiency of the 15 ESEPCEs using the second form of the stage stochastic incremental DEA model, 9 of the 15 enterprises under the two confidence levels are efficient, indicating that these 9 ESEPCEs are in a better position to invest in the evaluated stage. It shows that 9 ESEPCEs are capable of allocating resources sensibly and increasing resource utilization efficiency according to market demand and their actual situation. Meanwhile, the enterprises have strong risk management abilities and can effectively cope with all kinds of risks.

Among the 6 inefficient enterprises, the enterprises with stage investment efficiency from low to high calculated by the stage stochastic increment DEA model under two confidence levels are Fuchun Environmental Protection, Xizi Clean Energy, Infore Environment, Zhongshan Public Utility, GRG Metrology & Test Group, and Centre Testing. The inefficiency of investment indicates that the investment status of the enterprise is not good, and it is necessary to adjust the investment input to obtain the maximum investment benefit. Fuchun Environmental Protection has the least efficient investment in the stage 2018–2022, and the stage investment efficiency under the two confidence levels is 0.2174 and 0.2096, respectively. The reason is that the enterprise continues to expand its industrial space through mergers and acquisitions during this stage, which requires significant investment. Meanwhile, with the continuous increase in operating income, the costs associated with it also expand. And the pressure of debt repayment is greater in the short term, all of which leads to lower investment efficiency for the enterprise at this stage.

The second lowest ranking is Xizi Clean Energy. The stage investment efficiency under the two confidence levels is 0.3578 and 0.3408. The reasons for the stage investment inefficiency mainly include financial pressure, lack of policy support, insufficient resource integration, limited governance level, market risk, technological bottlenecks, etc. These factors jointly affect the investment decisions and implementation effects of enterprises, resulting in low investment efficiency. Infore Environment and Zhongshan Public Utility, as two private enterprises, are also inefficient at two confidence levels of stage investment evaluation. One of the reasons for the stage investment inefficiency is due to the nature of the enterprises. Private enterprises face the main problems of insufficient capital and difficulty in financing, thus limiting their ability to invest and respond to market changes. In addition, Infore Environment carried out a ten-billion-dollar merger and acquisition of Zhonglian Environment in 2018, which led to an increase in the internal operating costs of the enterprise. This caused fluctuations in the internal control and risk management mechanisms, leading to heightened risks associated with investments. The last two enterprises, GRG Metrology & Test Group and Centre Testing, are also inefficient

Table 4
Stage investment efficiency of stage stochastic incremental DEA model and stage incremental DEA model.

Enterprises	Stage stochastic incremental DEA model		Stage incremental DEA model [27]
	$\alpha = \beta = 0.10$	$\alpha = \beta = 0.05$	
Zhongshan Public Utility	0.5932	0.5875	0.6296
City Development Environment	1.0000	1.0000	1.0000
UniTTEC	1.0000	1.0000	1.0000
Infore Environment	0.4165	0.4058	0.7157
Zhefu Holding Group	1.0000	1.0000	1.0000
Grammy	1.0000	1.0000	1.0000
Fuchun Environmental Protection	0.2174	0.2096	0.6042
Rainbow Heavy Industries	1.0000	1.0000	1.0000
Xizi Clean Energy	0.3578	0.3408	0.7470
Clean Environment	1.0000	1.0000	1.0000
Huahong Technology	1.0000	1.0000	1.0000
SDL Technology	1.0000	1.0000	1.0000
GRG Metrology & Test Group	0.7124	0.6914	0.6813
Southern Power Grid Energy	1.0000	1.0000	1.0000
Centre Testing	0.9132	0.8900	0.9264

in terms of stage investment efficiency at two confidence levels. This indicates that these two enterprises need to adjust their investment inputs to maximize their stage investment efficiency. However, compared with the other inefficient enterprises, the stage investment efficiency of these two enterprises is relatively high. A detailed examination of the reasons for the low efficiency found that part of the problem stems from the lack of internal management of the enterprise, while the other part is inevitably restricted by the fluctuation of the economic environment.

As seen in Table 4, it appears that the efficient and inefficient stage investment efficiencies, which are computed using the stage stochastic incremental DEA model at 90% and 95% confidence levels, closely resemble those obtained from the stage incremental DEA model. Among the 15 stage investment efficiencies calculated using the stage stochastic incremental DEA model, 9 enterprises have better stage investment efficiency, and 6 enterprises have worse stage investment efficiency. This case indicates that the two kinds of models show similar prediction accuracy for evaluating the enterprises' stage investment efficiency in stage investment efficiency evaluation. This not only proves the validity of the proposed stage stochastic incremental DEA model but also solves the problem of evaluating stage investment efficiency when the inputs–outputs are stochastic.

In summary, the stage investment situation of the 15 enterprises in the stage 2018–2022 is generally good. Among them, 9 enterprises have efficient stage investment efficiency, while the remaining 6 enterprises have inefficient efficiency. 4 of the 6 inefficient enterprises have extremely poor investment efficiency. The reason for the low stage investment efficiency comes not only from the internal enterprise but also from the external environment. The above enterprises need to start from various aspects, including improved market information openness, expanded financing methods, strengthened internal management, and strengthened supervision. Only through these measures can enterprises better grasp market opportunities, reduce investment risks, and improve investment efficiency.

5. Conclusions

Stage evaluation becomes particularly crucial in practical management. When conducting stage evaluations, the incremental inputs–outputs after the difference can reflect the stage characteristics more directly than the real-valued inputs–outputs. The inputs and outputs selected in the stage evaluation may be subject to some uncertainty due to a variety of factors. These uncertainties may result in incremental inputs and incremental outputs deviating from expectations. Considering the randomness of incremental inputs–outputs, the deviation of evaluation results caused by data measurement errors and outliers can be corrected to a certain extent and improve the accuracy of stage evaluation.

Considering the aforementioned factors, we have suggested two different kinds of stage stochastic incremental DEA models for assessing stage performance. It studies the stage stochastic incremental DEA model, where inputs are determined and outputs are stochastic, and extends it to stage stochastic incremental DEA models, where both inputs and outputs are stochastic. The two kinds of stage stochastic incremental DEA models are applied to the stage financing and stage investment efficiency evaluation of ESEPCEs, respectively. The purpose is to examine the stage financing performance and stage investment performance of the enterprises with stochastic inputs and outputs. The green industry can better manage market uncertainty and achieve sustainable development by analyzing the stage financing and investment efficiency of ESEPCEs in a stochastic environment. It can also optimize stage financing and investment strategies, enhance risk resistance, encourage industrial upgrading and transformation, increase enterprise value, and serve as a guide for managers creating the next stage of the business plan.

The preceding analysis revealed that the two kinds of stage stochastic incremental DEA models more accurately reflect stage performance. The stage financing situation of the 15 ESEPCEs in the stage 2018–2022 is generally good. 2 of the 5 inefficient enterprises have extremely poor stage financing efficiency. The adverse financing effect is mainly caused by the internal factors of the enterprise, but some of it is also affected by the external environment. Similarly, the second kind of stage stochastic incremental DEA model is applied to assess the stage investment situation of 15 ESEPCEs different from those mentioned above. The results show that 9 out of 15 enterprises have better stage investment efficiency, indicating that the enterprises can obtain larger returns with smaller inputs in their investment activities. The reason for the low stage investment efficiency comes not only from the internal enterprise but also from the external environment. The above enterprises need to start from various aspects, strengthen internal management, expand financing channels, enhance market information transparency, and strengthen supervision.

In conclusion, ESEPCEs, to improve the efficiency of stage financing and investment, need to reasonably plan the financing methods, strengthen financial management and scientific decision-making, and optimize the capital structure.

In this paper, the randomness of input–output is further considered on the basis of the optimistic perspective of the stage incremental DEA model. And considering the randomness of inputs and outputs from the pessimistic perspective of the stage incremental DEA model, it can be included in the subsequent study of the stage DEA model. Furthermore, inputs–outputs may be expressed as fuzzy values in actual production management, so it is necessary to further consider the stage fuzzy DEA model in the future.

CRediT authorship contribution statement

Bo-wen Wei: Writing – original draft, Formal analysis, Data curation. **Yi-yi Ma:** Visualization, Validation, Software. **Ai-bing Ji:** Supervision, Resources, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The authors do not have permission to share data.

Acknowledgments

This work has received partial support from the National Statistical Science Research Project, China (2023LY025).

References

[1] Topcu TG, Triantis K. An ex-ante dea method for representing contextual uncertainties and stakeholder risk preferences. Ann Oper Res 2022;1–29.
[2] Ripoll-Zarraga AE, Lozano S. A centralised dea approach to resource reallocation in Spanish airports. Ann Oper Res 2020;288:701–32.
[3] Charnes A, Cooper WW, Rhodes E. Measuring the efficiency of decision making units. European J Oper Res 1978;2:429–44.
[4] Banker RD, Charnes A, Cooper WW. Some models for estimating technical and scale inefficiencies in data envelopment analysis. Manag Sci 1984;30:1078–92.
[5] Dehnokhalaji A, Khezri S, Emrouznejad A. A box-uncertainty in dea: A robust performance measurement framework. Expert Syst Appl 2022;187:115855.
[6] Omrani H, Alizadeh A, Emrouznejad A, Teplova T. A robust credibility dea model with fuzzy perturbation degree: An application to hospitals performance. Expert Syst Appl 2022;189:116021.
[7] Cordero JM, Díaz-Caro C, Pedraja-Chaparro F, Tzeremes NG. A conditional directional distance function approach for measuring tax collection efficiency: evidence from Spanish regional offices. Int Trans Oper Res 2021;28:1046–73.
[8] Li Y, Shi X, Emrouznejad A, Liang L. Environmental performance evaluation of chinese industrial systems: a network sbm approach. J Oper Res Soc 2018;69:825–39.
[9] Tone K. A slacks-based measure of efficiency in data envelopment analysis. European J Oper Res 2001;130:498–509.
[10] Sengupta JK. Data envelopment analysis for efficiency measurement in the stochastic case. Comput Oper Res 1987;14:117–29.
[11] Land KC, Lovell CK, Thore S. Chance-constrained data envelopment analysis. Manag Decis Econom 1993;14:541–54.
[12] Sengupta JK. Efficiency analysis by stochastic data envelopment analysis. Appl Econ Lett 2000;7:379–83.
[13] Huang Z, Li SX. Stochastic dea models with different types of input–output disturbances. J Product Anal 2001;15:95–113.
[14] Khodabakhshi M, Asgharian M. An input relaxation measure of efficiency in stochastic data envelopment analysis. Appl Math Model 2009;33:2010–23.
[15] Lotfi FH, Nematollahi N, Behzadi MH, Mirbolouki M, Moghaddas Z. Centralized resource allocation with stochastic data. J Comput Appl Math 2012;236:1783–8.
[16] Nazari A, Behzadi MH. Asymptotic distribution of the sum of skew-normal random variables: Application in data envelopment analysis. Iran J Sci Technol Trans A Sci 2017;41:199–207.
[17] Izadikhah M, Saen RF. Assessing sustainability of supply chains by chance-constrained two-stage dea model in the presence of undesirable factors. Comput Oper Res 2018;100:343–67.
[18] Jradi S, Ruggiero J. Stochastic data envelopment analysis: A quantile regression approach to estimate the production frontier. European J Oper Res 2019;278:385–93.
[19] Tavassoli M, Saen RF, Zanjirani DM. Assessing sustainability of suppliers: A novel stochastic-fuzzy dea model. Sustain Product Consump 2020;21:78–91.
[20] Kao C, Liu ST. Stochastic efficiencies of network production systems with correlated stochastic data: the case of Taiwanese commercial banks. Ann Oper Res 2022;315:1151–74.
[21] Wanke P, Rojas F, Tan Y, Moreira J. Temporal dependence and bank efficiency drivers in oecd: A stochastic dea-ratio approach based on generalized auto-regressive moving averages. Expert Syst Appl 2023;214:119120.
[22] Yaghoubi A, Fazli S. Bank efficiency forecasting model based on the modern banking indicators using a hybrid approach of dynamic stochastic dea and meta-heuristic algorithms. Iran J Manage Stud 2022;15.
[23] Gan L, Wan X, Ma Y, Lev B. Efficiency evaluation for urban industrial metabolism through the methodologies of emergy analysis and dynamic network stochastic block model. Sustainable Cities Soc 2023;90:104396.
[24] Kumar A, Anand N, Batra V. Trends in Indian private sector bank efficiency: non-stochastic frontier dea window analysis approach. J Asian Finance Econom Bus 2020;7:729–40.
[25] Liu F, Li L, Ye B, Qin Q. A novel stochastic semi-parametric frontier-based three-stage dea window model to evaluate China's industrial green economic efficiency. Energy Econ 2023;119:106566.
[26] Molinos-Senante M, Maziotis A, Sala-Garrido R, Arce MM. A stochastic meta-frontier approach for analyzing productivity in the english and welsh water and sewerage companies. Decis Anal J 2023;6:100185.
[27] Ji Ab, Wei Bw, Ma Yy. Incremental data envelopment analysis model and applications in sustainable efficiency evaluation. Comput Econ 2023;1–26.
[28] Cheng J, Lisser A. A second-order cone programming approach for linear programs with joint probabilistic constraints. Oper Res Lett 2012;40:325–8.

Bo-wen Wei, born in August 1995, is a doctoral candidate in Hebei University. Her main research interests are performance evaluation, machine learning and panel data.

Yi-yi Ma, born in October 1997, is a master's degree student in Hebei University. Her main research interests are machine learning and panel data.

Ai-bing Ji, born in December 1965, currently serves as a professor and doctoral supervisor in Hebei University. His primary research interests include machine learning, panel data analysis, and the application of fuzzy analysis for uncertainty prediction and decision-making.