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Optimising hierarchical emergency logistics network under ambiguous demand and transportation cost

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ABSTRACT

This article studies the response problem of an emergency logistics network with a decision hierarchy relationship under uncertainty. To account for the partial distribution information about uncertain demand and transportation costs, we construct a moment-based ambiguity set based on limited historical data, where the pivot variable method is employed to determine the confidence interval of the mean value. Based on the constructed ambiguity set, we develop a novel distributionally robust bi-level post-disaster emergency logistics location-routeing model. By exploiting the structural characteristic, chance-constrained models under box-ellipsoid and budget perturbation sets are reformulated as bi-level mixed-integer conic programming models. To accelerate the solution procedure, the bi-level models are further converted into single-level ones via Karush-Kuhn-Tucker condition, which can be directly solved to optimality using CPLEX software. Supply risk value for each supplier is obtained by applying analytic hierarchy process. We conduct extensive experiments using the Iranian flood as the case study to address the computational performance of our proposed optimisation method.

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1. Introduction

Disasters can leave thousands of people homeless and cause massive loss of life and negative socio-economic impacts. Natural disasters have become increasingly common (Kara and Savaşer 2017; Kovács and Falagara Sigala 2021). For example, in 2016, 1.4 million people (12% of Haiti's population) required assistance because of Hurricane Matthew. And in 2018, 315 major natural disasters occurred worldwide, causing $131.7 billion in economic damage and affecting 68.5 million people, who were in need of basic survival supplies and essential services (Guha-Sapir 2019). The devastating loss of life caused by major disasters highlights the importance of devising an effective post-disaster emergency management scheme (Noham and Tzur 2018; Zhang et al. 2022a). Under such circumstances, exploring the development of an appropriate emergency logistics network is critical to mitigate the

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impact of disasters and save lives. The response phase is one of the four critical phases of disaster emergency management. It begins after a disaster, usually lasts several weeks, and involves several activities (Shokr, Jolai, and Bozorgi-Amiri 2022). For instance, it is necessary to establish distribution centres at optimal locations for the efficient storage and transfer of relief items. The dispatching of vehicles and the meticulous planning of their routes are crucial elements in the transportation of items from distribution centres to disaster-stricken areas (Cheng et al. 2022). Additionally, the risks associated with purchasing relief items from suppliers need to be considered to effectively provide victims with what is required (Chen et al. 2022).

When designing an emergency logistics network, decisions regarding operational matters are made at two distinct levels. The Disaster Management Organization (DMO) and the International Federation of Red Cross (IFRC) act as decision makers at the upper and lower level, respectively. For emergency logistics to function optimally, decision makers should be able to optimise their respective objectives independently and cooperate with each other (Haeri et al. 2020). DMO dispatches vehicles, plans their routes, and determines the appropriate amount of supplies to be transported to cater to the requisites of the impacted regions (Jobe 2011). The IFRC is responsible for purchasing goods from suppliers and supplying relief items through public donations (Chen et al. 2022). It makes sense to set up the emergency response problem as a structure with the hierarchical relationship.

One of the main challenges in developing an efficient emergency logistics plan lies in managing uncertainties. When a disaster strikes, the transportation infrastructure is at risk of damage, and transportation costs become uncertain (Wang, Liu, and Pei 2023; Ye, Jiao, and Yan 2020). Despite the availability of disaster prediction systems, accurate assessment of damage remains a complex task, leading to uncertainty in demand (Eftekhar, Jeannette Song, and Webster 2022; Sabouhi et al. 2019). The robust optimisation (RO) approach is a reasonable choice when faced with uncertainty and lack of information about demand and transportation cost distributions, and is generally referred to as an overly conservative solution. In recent years, the distributionally robust optimisation (DRO) method has received attention as a powerful decision-making tool under uncertainty if supported by available data to avoid an overly conservative solution (Ghosal and Wiesemann 2020; Shang et al. 2021; Yin, Liu, and Chen 2023). Moment information on uncertain parameters, such as demand and transportation costs, can be estimated using statistical techniques to aid decision makers in optimising the rescue of victims. Therefore, we address the uncertainties regarding demand and transportation costs in emergency logistics problems and construct ambiguity sets using partial distribution information.

The above considerations encourage us to explore the following questions. How can a post-disaster location-routeing model that integrates the hierarchical relationship and uncertainties in demand and transportation costs be proposed? How can ambiguity sets be constructed when only limited historical data is available? How can the supply risk parameter be quantified? What are the effects of uncertainty on optimal decision making?

To address these research questions, we present a distributionally robust bi-level post-disaster emergency logistics location-routeing (PELL) model with ambiguous chance constraints, where only partial distribution information is available regarding the distributions of uncertain demand and transportation costs. The upper-level objective is to minimise transportation time, while the lower-level objective is to minimise supply risk. Due to the non-convexity of the chance-constrained model, we approximate it as mixed-integer second-order conic and mixed-integer linear programming models utilising partial distribution information. Analytic hierarchy process (AHP) is employed to quantify the level of supply risk. Moreover, comparative analyses are carried out to verify the performance of the DRO model. The main contributions of this study are summarised as follows.

- We focus on addressing the multi-supplier, multi-centre, multi-period emergency logistics problem under transportation cost and demand uncertainties, where the hierarchical decision relationships are established between DMO and IFRC. Based on partial moment information on random transportation cost and demand, a new distributionally robust bi-level model is developed to ensure the timely delivery of emergency supplies to disaster areas.
- According to limited historical data, the pivot variable method is employed to estimate the confidence interval when constructing the ambiguity set of the mean value. Computationally tractable forms about the ambiguous chance constraints are derived under box ∩ ellipsoid perturbation set and budget perturbation set. We then reformulate the bi-level model into a single-level mixed-integer second-order conic/linear programming model by Karush-Kuhn-Tucker (KKT) condition.
- The validity of the proposed method is verified through a case study of the Iranian flood. The computational results indicate that higher budget levels lead to increased inventory at the distribution centre, consequently reducing shipping time. Additionally, comparative analyses are carried out between the decision schemes of the DRO model and the RO model, as well as stochastic optimisation (SO) model. Numerical results demonstrate that DRO model reduces conservativeness compared to the RO model. In contrast to SO model, our DRO model can hedge against distribution ambiguity.

The remainder of this paper is organised as follows. Section 2 provides a literature review of relevant studies on emergency logistics problems. Section 3 outlines the new distributionally robust bi-level PELL model. Section 4 details the process for obtaining the safe approximation of the DRO model. Section 5 presents the numerical results of the proposed model and provides insights for management decision-making. Finally, conclusions and prospects for future research are discussed in Section 6. All proofs are relegated to Appendix 1.

2. Literature review

This study is associated with emergency logistics location-routeing and allocation problem, and methodologies for dealing with uncertainty in emergency logistics. Thus, these problems are reviewed in the following subsections.

2.1. Emergency logistics location-routeing and allocation problem

The importance of emergency logistics location-routeing and allocation problem has been recognised by both researchers and practitioners (Kundu, Sheu, and Kuo 2022). Related research has been based on a single-level modelling framework. For example, Knott (1987) was among the first to study emergency logistics problem and proposed a model to reduce demand loss using limited supply and different vehicles. Sheu (2007) proposed a hybrid fuzzy clustering optimisation common distribution method for solving the problem of emergency logistics distribution to meet the urgent needs of disaster areas. Huang, Kim, and Menezes (2010) explored the location problem for emergency logistics and developed an algorithm to recognise locations for the relief chain. Sheu (2010) developed a model for relief need prediction and allocation under incomplete information for large-scale emergency logistics operation problems. Huang, Smilowitz, and Balcik (2012) proposed a routeing-allocation model in emergency logistics considering three key elements: efficiency, efficacy, and fairness. Rath and Gutjahr (2014) considered a location-routeing model in emergency logistics systems and applied math-heuristic technique to solve the model. More recently, Vahdani et al. (2018) presented a multi-objective, multi-commodity, and multi-period formulation to improve the safety of rescuers, the possibility of on-time vehicle arrival, and uninterrupted road repair. Sabouhi et al. (2019) examined a multi-objective programming model for locating interchange points and shelters to solve the problem of transporting the injured to hospitals and evacuating people to shelters. Yu et al. (2019) presented a nonlinear integer programming model and developed a rollout algorithm to address the issues of efficiency, effectiveness, and fairness in emergency logistics. Ghaffari et al. (2020) considered a supply chain network consisting of suppliers, distribution centres and demand points and proposed a mixed-integer programming model in which a particle swarm optimisation algorithm was designed to solve the large-scale problem. Akbari and Shiri (2022) conducted research on the relief distribution problem aiming to minimise delays at critical nodes following road network damage caused by a disaster. In addition, some researchers have studied demand forecasting and central site reliability in emergency management. For example, Ghasemi and Babaeinesami (2019) used a fuzzy inference system to predict the demand for relief supplies under different scenarios, which was tested in a case study of Tehran city. Ahmadi Choukolaei et al. (2021) evaluated the location of affected areas and the optimal points proposed by the Geographic Information System based on 18 criteria, which were weighted by applying the triangular fuzzy aggregation method. Hosseini et al. (2022) evaluated the performance of emergency centres based on health protocols with criteria that considered emergency centres prevention and vehicle operation, in which the criteria were weighted using the triangular fuzzy aggregation method. Choukolaei, Ghasemi, and Goodarzian (2023) assessed the efficiency and sustainability of disaster management centres in the disaster response phase, using fuzzy Delphi method to classify research criteria and triangular fuzzy aggregation method to classify weight criteria.

There are also researchers focussing on bi-level models for addressing emergency logistics problems. Kamyabniya et al. (2019) investigated a two-stage mechanism capable of coordinating two heterogeneous rescue organisations in a network such that their interests and goals are satisfied, where the first stage is a bi-level mixed-integer linear model. Li and Teo (2019) demonstrated a bi-level formulation for the repair of road networks and the dispatch of materials in the aftermath of a disaster. Gao (2022) proposed a bi-level stochastic mathematical framework addressing the multi-commodity rebalancing quandary in emergency logistics. This model offered optimal schemes for commodity transport allocation and vehicle transport quantities. Ghasemi et al. (2022a) designed a secondary blood supply chain network taking into account the uncertain nature of production costs, transportation costs, and demand. They developed a two-stage bi-level model with the goal of minimising total costs and maximising donor utility. They used a hybrid planning approach to tackle uncertainty and validated the model using a real case. The above-mentioned studies focus on the location of the distribution centre or the vehicle routeing problem, whereas the current study investigates the two problems simultaneously. Additionally, we explicitly capture the uncertainties of demand and transportation cost to formulate the PELL model, and the obtained robust optimal solution can immunise against uncertainty when only partial distribution information is available.

2.2. Methodologies for dealing with uncertainty in emergency logistics

Two primary techniques, namely SO and RO, have been proposed to tackle uncertainty. In the context of SO, uncertain parameters are modelled as random variables with known probability distributions (Alem, Clark, and Moreno 2016; Gharib, Fatemi Ghomi, and Jolai 2022; Keshvari Fard, Ljubić, and Papier 2022). For example, Elçi and Noyan (2018) adopted a mean-risk two-stage SO approach to study emergency logistics problems with uncertainties of demand and transportation network conditions. Noyan, Meraklı, and Küçükyavuz (2019) conducted a study on the two-stage optimisation problem associated with managing the last-mile relief distribution process in the face of uncertainties arising from demand and network-related factors. Oksuz and Satoglu (2020) explored a two-stage stochastic programming model to tackle the problem of medical centre location while accounting for uncertainties in demand, casualty type, distance, and hospital capacity. Ghasemi, Goodarzian, and Abraham (2022) presented a multi-objective stochastic programming model for solving an emergency logistics network design problem in the event of an earthquake. The proposed model was solved using ϵ-constraint method and three meta-heuristic algorithms for a case study. Ghasemi et al. (2022b) formulated a two-stage stochastic programming model for designing a location-routeing-inventory problem under seismic conditions, in which the contribution was to design a multi-objective stochastic fractal search algorithm to solve the second stage model. They considered a case study of the potential earthquake in Tehran and showed that the proposed model accurately reflected the performance of the actual system. Rodríguez-Espíndola (2023) devised a stochastic programming model to optimise decision-making options in the emergency response scenario. This model took into consideration facility location, purchase, prepositioning, and distribution while knowing the probability distribution of stochastic demand. To address emergency logistics problems, some researchers have introduced RO models, which confine uncertain parameters to a particular set of uncertainties without any distributional assumptions. For example, Hu, Liu, and Hua (2016) studied the emergency logistics problem by incorporating transportation costs and demand uncertainty into a RO model. Ni, Shu, and Song (2018) utilised RO framework to investigate the prepositioning of relief goods and the post-disaster transportation problem, considering uncertainties such as demand, surplus supplies, and arc capacity. Balcik and Yanıkoğlu (2020) formulated a post-disaster needs assessment model considering uncertain travel time restricted in a coaxial box uncertainty set.

Given the potential tradeoff between stochastic and robust frameworks, DRO has recently received significant attention. A few studies have considered DRO in emergency logistics because developing computationally tractable solutions is quite challenging. Zhang et al. (2022b) studied a DRO multi-objective last-mile transportation model for emergency logistics and then transformed the DRO model into second-order conic programming. Zhang et al. (2021) provided a joint chance constraint model for the emergency logistics problem, constructed an ambiguity set under random travel time, and subsequently developed conic programming utilising worst-case conditional value-at-risk approximation. Yang et al. (2023) introduced a DRO location-allocation model that accounts for uncertainty in both contingency demand and resource supply time. The model constructed an ambiguity set according to information regarding support, means, and deviations. X. Wang et al. (2021) considered a target-oriented multi-period location-transportation problem and developed a DRO model to hedge against demand uncertainty. They proposed benders decomposition method to solve the approximate form of robust counterpart. D. Wang et al. (2023) considered supplier selection and inventory prepositioning as well as procurement and delivery in response. To counteract the ambiguity of the demand probability distribution, they proposed a two-stage DRO model to address the emergency response problem. Different from previous studies, our study utilises the DRO technique to hedge model uncertainty and employs the pivot variable method to construct ambiguity sets that rely on partial moment distribution of demand and transportation costs. To the best of our knowledge, very few studies provide a practical solution to fully support the bi-level emergency logistics location-routeing problem based on demand and transportation cost moment information ambiguity.

2.3. Research gap

Table 1 summarises the main differences between our research and the existing study. As shown in Table 1, only a few studies have addressed bi-level multi-period location-routeing problem. In the context of the emergency logistics network problem, the majority of existing literature adopts SO and RO to mitigate uncertainty while few studies have been conducted on the utilisation of DRO. Finally, a noteworthy observation is that most studies address demand uncertainty, whereas none extend their investigation to encompass uncertainty in both demand and transportation costs and construct ambiguity sets using the exponential moment information of the distribution. To address these gaps, the bi-level multi-period location-routeing model is proposed for designing an emergency relief logistics network. We apply DRO to hedge against the distributional ambiguity of demand and transportation costs, thereby avoiding the excessive conservativeness limitation of RO and the need for precise probability distributions in the SO.

3. Distributionally robust bi-level PELL model

The detailed mathematical formulation of the distributionally robust bi-level PELL model is provided in this section. In the beginning, the main notations used are listed in Table 2.

3.1. Problem statement

Natural disasters have the potential to cause substantial loss of life and extensive damage to property. Effective management of emergency logistics is essential for safeguarding human lives and mitigating the catastrophic aftermath. Thus, at the beginning of the response phase, suitable routes are arranged, and the network is designed to distribute relief items

Table 1. Summary of the existing relevant literature.

Reference	Bi-level	location-routeing	Multi-period	Uncertain parameter: Demand	Uncertain parameter: Transportation cost	Optimisation method
Huang, Kim, and Menezes (2010)	×	×	×	×	×	Deterministic
Elçi and Noyan (2018)	×	×	×	✓	✓	SO
Vahdani et al. (2018)	×	✓	✓	×	×	Deterministic
Li and Teo (2019)	✓	×	✓	×	×	Deterministic
Balcik and Yanıkoğlu (2020)	×	×	×	×	×	RO
Oksuz and Satoglu (2020)	×	×	×	✓	×	SO
Zhang et al. (2022b)	×	×	×	✓	×	DRO
Akbari and Shiri (2022)	×	✓	×	×	×	Deterministic
Gao (2022)	✓	×	×	✓	×	SO
Ghasemi et al. (2022a)	✓	×	✓	✓	✓	Hybrid method
Ghasemi et al. (2022b)	×	✓	✓	✓	×	SO
Ghasemi, Goodarzian, and Abraham (2022)	×	✓	×	✓	×	SO
Yang et al. (2023)	×	×	✓	✓	×	DRO
This article	✓	✓	✓	✓	✓	DRO

(RIs) to affected people. To describe the transportation and relief allocation aspects of emergency logistics clearly, Figure 1 is applied to represent this network.

In Figure 1, the emergency logistics activity from the supplier to the IFRC central warehouse, later to the distribution centres (DCs) and finally to the need sites (NSs) is considered. The supplier consists of three components: a central IFRC warehouse, external suppliers (ESs), and public donations. The IFRC central warehouse is a permanent facility where relief items can be stored and updated. The IFRC organisation acts as a decision maker and aims to collect goods from these three sub-warehouses to the IFRC's warehouse. DCs play a transit role and are located near the NSs. The endpoint of emergency logistics is NSs, where RIs are distributed directly to the victims. DMO is responsible for transporting and distributing RIs to meet the needs of survivors.

In this context, DMO is considered to be the leader and IFRC is the follower. The decision makers of the upper-level and lower-level optimisation problems correspond to the leader and follower, respectively. The DMO first makes a decision and, after observing the DMO's decision, the IFRC reacts. Thus, the IFRC optimises his/her objective by considering the decision made by the DMO. In this study, the emergency logistics transportation and relief distribution problem is described as a bi-level model. More specifically, the DMO with higher authority decides on the transportation and distribution options for emergency logistics and is concerned with reducing transportation time. At the lower level, the IFRC organisation is responsible for supply risk minimisation and decides on the number of relief items to be provided to the IFRC in emergency logistics by ESs and public donations. In addition, the general assumptions are summarised as follows.

Table 2. Notations.

Notations	Detailed definitions
Indices and sets	
$[I]$	Set of external supplier, $i \in [I]$;
$[J]$	Set of IFRC warehouse, $j \in [J]$;
$[K]$	Set of distribution centre, $k \in [K]$;
$[C]$	Set of need site, $m, n \in [C]$;
$[R]$	Set of relief item, $r \in [R]$;
$[T]$	Set of time periods, $t \in [T]$;
$[V]$	Set of vehicles, $v \in [V]$;
$[L]$	Set of dimensions of the perturbation vector $\boldsymbol{\zeta}$, $l \in [L]$;
$[E]$	Set of dimensions of the perturbation vector $\boldsymbol{\eta}_{mrt}$, $e \in [E]$.
Deterministic parameters	
F_k	The cost for opening a distribution centre $k \in [K]$;
C_{irt}	Procuring cost of relief item $r \in [R]$ from external supplier $i \in [I]$ in period $t \in [T]$;
HR_{jr}	Holding inventory cost for a unit relief item $r \in [R]$ at IFRC centre $j \in [J]$;
HD_{kr}	Holding inventory cost for a unit relief item $r \in [R]$ at distribution centre $k \in [K]$;
te^1_{ij}	Transport time from external supplier $i \in [I]$ to IFRC centre $j \in [J]$;
te^2_{jk}	Transport time from IFRC centre $j \in [J]$ to distribution centre $k \in [K]$;
te^3_{mn}	Transport time between need site nodes and between need site and distribution centre
A^1_{irt}	Capacity of external supplier $i \in [I]$ for relief item $r \in [R]$ in $t \in [T]$;
A^2_{jt}	Capacity of IFRC centre $j \in [J]$ in $t \in [T]$;
A^3_{kt}	distribution centre $k \in [K]$ storage capacity in $t \in [T]$;
A^4	Capacity per vehicle;
f^1_{ir}	Risk of external supplier $i \in [I]$ for relief item $r \in [R]$;
f^2_r	Public donation risk for relief item $r \in [R]$;
W_r	Weight of one unit of relief item $r \in [R]$;
δ_r	Order of relief item $r \in [R]$;
d_{mn}	Distance between need sites and distribution centre and between two nodes within need sites;
$TD_{\max}$	Limitations on the number of distribution centres;
B	Total funding level;
xl	The effectiveness of public donations;
M	A big number.
Uncertain parameters	
D_{mrt}	Amount of demand for relief item $r \in [R]$ in need site $m \in [C]$ in period $t \in [T]$
TC^1_{ijr}	Transportation cost for a unit relief item $r \in [R]$ from external supplier $i \in [I]$ to IFRC centre $j \in [J]$;
TC^2_{jkr}	Transportation cost for a unit relief item $r \in [R]$ from IFRC's warehouse $j \in [J]$ to distribution centre $k \in [K]$;
TC^3	Unit transportation cost per vehicle between need site nodes.
upper-level decision variables	
LOC_k	1, if distribution centre is set at location k; 0, otherwise;
X^1_{jkrt}	Amount of relief item $r \in [R]$ transported from IFRC $j \in [J]$ to distribution centre $k \in [K]$ in $t \in [T]$;
X^2_{mrvt}	Number of relief item $r \in [R]$ transported to need site $m \in [M]$ with vehicle $v \in [V]$ in $t \in [T]$;
X^3_{jrt}	Inventory of relief item $r \in [R]$ at IFRC $j \in [J]$ in $t \in [T]$;
X^4_{krt}	Inventory of relief item $r \in [R]$ at distribution centre $k \in [K]$ in $t \in [T]$;
Q^3_{mnkvt}	1, vehicle $v \in [V]$ visits need site $m \in [C]$ after visiting need site $n \in [C]$ on the route of distribution centre $k \in [K]$, and 0, otherwise,
U^1_{ijt}	1, if the relief item is delivered from external supplier $i \in [I]$ to IFRC $j \in [J]$, and 0, otherwise,
U^2_{jkt}	1, if the relief item is transferred from IFRC centre $j \in [J]$ to distribution centre $k \in [K]$;
H_{mkt}	1, if need site $m \in [C]$ is assigned to distribution centre $k \in [K]$;
G_{mvt}	Auxiliary variable used to eliminate sub-tour.
lower-level decision variables	
Y^1_{ijrt}	Amount of relief item $r \in [R]$ transferred from external supplier i to IFRC $j \in [J]$ in $t \in [T]$;
Y^2_{jrt}	Amount of public donations of relief item $r \in [R]$ to IFRC $j \in [J]$ in $t \in [T]$.

- The transportation system for sending humanitarian aid allows for unlimited vehicles but is homogeneous with a predetermined capacity. This is an assumption in many emergency rescue problems, because there is a uniform standard for the configuration of rescue vehicles (Wen et al. 2010).

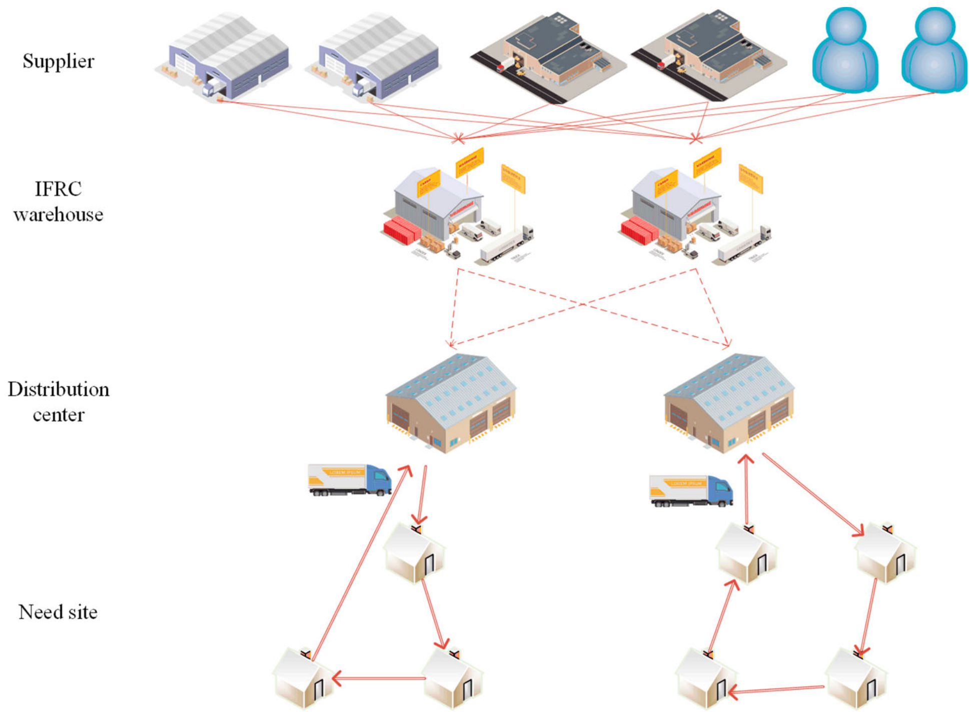


Figure 1. The structure of the emergency logistic network.

- Each trip starts and ends at a DC and each NS can only receive relief from one DC during a trip. This assumption is based on actual situations.
- The distance and time between all points remain constant.
- DCs have limited capacities, which may vary for each centre (Khanchehzarrin et al. 2022).
- The fixed cost of setting up DCs differs across centres.
- The ambiguity of information concerning demand and transportation cost distribution is described by ambiguity sets, where the mean values of demand and transportation cost are located in the confidence intervals.

3.2. Uncertainty in demand and transportation cost

The needs of affected areas and transportation cost after a disaster are uncertain. This section focuses on the ambiguity set for random demands. The other set for transportation cost is defined in a similar manner. In practice, the population of the impacted region and the standard of RIs required per person are typically utilised to gauge the actual needs of the affected area after a disaster. Although the point-estimation approach is easy to implement, its efficiency remains questionable. If the actual scenario is significantly different from the predicted one, the quality of the obtained solution may be significantly reduced or even unacceptable. Here, we suppose that distributions are not perfectly known and consider the case where only mean information is used to construct ambiguity set.

Assume that D_{mrt} is subject to perturbations around their nominal value and belongs to the uncertainty set $\mathcal{U}^2_{mrt}$.

$$\mathcal{U}^2_{mrt} = \left\{ D_{mrt} | D_{mrt}(\boldsymbol{\eta}_{mrt}) = D^0_{mrt} + \sum_{e\in[E]} \eta_{mrte} D_{mrte} \right\},$$

where $D_{mrt}(\boldsymbol{\eta}_{mrt})$ represents the uncertain demand, D^0_{mrt} represents the nominal value, D_{mrte} repersents perturbation value, and $\boldsymbol{\eta}_{mrt} = (\eta_{mrt1}, \ldots, \eta_{mrtE})$ is stochastic perturbation vector.

Specifically, the stochastic perturbation vector $\boldsymbol{\eta}_{mrt}$ possesses the following properties (see, Ben-Tal, El Ghaoui, and Nemirovski 2009):

P.1. η_{mrte}, $e = 1, \ldots, E$, are mutually independent random variables;

P.2. The distribution $\mathbb{P}_{\eta_{mrte}}$ of random variable η_{mrte} $(e = 1, \ldots, E)$ satisfies the following inequalities:

$$\int \exp ts d\mathbb{P}_{\eta_{mrte}}(s) \leq \exp \left\{ \max[\mu^-_{mrte}, \mu^+_{mrte}] + \frac{1}{2}(\sigma_{mrte})^2 t^2 \right\}, \quad \forall t \in R,$$

with the known constants $\mu^-_{mrte} \leq \mu^+_{mrte}$ and $\sigma_{mrte} \geq 0$. The property P.2 is a description of the distribution information in which the exponential moment $\int \exp ts d\mathbb{P}_{\eta_{mrte}}(s)$ of the true distribution is controlled by an upper bound function. Moreover, we assume that the support set of the probability distribution $\mathbb{P}_{\eta_{mrte}}$ is $[-1, 1]$ and the expectation of the random perturbation variable belongs to $[\mu^-_{mrte}, \mu^+_{mrte}]$. The ambiguity set $\mathscr{P}_{mrte}$ of distribution $\mathbb{P}_{\eta_{mrte}}$ takes the following form:

$$\mathscr{P}_{mrte} = \left\{ \mathbb{P}_{\eta_{mrte}} : \quad \begin{array}{l} \mathbb{E}_{\mathbb{P}_{\eta_{mrte}}}(\eta_{mrte}) \subseteq [\mu^-_{mrte}, \mu^+_{mrte}] \subseteq [-1, 1] \\ \mathbb{P}_{\eta_{mrte}}(\eta_{mrte} \in [-1, 1]) = 1 \end{array} \right\}. \tag{1}$$

To apply the distribution information set characterised by P.1 and P.2, we should utilise the prior knowledge of distributions of η_{mrte} to translate into specific values of the parameters μ^-_{mrte}, μ^+_{mrte} in P.2. In constructing demand ambiguity set, we use historical data for some regions to count the demand. For other areas where data are lacking, we estimate demand by assuming demand per affected population and using historical data. Confidence intervals for the mean of demand can then be obtained using interval estimation method. More specifically, the parameters μ^-_{mrte}, μ^+_{mrte}, and σ_{mrte} associated with the constructed ambiguity set are estimated in the following steps.

Step 1: The pivot variable method is used to obtain confidence interval for mean $\bar{\mu}_{mrt}$ of the demand. When the overall variance is unknown, the pivot variable obeys a t-distribution, i.e. $\frac{\bar{x}_{mrt} - \bar{\mu}_{mrt}}{s_{mrt}}\sqrt{n} \sim t(n-1)$. Based on the upper quantile, we have

$$\Pr\left(-t_{\frac{\alpha}{2}}(n-1) \leq \frac{\bar{x}_{mrt} - \bar{\mu}_{mrt}}{s_{mrt}}\sqrt{n} \leq t_{\frac{\alpha}{2}}(n-1) \right) = 1 - \alpha.$$

Thus the confidence interval for $\bar{\mu}_{mrt}$ with a confidence level $1 - \alpha$ is

$$\bar{\mu}_{mrt} \in \left[\bar{x}_{mrt} - \frac{s_{mrt}}{\sqrt{n}} t_{\frac{\alpha}{2}}(n-1), \bar{x}_{mrt} + \frac{s_{mrt}}{\sqrt{n}} t_{\frac{\alpha}{2}}(n-1) \right],$$

where $\bar{x}_{mrt}$ denotes the sample mean, s_{mrt} is the sample standard deviation, and $1-\alpha$ is the confidence level. The confidence interval of the random perturbation variables η_{mrte} is then obtained by the above interval.

Step 2: Based on $\mathscr{P}_{mrte}$, we can determine the value of σ_{mrte} in P.2 as follows,

$$\sigma_{mrte} = \max_{\hat{t}} \sqrt{\left(2\ln(\cosh(\hat{t}) + \mu_{mrte}\sinh(\hat{t})) - 2\max[\mu^{-}_{mrte}\hat{t}, \mu^{+}_{mrte}\hat{t}]\right)/\hat{t}^2}.$$

The above equation is derived as follows (see, Nemirovski and Shapiro 2007).

Process 1: According to P.2, parameters $\mu^{-}_{mrte}, \mu^{+}_{mrte}, \sigma_{mrte}$ satisfy the following inequalities,

$$\int \exp\{\hat{t}\eta_{mrte}\}d\mathbb{P}(\eta_{mrte}) \leq \exp\left\{\max[\mu^{-}_{mrte}\hat{t}, \mu^{+}_{mrte}\hat{t}] + \frac{1}{2}(\sigma_{mrte})^2\hat{t}^2\right\}, \quad \forall \hat{t} \in R, e \in [E],$$

where μ_{mrte} is the actual mean of $\mathbb{P}_{\eta_{mrte}}$.

Process 2: Consider the function,

$$\Phi(\eta_{mrte}) = \exp\{\hat{t}\eta_{mrte}\} - \sinh(\hat{t})\eta_{mrte}, -1 \leq \eta_{mrte} \leq 1.$$

When t is given, it is convex on $[-1, 1]$ and reaches a maximum at one of the endpoints of the segment. Because $\Phi(-1) = \Phi(1) = \cosh(\hat{t})$, the following inequality holds:

$$\begin{aligned}\int \exp\{\hat{t}\eta_{mrte}\}d\mathbb{P}(\eta_{mrte}) &= \int \Phi(\eta_{mrte})d\mathbb{P}(\eta_{mrte}) + \mu_{mrte}\sinh(\hat{t}) \\ &\leq \max_{-1\leq\eta_e\leq1} \Phi(\eta_{mrte}) + \mu_{mrte}\sinh(\hat{t}) = \cosh(\hat{t}) + \mu_{mrte}\sinh(\hat{t}).\end{aligned}$$

Process 3: By calculation, one has

$$\begin{aligned}\sup_{\mathbb{P}_{\eta_{mrte}}\in\mathscr{P}_{mrte}} \left\{\int \exp\{\hat{t}\eta_{mrte}\}d\mathbb{P}(\eta_{mrte})\right\} &= \cosh(\hat{t}) + \mu_{mrte}\sinh(\hat{t}) \\ &\leq \exp\left\{\max[\mu^{-}_{mrte}\hat{t}, \mu^{+}_{mrte}\hat{t}] + \frac{1}{2}(\sigma_{mrte})^2\hat{t}^2\right\}.\end{aligned}$$

It follows that

$$\sigma_{mrte} = \max_{\hat{t}} \sqrt{\left(2\ln(\cosh(\hat{t}) + \mu_{mrte}\sinh(\hat{t})) - 2\max[\mu^{-}_{mrte}\hat{t}, \mu^{+}_{mrte}\hat{t}]\right)/\hat{t}^2}.$$

Similarly, for transportation cost TC^1_{ijr}, TC^2_{jkr}, and TC^3, the distribution information set characterised by $\bar{P}$.1 and $\bar{P}$.2 is represented as follows:

$\bar{P}$.1. $\zeta_l, l = 1, \ldots, L$, are mutually independent random variables;
$\bar{P}$.2. The distribution $\mathbb{P}_{\zeta_l}$ of random variable ζ_l $(l = 1, \ldots, L)$ satisfies the following inequalities:

$$\int \exp tsd\mathbb{P}_{\zeta_l}(s) \leq \exp\left\{\max[\mu^{-}_l, \mu^{+}_l] + \frac{1}{2}(\sigma_l)^2t^2\right\}, \quad \forall t \in R,$$

with known constants $\mu_l^- \leq \mu_l^+$ and $\sigma_l \geq 0$. Moreover, we assume that the support of the probability distribution $\mathbb{P}_{\zeta_l}$ is $[-1, 1]$ and the expectation of the random perturbation variable belongs to $[\mu_l^-, \mu_l^+]$. The ambiguity set $\mathscr{P}_l$ is defined as follows.

$$\mathscr{P}_l = \left\{ \mathbb{P}_{\zeta_l} : \begin{array}{l} \mathbb{E}_{\mathbb{P}_{\zeta_l}}(\zeta_l) \subseteq [\mu_l^-, \mu_l^+] \subseteq [-1, 1] \\ \mathbb{P}_{\zeta_l}(\zeta_l \in [-1, 1]) = 1 \end{array} \right\}. \tag{2}$$

3.3. Formulation of upper-level location-routeing problem

3.3.1. Objective function

According to notations, the objective of effectiveness is to minimise the following overall transportation time:

$$\sum_{i,j,t} t\,e_{ij}^1 U_{ijt}^1 + \sum_{j,k,t} t\,e_{jk}^2 U_{jkt}^2 + \sum_{m,n \in [C] \cup \{0\}} \sum_{k,v,t} t\,e_{mn}^3 Q_{mnkvt}^3, \tag{3}$$

where the first term represents the cumulative transportation time from ES i to IFRC centre j, the second term represents the cumulative transportation time from IFRC centre j to DC k, and the third term is the sum of transportation time of goods from DC to NS.

3.3.2. Deterministic constraints

Constraints (4) indicate the balance in DC. The capacity limits of DCs are represented by constraints (5). Constraint (6) takes into account the maximum number of DCs. Constraints (7) provide that if the DC is established, it sends RIs to the NS; otherwise, it does not. Constraints (8) guarantee that each customer is assigned to just one DC.

$$\sum_{j} X_{jkrt}^1 + X_{kr(t-1)}^4 = \sum_{m,v} X_{mrvt}^2 + X_{krt}^4, \quad \forall k, r, t, \tag{4}$$

$$\sum_{j,r} X_{jkrt}^1 + \sum_{r} X_{kr(t-1)}^4 \leq A_{kt}^3 LOC_k, \quad \forall k, t, \tag{5}$$

$$\sum_{k} LOC_k \leq TD_{\max}, \tag{6}$$

$$\sum_{m,t} H_{mkt} \leq M * LOC_k, \quad \forall\, k, \tag{7}$$

$$\sum_{k} H_{mkt} = 1, \quad \forall\, m, t. \tag{8}$$

Constraints (9) are related to the route continuity. Constraints (10) ensure that if customer n is allocated to DC k, it will be served by DC k. Constraints (11) are designed to remove sub-tours. Constraints (12) state that if a vehicle does not travel to the NS, then the vehicle should not ship RIs to the NS. Otherwise, the RIs are delivered to the NS. Constraints (13) are associated with vehicle capacity. Constraints (14) make sure that vehicle can be used at most once. Constraints (15)–(18) denote the route usage. The decision variables of upper-level problem are defined by constraints (19)–(20).

$$\sum_{m \in [C] \cup \{0\}} Q_{mnkvt}^3 = \sum_{m \in [C] \cup \{0\}} Q_{nmkvt}^3, \quad \forall\, k, v, n \in [C] \cup \{0\}, t, \tag{9}$$

$$\sum_{m\in[C]\cup\{0\}}\sum_{v} Q^3_{mnkvt} \leq M * H_{nkt}, \quad \forall\, n,k,t, \tag{10}$$

$$G_{nvt} - G_{mvt} + A^4 \sum_{k} Q^3_{nmkvt} \leq A^4 - 1, \quad \forall\, m,n,v,t, \tag{11}$$

$$X^2_{mrvt} \leq M \sum_{n\in[C]\cup\{0\}}\sum_{k} Q^3_{nmkvt}, \quad \forall\, m,v,r,t, \tag{12}$$

$$\sum_{r,m} W_r X^2_{mrvt} \leq A^4, \quad \forall\, v,t, \tag{13}$$

$$\sum_{m}\sum_{k} Q^1_{0mkvt} \leq 1, \quad \forall\, v,t, \tag{14}$$

$$U^1_{ijt} \leq M \sum_{r} Y^1_{ijrt}, \quad \forall\, i,j,t, \tag{15}$$

$$U^1_{ijt} \geq \frac{\sum_r Y^1_{ijrt}}{M}, \quad \forall\, i,j,t, \tag{16}$$

$$U^2_{jkt} \leq M \sum_{r} X^1_{jkrt}, \quad \forall\, j,k,t, \tag{17}$$

$$U^2_{jkt} \geq \frac{\sum_r X^1_{jkrt}}{M}, \quad \forall\, j,k,t, \tag{18}$$

$$X^1_{jkrt}, X^2_{mrvt} \text{ are non} - \text{negative integer variables}, \quad \forall\, j,k,r,m,v,t, \tag{19}$$

$$Q^3_{mnkvt}, U^1_{ijt}, U^2_{jkt}, H_{mkt} \in \{0,1\}, G_{mvt} \geq 0, \quad \forall\, i,j,k,m,n,v,t. \tag{20}$$

3.3.3. Ambiguous chance constraints

Disasters can disrupt transportation networks and the transportation cost on the path is uncertain. We assume that the stochastic perturbation variable ζ_l obeys probability distribution $\mathbb{P}_{\zeta_l}$. For this, constraints (21) ensure that the total cost can be maintained within the available funding level with a certain probability $1-\epsilon$:

$$\begin{aligned}\mathbb{P}_{\zeta_l}\Bigg\{ & CO + CP + \sum_{i,k,r,t} TC^1_{ijr}(\boldsymbol{\zeta})Y^1_{ijrt} + \sum_{j,k,r,t} TC^2_{jkr}(\boldsymbol{\zeta})X^1_{jkrt} \\ & + \sum_{m,n\in[C]\cup\{0\}}\sum_{k,v,t} TC^3(\boldsymbol{\zeta})d_{mn}Q^3_{mnkvt} + CH \leq B\Bigg\} \geq 1-\epsilon, \quad \forall \mathbb{P}_{\zeta_l} \in \mathscr{P}_l,\end{aligned} \tag{21}$$

where ϵ is the probability that the cost exceeds the funding level, $CO = \sum_k LOC_k F_k$ is the cost of opening DCs, $CP = \sum_{i,j,r,t} C_{irt} Y^1_{ijrt}$ represents the purchasing cost, the three items with uncertain parameters are transportation cost, and $CH = \sum_{j,r,t} HR_{jr} X^3_{jrt} + \sum_{k,r,t} HD_{kr} X^4_{krt}$ represents the cost of holding inventory.

The perturbation variable η_{mrte} obeys probability distribution $\mathbb{P}_{\eta_{mrte}}$. Constraints (22) denote that the probability that all demands at the need site can be fully satisfied exceeds $1 - \epsilon_{mrt}$:

$$\mathbb{P}_{\eta_{mrte}} \left\{ \sum_v X^2_{mrvt} \geq D_{mrt}(\eta_{mrt}) \right\} \geq 1 - \epsilon_{mrt}, \quad \forall m, r, t, \forall \mathbb{P}_{\eta_{mrte}} \in \mathscr{P}_{mrte}. \tag{22}$$

3.4. Formulation of lower-level supply problem

The lower-level formulation is described by Equations (23)–(28). Equation (23) describes the objective function of the lower-level problem as the minimisation of supply risk of both ESs and public donations. Constraints (24) indicate that the total efficacy of public donations satisfies a certain level. Constraints (25) denote the balance constraints at the IFRC centre. Constraints (26) consider the supply ability of RI r by ES i. Constraints (27) are relevant to the storage capacity of the IFRC centre. Constraints (28) define decision variables of the lower level.

$$\min \quad \sum_{i,j,r,t} f^1_{ir} Y^1_{ijrt} + \sum_{j,r,t} f^2_r Y^2_{jrt} \tag{23}$$

$$\text{s.t.} \quad \sum_{r,j,t} \delta_r Y^2_{jrt} \geq xl, \tag{24}$$

$$\sum_i Y^1_{ijrt} + X^3_{jr(t-1)} + Y^2_{jrt} = \sum_k X^1_{jkrt} + X^3_{jrt}, \quad \forall j, r, t, \tag{25}$$

$$\sum_j Y^1_{ijrt} \leq A^1_{irt}, \quad \forall i, r, t, \tag{26}$$

$$\sum_{i,r} Y^1_{ijrt} + \sum_r Y^2_{jrt} + \sum_r X^3_{jr(t-1)} \leq A^2_{jt}, \quad \forall j, t, \tag{27}$$

$$Y^1_{ijrt}, Y^2_{jrt} \geq 0, \quad \forall i, j, r, t. \tag{28}$$

3.5. Distributionally robust bi-level mixed-integer programming model formulation

The following distributionally robust bi-level model is formally built under the premise of the presented upper-level and lower-level problems:

$$\begin{aligned} \min \quad & (3) \\ \text{s.t.} \quad & (4) - (22) \\ & Y^1_{ijrt}, Y^2_{jrt} \in \operatorname{argmin} \left\{ \sum_{i,j,r,t} f^1_{ir} Y^1_{ijrt} + \sum_{j,r,t} f^2_r Y^2_{jrt} \; : \; (24) - (28) \right\}. \end{aligned} \tag{29}$$

The proposed model is a bi-level semi-infinite programming model, which leads to serious computational difficulties. Hence, we will discuss how to reformulate the distributionally robust bi-level PELL model into a tractable formulation in the next section.

4. Solution strategies for distributionally robust bi-level PELL model

To effectively solve the presented model, robust counterpart approximation (RCA) is applied to reformulate the model into safe approximation formulation. KKT condition is employed to reformulate the presented model into a single-level one.

4.1. Approximation formulations of ambiguous chance constraints

The model with chance constraints (21) and (22) is computationally difficult to implement owing to its non-convexity and the requirement for a substantial number of samples to achieve good performance. Nevertheless, the distribution of random cost and demand is usually not completely known. To address these challenges, we use RCA to represent the chance-constrained model as a bi-level mixed-integer second-order conic and linear programming, respectively, which exploits partial distributional information. We now discuss suitable computable methods to solve the PELL problem.

The RCA under box ∩ ellipsoid perturbation set:

To enhance clarity in the presentation, additional variables are introduced.

$$TC^0 = \sum_{i,j,r} TC^{10}_{ijr} Y^1_{ijr} + \sum_{j,k,r} TC^{20}_{jkr} X^1_{jkr} + \sum_{m,n \in [L] \cup \{0\}} \sum_{k,v} TC^{30} d_{mn} Q^3_{mnkr},$$

$$TC_l = \sum_{i,j,r} TC^1_{ijrl} Y^1_{ijr} + \sum_{j,k,r} TC^2_{jkrl} X^1_{jkr} + \sum_{m,n \in [C] \cup \{0\}} \sum_{k,v} TC^3_l d_{mn} Q^3_{mnkr}.$$

Theorem 4.1 provides a safe approximation result for constraints (21).

Theorem 4.1: *For ambiguous chance constraints* (21)*, assume that the uncertain transportation cost $TC^1_{ijr}(\boldsymbol{\zeta})$ is represented by the affine sum of perturbation variables, i.e. $TC^1_{ijr}(\boldsymbol{\zeta}) = TC^{10}_{ijr} + \sum_{l \in [L]} \zeta_l TC^1_{ijrl}$. If the probability distribution of random variables ζ_l belongs to ambiguity set* (2)*, then the following convex algebraic system is a robust counterpart approximation of ambiguous chance constraints* (21)*,*

$$\begin{aligned}
&TC^0 + \sum_k F_k LOC_k + \sum_{i,j,r} C_{ir} Y^1_{ijr} + \sum_{j,r,t} HR_{jr} X^3_{jrt} + \sum_{k,r,t} HD_{kr} X^4_{krt} - B = u^0 + z^0,\\
&TC_l = u_l + z_l, \quad \forall\, l \in [L],\\
&u^0 + \sum_{l \in [L]} |u_l| \le 0, \\
&z^0 + \sum_{l \in [L]} \max[\mu_l^- z_l, \mu_l^+ z_l] + \sqrt{2\ln(1/\epsilon)} \sqrt{\sum_{l \in [L]} (\sigma_l)^2 (z_l)^2} \le 0,
\end{aligned} \tag{30}$$

where

$$\sigma_l = \max_t \sqrt{\left(2\ln(\cosh(t) + \mu_l \sinh(t)) - 2\max[\mu_l^- t, \mu_l^+ t]\right)/t^2}. \tag{31}$$

Next, we provide the computationally tractable formulations of ambiguous chance constraints (22).

Theorem 4.2: *For ambiguous chance constraints* (22) *on demand satisfaction, assume that the uncertain demand* $D_{mrt}(\boldsymbol{\eta}_{mrt})$ *is represented by the affine sum of perturbation variables* η_{mrte}, *i.e.* $D_{mrt}(\boldsymbol{\eta}_{mrt}) = D^0_{mrt} + \sum_{e\in[E]} \eta_{mrte} D_{mrte}$. *If the probability distribution of random perturbation variables* η_{mrte} *belongs to the ambiguity set* (1), *then the following convex algebraic system is a robust counterpart approximation of ambiguous chance constraints* (22),

$$\begin{aligned}
&D^0_{mrt} - \sum_{v} X^2_{mrvt} = u^0_{mrt} + z^0_{mrt}, \quad \forall\, m, r, t,\\
&D_{mrte} = u_{mrte} + z_{mrte}, \quad \forall\, e \in [E], m, r, t,\\
&u^0_{mrt} + \sum_{e\in[E]} |u_{mrte}| \le 0, \quad \forall\, m, r, t,\\
&z^0_{mrt} + \sum_{e\in[E]} \max[\mu^-_{mrte} z_{mrte}, \mu^+_{mrte} z_{mrte}] + \sqrt{2\ln(1/\epsilon)}\sqrt{\sum_{e\in[E]} (\sigma_{mrte})^2 (z_{mrte})^2} \le 0, \quad \forall\, m, r, t,
\end{aligned} \tag{32}$$

where

$$\sigma_{mrte} = \max_{\hat{t}} \sqrt{\left(2\ln(\cosh(\hat{t}) + \mu_{mrte}\sinh(\hat{t})) - 2\max[\mu^-_{mrte}\hat{t}, \mu^+_{mrte}\hat{t}]\right)/\hat{t}^2}. \tag{33}$$

The RCA under budget perturbation set:

We obtain the following results by analysing the relationship between budget perturbation set and box ∩ ellipsoid perturbation set.

Theorem 4.3: *For ambiguous chance constraints* (21), *assume that the uncertain transportation cost* $TC^1_{ijr}(\boldsymbol{\zeta})$ *is represented by the affine sum of perturbation variables, i.e.* $TC^1_{ijr}(\boldsymbol{\zeta}) = TC^{10}_{ijr} + \sum_{l\in[L]} \zeta_l TC^1_{ijrl}$. *If the probability distribution of random perturbation variables* ζ_l *belongs to ambiguity set* (2), *then vectors* $\boldsymbol{Y}^1_{ijr}$, $\boldsymbol{X}^1_{jkr}$, *and* $\boldsymbol{Q}^3_{mnkr}$ *satisfy* (21) *if there exists* $(\boldsymbol{\kappa}, \boldsymbol{\nu})$ *such that* $(\boldsymbol{Y}^1_{ijr}, \boldsymbol{X}^1_{jkr}, \boldsymbol{Q}^3_{mnkr}, \boldsymbol{\kappa}, \boldsymbol{\nu})$ *satisfies the following linear system:*

$$\begin{aligned}
&TC^0 + \sum_{k} F_k LOC_k + \sum_{i,j,r} C_{ir} Y^1_{ijr} + \sum_{j,r,t} HR_{jr} X^3_{jrt} + \sum_{k,r,t} HD_{kr} X^4_{krt} - B = \kappa^0 + \nu^0,\\
&TC_l = \kappa_l + \nu_l, \quad \forall\, l \in [L],\\
&\kappa^0 + \sum_{l\in[L]} |\kappa_l| \le 0,\\
&\nu^0 + \sum_{l\in[L]} \max[\mu^-_l \nu_l, \mu^+_l \nu_l] + \sqrt{L}\sqrt{2\ln(1/\epsilon)} \max_{1\le l\le L} \sigma_l \nu_l \le 0,
\end{aligned} \tag{34}$$

where

$$\sigma^l = \max_{t} \sqrt{\left(2\ln(\cosh(t) + \mu_l \sinh(t)) - 2\max[\mu^-_l t, \mu^+_l t]\right)/t^2}. \tag{35}$$

Computationally tractable formulations of constraints (22) are provided by the following Theorem 4.4.

Theorem 4.4: *For ambiguous chance constraints* (22) *on demand satisfaction, assume that the uncertain demand* $D_{mrt}(\boldsymbol{\eta}_{mrt})$ *is represented by the affine sum of perturbation variables,* *i.e.* $D_{mrt}(\boldsymbol{\eta}_{mrt}) = D^0_{mrt} + \sum_{e\in[E]} \eta_{mrte} D_{mrte}$. *If the probability distribution of random perturbation variables* η_{mrte} *belongs to ambiguity set* (1), *then vector* $\boldsymbol{X}^2_{mrvt}$ *satisfies* (22) *if there exists* $(\boldsymbol{\kappa}_{mrt}, \boldsymbol{\nu}_{mrt})$ *such that* $(\boldsymbol{X}^2_{mrvt}, \boldsymbol{\kappa}_{mrt}, \boldsymbol{\nu}_{mrt})$ *satisfies the following linear system:*

$$\begin{aligned}
&D^0_{mrt} - \sum_v X^2_{mrvt} = \kappa^0_{mrt} + \nu^0_{mrt}, \quad \forall\, m, r, t, \\
&D_{mrte} = \kappa_{mrte} + \nu_{mrte}, \quad \forall\, e \in [E], m, r, t, \\
&\kappa^0_{mrt} + \sum_{e\in[E]} |\kappa_{mrte}| \le 0, \quad \forall\, m, r, t, \\
&\nu^0_{mrt} + \sum_{e\in[E]} \max[\mu^-_{mrte}\nu_{mrte}, \mu^+_{mrte}\nu_{mrte}] + \sqrt{E}\sqrt{2\ln(1/\epsilon)} \max_{1\le e\le E} \sigma_{mrte}\nu_{mrte} \le 0, \quad \forall\, m, r, t,
\end{aligned} \tag{36}$$

where

$$\sigma_{mrte} = \max_{\hat{t}} \sqrt{\left(2\ln(\cosh(\hat{t}) + \mu_{mrte}\sinh(\hat{t})) - 2\max[\mu^-_{mrte}\hat{t}, \mu^+_{mrte}\hat{t}]\right)/\hat{t}^2}. \tag{37}$$

Remark 4.1: The proofs of Theorems 4.3 and 4.4 show that the budget robust counterpart (RC) is more conservative than the box ∩ ellipsoid RC. However, the budget RC is formulated by a linear constraint system. In contrast, the box ∩ ellipsoid RC results in a conic quadratic formulation, which requires high computability.

4.2. Transformation of bi-level PELL model

The dual expression of the lower-level model is formulated in the following expression.

$$\begin{aligned}
\max \quad & xl * \omega^1 + \sum_{j,r,t}\left(\sum_k X^1_{jkrt} + X^3_{jrt} - X^3_{jr(t-1)}\right)\omega^2_{jrt} \\
& + \sum_{i,r,t} A^1_{irt}\omega^3_{irt} + \sum_{j,t}\left(A^2_{jt} - \sum_r X^3_{jr(t-1)}\right)\omega^4_{jt}
\end{aligned} \tag{38}$$

$$\text{s.t.} \quad \omega^2_{jrt} + \omega^3_{irt} + \omega^4_{jt} \le f^1_{ir}, \quad \forall\, i, j, r, t, \tag{39}$$

$$\delta_r\omega^1 + \omega^2_{jrt} + \omega^4_{jt} \le f^2_r, \quad \forall\, j, r, t, \tag{40}$$

$$\omega^1 \ge 0, \omega^3_{irt}, \omega^4_{jt} \le 0, \omega^2_{jrt} \text{ is free}, \quad \forall\, i, j, r, t. \tag{41}$$

According to KKT condition, the model under box ∩ ellipsoid perturbation set is transformed into an equivalent single-level non-linear formulation.

$$\min \quad (3) \tag{42}$$

$$\text{s.t.} \quad Y^1_{ijrt}(\omega^2_{jrt} + \omega^3_{irt} + \omega^4_{jt} - f^1_{ir}) = 0, \quad \forall\, i, j, r, t, \tag{43}$$

$$Y^2_{jrt}(\delta_r\omega^1 + \omega^2_{jrt} + \omega^4_{jt} - f^2_r) = 0, \quad \forall\, j, r, t, \tag{44}$$

$$\omega^1\left(\sum_{r,j,t}\delta_r Y^2_{jrt} - xl\right) = 0, \tag{45}$$

$$\omega_{irt}^{3}\left(\sum_{j} Y_{ijrt}^{1}-A_{irt}^{1}\right)=0, \quad \forall\, i, r, t, \tag{46}$$

$$\omega_{jt}^{4}\left(\sum_{i,r} Y_{ijrt}^{1}+\sum_{r} Y_{jrt}^{2}+\sum_{r} X_{jr(t-1)}^{3}-A_{jt}^{2}\right)=0, \quad \forall\, j, t, \tag{47}$$

$$(4)-(20),\ (24)-(28),\ (30),\ (32),\ (39)-(41). \tag{48}$$

To solve the nonlinearity mentioned in the single-level nonlinear model (42)–(48), auxiliary variables are introduced into linearising constraints (43)–(47). Variables $\gamma_{ijrt}^{1} \in \{0,1\}$ are added and constraints (43) are replaced with the following constraints:

$$Y_{ijrt}^{1} \leq M(1-\gamma_{ijrt}^{1}), \quad \forall i, j, r, t, \tag{49}$$

$$\omega_{jrt}^{2}+\omega_{irt}^{3}+\omega_{jt}^{4}-f_{ir}^{1} \geq M\gamma_{ijrt}^{1}, \quad \forall\, i, j, r, t. \tag{50}$$

Similarly, $\gamma_{jrt}^{2} \in \{0,1\}$ is used as an auxiliary variable to handle the nonlinear problem with constraint (44). By combining $Y_{jrt}^{2} \geq 0$ and constraints (40), the linear constraints corresponding to constraints (44) are represented as follows:

$$Y_{jrt}^{2} \leq M(1-\gamma_{jrt}^{2}), \quad \forall j, r, t, \tag{51}$$

$$\delta_{r}\omega^{1}+\omega_{jrt}^{2}+\omega_{jt}^{4}-f_{r}^{2} \geq M\gamma_{jrt}^{2}, \quad \forall\, j, r, t. \tag{52}$$

Linearise constraints (45) with the definition of zero-one variable γ^{3}.

$$\omega^{1} \leq M(1-\gamma^{3}), \tag{53}$$

$$\sum_{r,j,t} \delta_{r} Y_{jrt}^{2}-xl \leq M\gamma^{3}. \tag{54}$$

The zero-one variable γ_{irt}^{4} is introduced to linearise the constraints (46).

$$\omega_{irt}^{3} \geq M(1-\gamma_{irt}^{4}), \quad \forall i, r, t, \tag{55}$$

$$\sum_{j\in[J]} Y_{ijrt}^{1}-A_{irt}^{1} \geq M\gamma_{irt}^{4}, \quad \forall i, r, t. \tag{56}$$

Using 0-1 variable γ_{jt}^{5}, we linearise constraints (47).

$$\omega_{jt}^{4} \geq M(1-\gamma_{jt}^{5}), \quad \forall\, j, t, \tag{57}$$

$$\sum_{i,r} Y_{ijrt}^{1}+\sum_{r} Y_{jrt}^{2}+\sum_{r} X_{jr(t-1)}^{3}-A_{jt}^{2} \geq M\gamma_{jt}^{5}, \quad \forall\, j, t. \tag{58}$$

An equivalent single-level mixed-integer second-order conic programming (MSCP) model is illustrated as follows.

$$\min \quad (3) \tag{59}$$

$$\text{s.t.} \quad \gamma_{ijrt}^{1}, \gamma_{jrt}^{2}, \gamma^{3}, \gamma_{irt}^{4}, \gamma_{jt}^{5} \in \{0,1\}, \tag{60}$$

$$(4)-(20),\ (24)-(28),\ (30),\ (32),\ (39)-(41),\ (49)-(58). \tag{61}$$

Similarly, we obtain the mixed-integer linear programming (MLP) model under budget perturbation set.

$$\min \quad (3) \tag{62}$$

$$\text{s.t.} \quad \gamma^1_{ijrt}, \gamma^2_{jrt}, \gamma^3, \gamma^4_{irt}, \gamma^5_{jt} \in \{0,1\}, \tag{63}$$

$$(4)-(20),\ (24)-(28),\ (34),\ (36),\ (39)-(41),\ (49)-(58). \tag{64}$$

5. Case study

In this section, the real case of Sari flood is carried out to demonstrate the validity of the presented models and insights are derived into the design of emergency logistics systems. First, we present the computational results based on real flood data in the Sari region of Iran. Next, we numerically analyse the sensitivity of important model parameters, including funding level, expectation information, and tolerance level. Finally, in situations where information regarding the distribution of random variables is only partially known, the comparison results show the credibility of the proposed DRO approach. All numerical experiments are performed with the IBM CPLEX 12.6.3 solver on an Inter(R) Core(TM) i5-8265U 1.80 GHz personal computer running under Windows 10 (64-bit) with 8 GB of RAM.

5.1. Problem description

Based on statistics and survey reports, Iran is one of the most disaster-prone countries in the world (Ghasemi et al. 2022a). Floods in 2019 affected the northern and northeastern regions of Iran, with Mazandaran and Golestan provinces being the most severely affected (Dodangeh et al. 2020). Therefore, Sari, the capital of Mazandaran Province, is selected as a case study to demonstrate the performance of the proposed model. In this study, four villages are considered as need sites. Table 3 reports the affected villages and the population in need of relief. Two potential sites in Sari are identified as candidates for DCs. Table 4 illustrates the capacity and cost of the DCs. In addition, two IFRC central warehouses are selected as permanent warehouses. Two ESs are considered. Figure 2 presents the location of the nodes.

The tables below provide the relevant data for the nodes. The input data are estimated based on data collected from Khanchehzarrin et al. (2022) to closely represent the actual situation. The capacity of suppliers and the cost of purchasing each RI are shown in Table 5. Table 6 describes the capacity, initial inventory and unit inventory holding cost at the IFRC warehouse. The transportation cost, time and distance between SC, IFRC and DC are shown

Table 3. Affected villages and the population at each NS.

Need sites	Village name	Number of people in need of RI	Need sites	Village name	Number of people in need of RI
NS1	Abmal	576	NS3	Panbeh Chooleh	796
NS2	Marzrud	712	NS4	Esfandan	758

Table 4. Data about the DC.

			Cost of holding inventory	
Distribution centre	Fixed cost	Capacity	water	food
DC1	330,000,000	58,000	5000	9000
DC2	440,000,000	60,000	6500	11,500

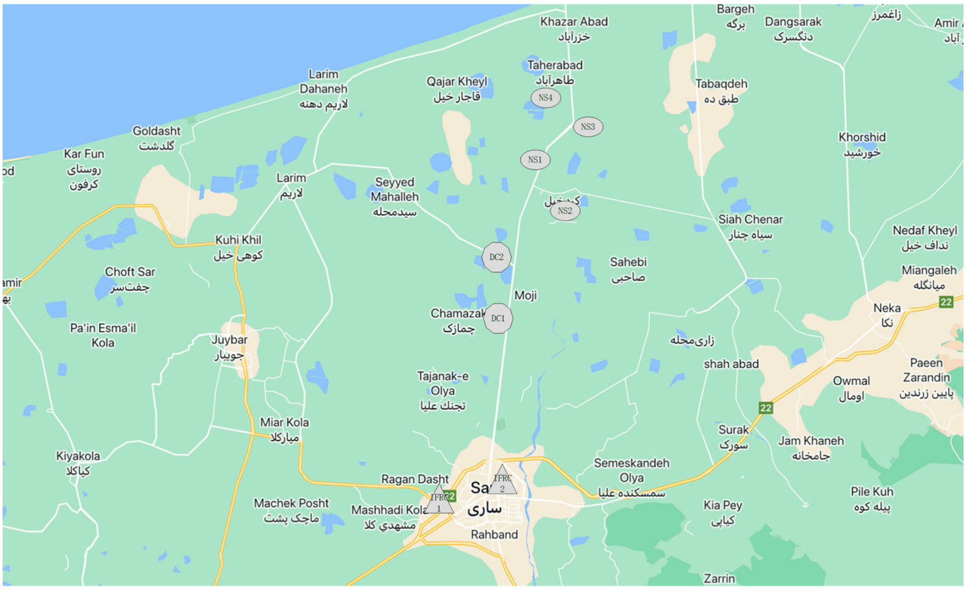


Figure 2. Locations of IFRC centres, possible distribution centres and need sites.

Table 5. Procurement RI cost and ES supply capacity.

		Capacity		Cost	
ES	Time period	RI1	RI2	RI1	RI2
ES1	T1	10,000	8000	18,000	230,000
	T2	9000	7500	19,700	248,000
ES2	T1	6000	4000	20,000	242,000
	T2	5500	6500	19,300	236,000

Table 6. Data required for IFRC warehouse.

	Cost of holding inventory		Capacity		Initial inventory	
IFRC	RI1	RI2	T1	T2	RI1	RI2
IFRC1	8000	13,000	30,000	30,000	1500	1800
IFRC2	7000	14,000	50,000	50,000	4200	1400

in Table 7. The distances and transportation time between the DC and the NS and between each need site are given in Table 8, which are taken from Google Map. The risk of public donations to supply relief item *r* is 0.80. Transportation cost of per vehicle per kilometer is 125235 (Rial/Km). The quantities of water and food required per person are considered to be 3 kg and 1.18 kg, respectively.

Table 7. Transportation data between ES, IFRC and DC nodes.

		Transportation cost						Transportation cost			
		Water	Relief food	Travel time	Distance			Water	Relief food	Travel time	Distance
ES1	IFRC1	300	550	9	1.7	IFRC1	DC1	600	750	20	12.6
	IFRC2	400	500	13	7.8		DC2	630	700	22	14.5
ES2	IFRC1	330	470	13	3.9	IFRC2	DC1	660	770	16	13.4
	IFRC2	240	280	4	1		DC2	620	800	18	16.1

Table 8. Transportation time and distance between DC and NS.

		Abmal	Marzrud	Panbeh Chooleh	Esfandan
DC1	Time	12	17	14	15
	Dis.	12	17	14	17
DC2	Time	11	15	13	13
	Dis.	10	16	13	15
Abmal	Time	0	8	6	10
	Dis.	0	6	3.5	5.8
Marzrud	Time	8	0	6	11
	Dis.	6	0	4.3	7.3
Panbeh Chooleh	Time	6	6	0	9
	Dis.	3.5	4.3	0	4.7
Esfandan	Time	10	11	9	0
	Dis.	5.8	7.3	4.7	0

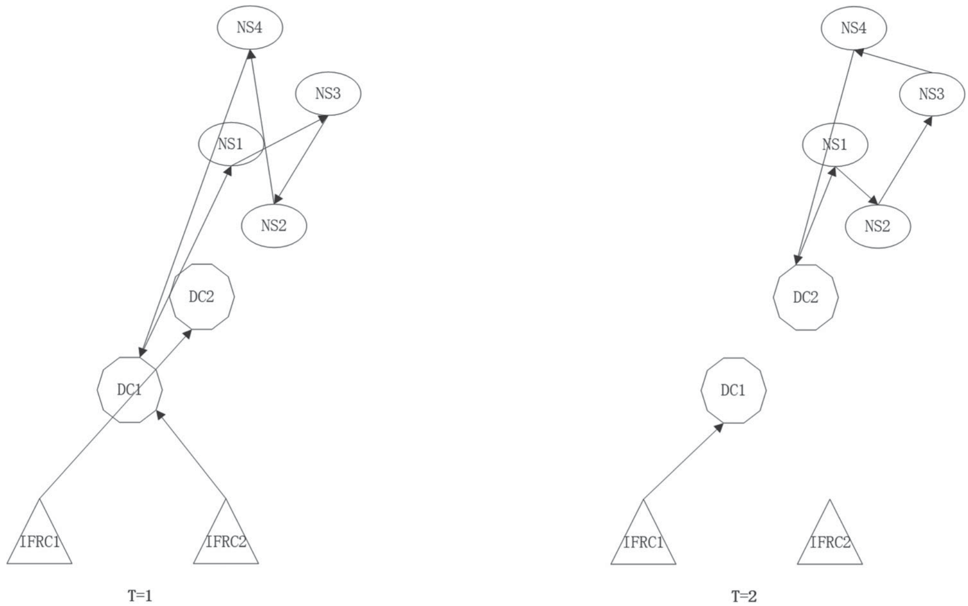
5.2. Estimating supply risk

At lower-level, one of the main tasks of the IFRC organisation is to purchase the required RIs for delivery. RIs from the same supplier vary regarding quality, delivery time, supply flexibility, and reputation. This requires that different risks are assigned to different suppliers. In this context, the process of assessing supply risk using AHP is described. The rationale is to decompose the factors relevant to the decision into several levels, such as the target level, the criteria level, and the solution level, and to calculate and compare the weights of different factors. The relevant description of AHP can be found in Feng, Liu, and Chen (2022). The process of AHP is summarised in Appendix 2.

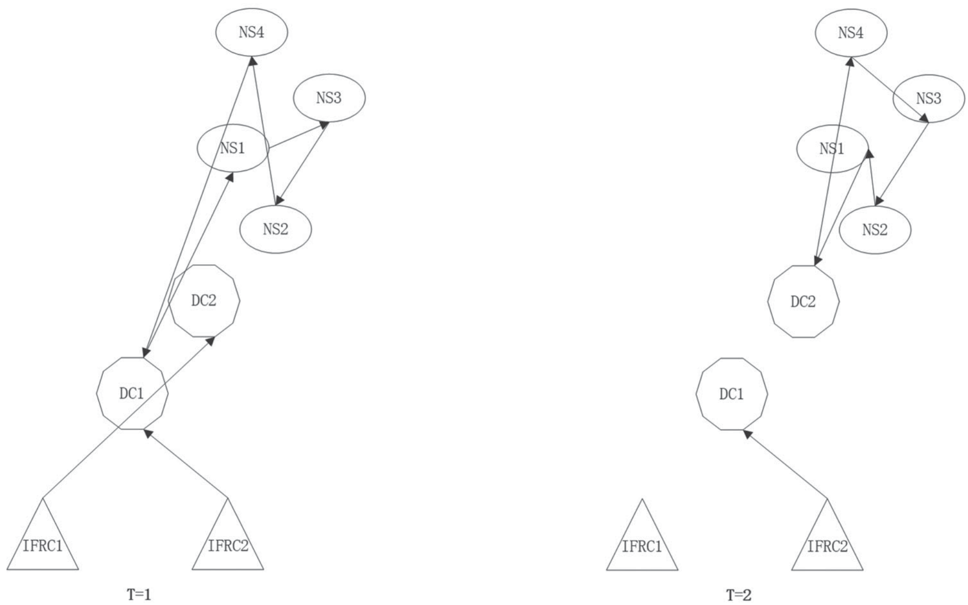
5.3. Calculation results of presented models

The computational results of the proposed DRO models are presented. We conduct experiments by fixing the confidence level as 95%, setting tolerance level as $\epsilon = 0.35$ and funding level as $B = 1{,}350{,}000{,}000$. σ_l and σ_{mrte} are obtained from Equations (31) and (33) as 0.9595 and 0.983, respectively. Furthermore, two factors are assumed to cause uncertainty in transportation costs and demand. The contributions of the lth and eth influences to uncertain transportation costs and demand are represented by the perturbation coefficients TC_l and D_{mrte}. In particular, the perturbation to uncertainty is considered to be 5% of the normal value. Moreover, the optimal transportation time for MSCP model and MLP model are 261 min and 269 min, respectively. The best responses of the proposed models are presented in Figures 3 and 4 and Table 9.

Figure 3 depicts the results of RIs assignment and vehicle paths. In the MSCP model, for the central warehouse selection, in the first period, IFRC1 and IFRC2 are reopened. This is

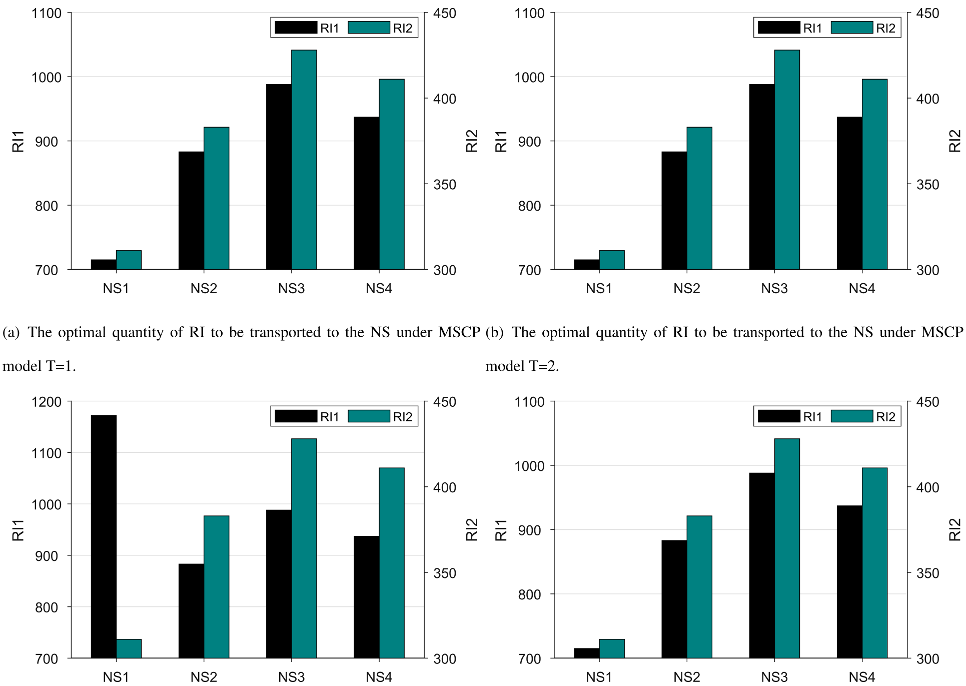


(a) Material assignment and vehicle routing under MSCP model.



(b) Material assignment and vehicle routing under MLP model.

Figure 3. Material assignment and vehicle routeing under DRO models. (a) Material assignment and vehicle routeing under MSCP model. (b) Material assignment and vehicle routeing under MLP model.



(a) The optimal quantity of RI to be transported to the NS under MSCP model T=1.

(b) The optimal quantity of RI to be transported to the NS under MSCP model T=2.

(c) The optimal quantity of RI to be transported to the NS under MLP model T=1.

(d) The optimal quantity of RI to be transported to the NS under MLP model T=2.

Figure 4. The optimal quantity of RI to be transported to the NS under DRO models. (a) The optimal quantity of RI to be transported to the NS under MSCP model T = 1. (b) The optimal quantity of RI to be transported to the NS under MSCP model T = 2. (c) The optimal quantity of RI to be transported to the NS under MLP model T = 1 and (d) The optimal quantity of RI to be transported to the NS under MLP model T = 2.

Table 9. Lower-level decision variables under DRO models.

Decision variable	ES	IFRC	DC	Commodity	Period	Value under MSCP model	Value under MLP model
Y1	1	1	–	1	1	8160	–
Y1	1	2	–	1	1	1012	–
Y1	2	1	–	1	1	–	6000
Y1	2	1	–	2	1	–	1263
Y1	2	2	–	2	1	–	232
Y1	2	2	–	1	2	–	2817
Y2	–	1	–	1	2	225	–
Y2	–	1	–	2	1	251	–
Y2	–	1	–	2	2	1532	–
Y2	–	2	–	2	1	132	–
Y2	–	2	–	1	2	–	484
Y2	–	2	–	2	2	–	1526

because the initial inventory of the DC is 0 during the first period, so the central warehouse is required to transport RIs to the DC. In the second period, no RIs are transported from the central warehouse to DC2 since DC2 has surplus inventory. Regarding vehicle routes, in the first period, vehicles travel from DC1 to NS1, 3, 2, and 4, respectively, and finally return to

Table 10. Computational results for MLP model at different scales.

Size I×J×K×C×R	Objective value	CPU time/s	Gap
2×2×2×4×2	269	46.46	2.6%
3×3×2×4×2	242	22.43	4.5%
4×4×4×5×3	657	200.69	8.5%
5×5×4×6×3	728	229.2	8.45%
8×8×4×6×3	738	311.08	6.96%
8×8×5×8×3	1658	3982.97	4.85%
10×10×5×8×3	1767	1428	5.46%
12×12×5×8×3	1927	213.95	3.88%
12×12×10×10×3	9260	716.69	1.69%
12×12×10×11×3	9305	5280.55	2.57%

DC. Unlike MSCP model, in the second period of the MLP model relief items are transferred from DC 2 to NS4, 3, 2, and 1.

Figure 4 shows the quantitative characteristics of the transportation. We use different bar colours to indicate the different RIs transported. In the MSCP model, it can be seen from the first period that the quantity of RI1 transported to NS1 is 715, which is greater than its demand. In the second period, the number of RI2 transported to NS3 is 428, which is also the number that meets its demand. This is also in line with the idea that humanitarian relief is people-oriented. In the MLP model, the quantity of RI1 shipped to NS1 in the first period is greater than that in the MSCP model, with a specific value of 1172. Table 9 shows the number of RIs purchased from ESs and provided by public donations. In the MSCP model, it is seen that supplier 2 delivers RI1 to central warehouses 1 and 2. RI2 is provided by public donations. From the MLP model, it is observed that RI1 and RI2 come from supplier purchases and public donations. And it is seen from Table 9 that supplier 2 provides supplies because supplier 1 has a greater supply risk than supplier 2, which is consistent with the results. Meanwhile, relief goods in central warehouse 2 are provided by public donors. With the reported optimal values and optimal solutions, consistent with intuition, we find that the MSCP model is less conservative than the MLP model. To further illustrate the performance of the developed DRO model, we utilise CPLEX solver to solve MLP model at various scales with default settings. Table 10 shows the calculation results for different-sized instances. It can be observed that an increase in instance size is accompanied by a corresponding rise in computation time. In particular, for all test instances, the gap values reported by CPLEX do not exceed 9%, which indicates that the obtained solutions are acceptable.

5.4. Sensitivity analysis

The solution may be affected by key model parameters. Here we investigate the effect of funding level, expectation information, and tolerance on transportation time.

5.4.1. Effect of funding level

Capital is an important limiting factor for planners and the subsection highlights the ensuing sensitivity study. To examine the influence of parameters on the reality of the situation, the objective is evaluated according to variation in the funding level B. The corresponding variation of the models is presented in Figure 5.

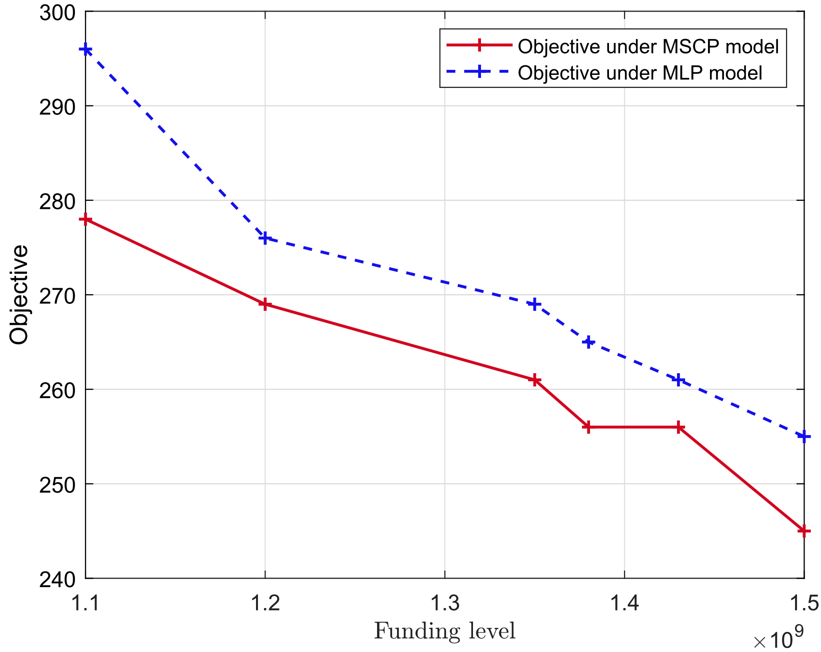


Figure 5. The influence of funding level B on the optimal value.

Figure 5 shows the variation of transportation time with funding level B for MSCP model and MLP model. To assess the sensitivity of the objective to funding level, different levels of funding are tested from 1,100,000,000 to 1,500,000,000. It is evident that the overall shipping time decreases as funds increase. This is because an increase in funding enables more purchases and more inventory, resulting in less DC allocation to the central warehouse and thus less time. For example, in the MSCP model, the transportation time is 269 min for the funding of 1,200,000,000, and 256 min for the funding of 1,380,000,000. In addition, we find that the total transportation time of MSCP model is less than that of the MLP model when the values of other parameters are set the same for both models. At this point, managers can choose the appropriate funding level to enhance the efficiency of PELL according to the actual situation.

5.4.2. Effect of the expectation information

In this subsection, we examine the effect of the robustness level of the ambiguity set on target values. Since parameters μ^-_{mrte}, μ^+_{mrte}, μ^-_l, and μ^+_l limit the scale of the ambiguity set, a sensitivity analysis is performed using the same data from Sections 5.1 and 5.2 with different μ^-_{mrte}, μ^+_{mrte}, μ^-_l, and μ^+_l to investigate the influence of ambiguity-control parameters on decision making. As previously mentioned, we adjust the confidence level to 90%, 95%, 98%, and 99%. By adjusting the choices of these parameters, we construct models that are robust to different degrees of uncertainty in the distribution parameters. This is because the critical value of the t-distribution varies with confidence level, leading to a change in the size of the ambiguity set. In this experiment, for MSCP and MLP, different funding and tolerance levels are chosen as $B = 1{,}200{,}000{,}000$, $\epsilon = 0.35$ and $B = 1{,}117{,}000{,}000$, $\epsilon = 0.22$, respectively.

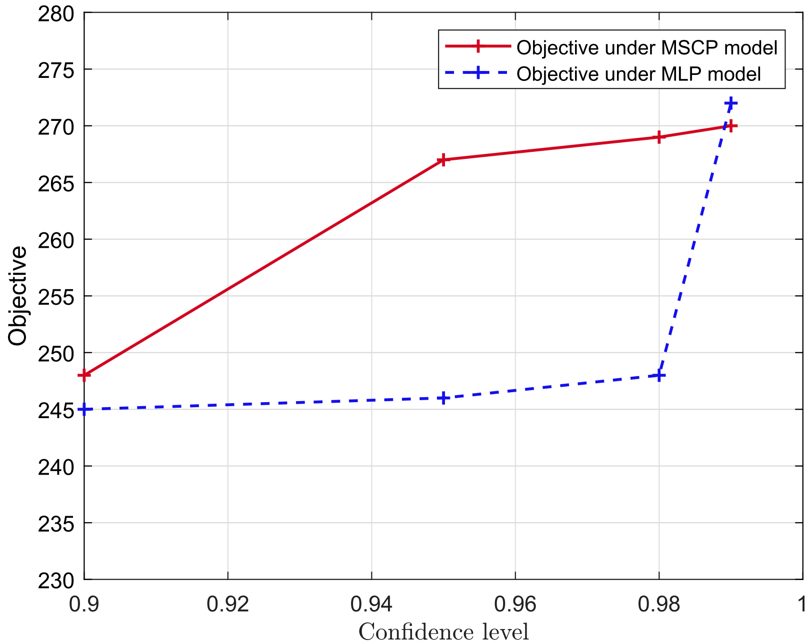


Figure 6. The influence of expectation information on the optimal value.

Figure 6 depicts how the target value is affected by the variation of confidence level. When the confidence level increases with constant sample size, the critical value of the t-distribution increases and the resulting confidence interval becomes larger. Therefore, the larger the confidence interval of the mean, the wider range of ambiguity that we consider in the unknown distribution about demand and transportation costs and become more conservative. From Figure 6, it can be observed that as confidence level increases, the time objective becomes larger and thus, the model becomes increasingly conservative. As a result, the distributionally robust approach becomes more cautious with increased ambiguity.

5.4.3. Effect of the tolerance level

Here, due to the conservativeness of MLP model, the impact of the tolerance level on the objective of MSCP model is tested. To evaluate the impact of probability ϵ, we set B and confidence level as 1,430,000,000 and 95%, respectively and change the probability $1-\epsilon$ from 68% to 99%, as shown in Figure 7. From a longitudinal perspective, it is intuitively clear that transportation time decreases with increasing ϵ. For example, the transportation time is 248 min at $\epsilon = 0.1$ and 245 min slightly greater than $\epsilon = 0.3$. This is because when the risk of total costs exceeding the funding level and the probability of supply being less than demand increase, vehicles are deployed on routes with lower transportation cost, which leads to an increase in transportation time.

According to the sensitivity analyses for ϵ, different tolerance levels exert certain effects on the objective value. That is, the target is sensitive to tolerance level. In such a case, decision-makers can rely on their personal experience and knowledge to confirm the probability level to rationally select vehicle routeing and deliver supplies to NSs quickly.

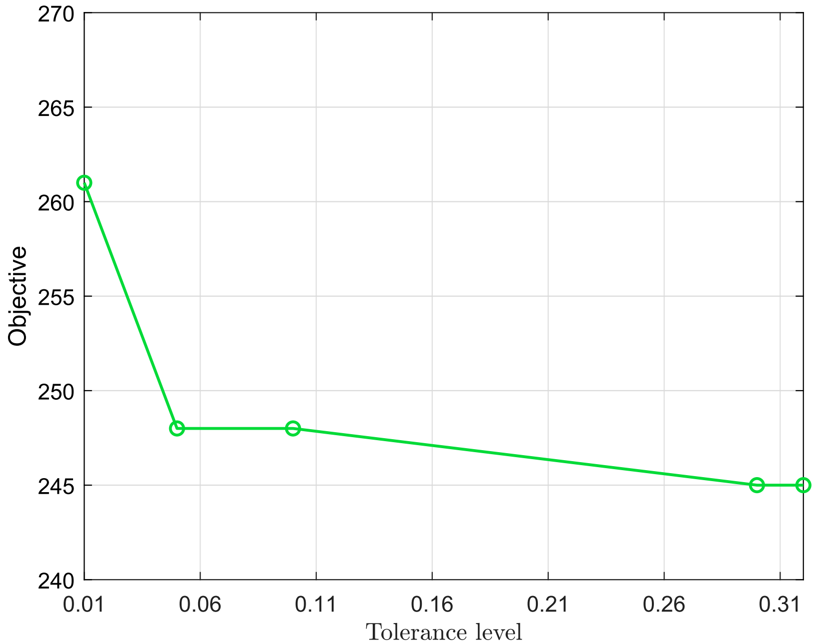


Figure 7. Optimal value of MSCP model with different ϵ.

5.5. Model comparison

5.5.1. Comparison with the RO model

Optimal routeing plan and transportation time are compared in two environments, one with partial demand and transportation cost distribution information, i.e. the MSCP model and the MLP model, and the other without distribution information. For the second case, we consider that $\mathscr{P}^1$ and $\mathscr{P}^2_{mrt}$ contain all distributions on a fixed support set, i.e. the distribution for uncertain parameters is free, in this case, the proposed DRO models reduce to the RO model. The mathematical form of the RO model is provided in Appendix 3. Consistent with Subsection 5.3, we set the funding level to 1,350,000,000. The target value of the DMO for the bi-level RO model is calculated to be 302 min. The best response is shown in Figure 8.

From the optimal routeing performance in Figures 3 and 8, we observe that the two models obtain different routeing under the same parameter settings. For example, In Figure 8, in the first period, the vehicles depart from DC1, reach NS1, 4, 2, and 3 in order, and finally return to the DC. In addition, unlike Figure 3, in the first period, relief items for DC1 are provided by central warehouse 1, and central warehouse 2 provides supplies for DC2.

To further compare the DRO models with the RO model, the funding level parameter is set to the value between 1,350,000,000 and 1,500,000,000 to explore the conservativeness of the model. The effect of B on its optimal outcome is presented in Figure 9. It is observed that the DRO models obtain better optimal objectives for the same funding level B from the representation of the optimal results in Figure 9. Specifically, MSCP model yields lower transportation time. This indicates that DRO models can provide a less conservative solution than RO model.

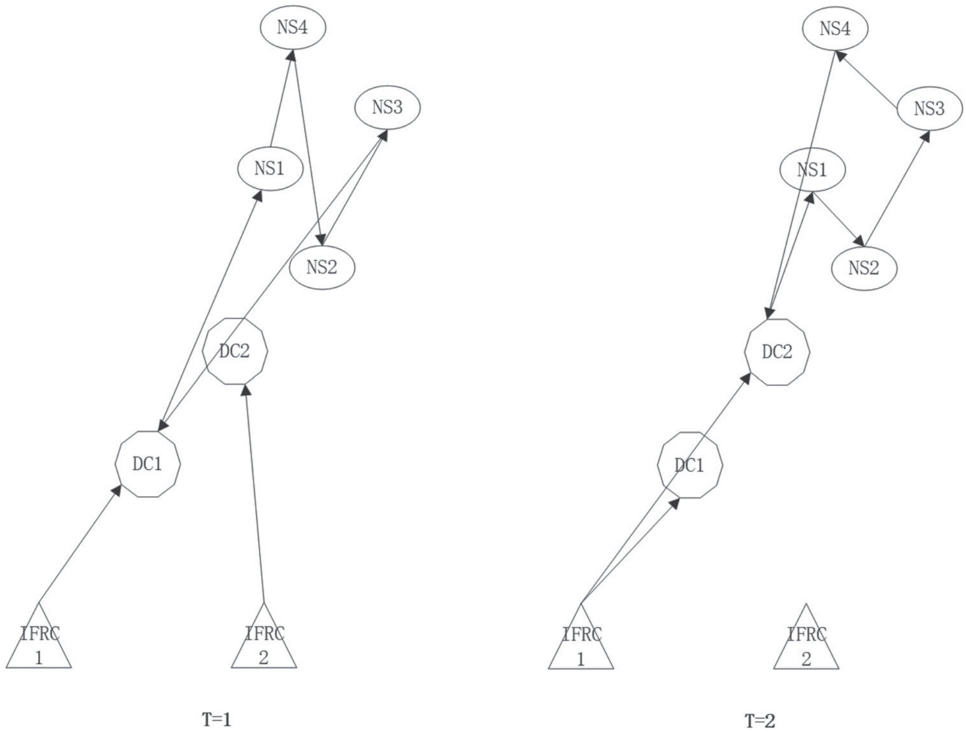


Figure 8. Material assignment and vehicle routeing under RO model.

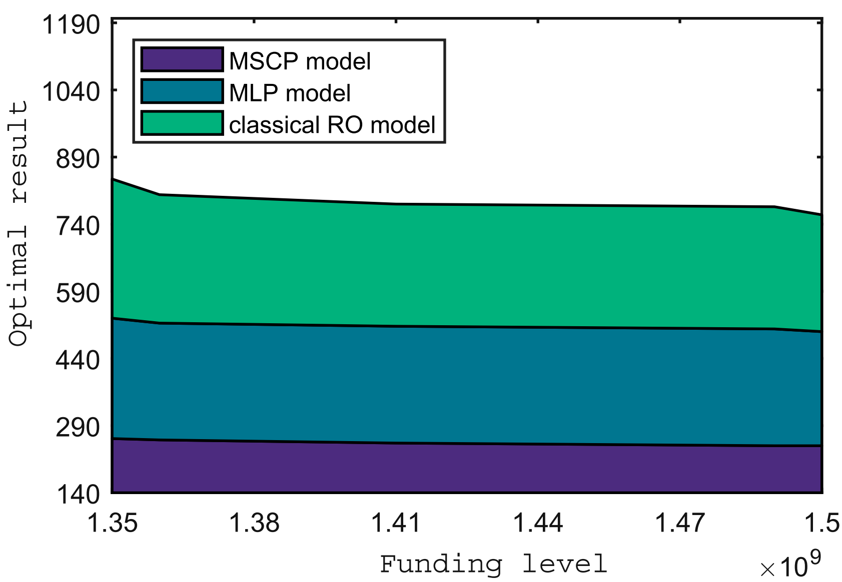


Figure 9. Comparing the optimal result of RO and DRO models with different *B*.

5.5.2. Comparison with SO model

In this section, we conduct a comparison study between the DRO model and the sample average approximation (SAA) model of the SO model. From a decision-making perspective, the DRO model optimises location-routeing decisions to achieve robust performance, where the distributions of demand and transportation costs vary within the constructed ambiguity sets. The SAA model, on the other hand, makes the optimal location-routeing decisions according to a number of samples from the specific distributions. In our experiments, the number of samples is set to 50.

The SAA model yields the optimal transportation time of 228 min. The SAA model provides a detailed scheme illustrated in Figure 10. Specifically, unlike the first period of the DRO model, the SAA model incorporates a routeing scheme that sequentially transports relief supplies from DC1 to NS4, 3, 2, 1, and back to DC1. Moreover, we employ the price of distributional robustness (PODR) to further analyse the distinction between the DRO model and SAA model. The expression for PODR is as follows:

$$\mathrm{PODR} = \frac{(\mathrm{DRO})^* - (\mathrm{SAA})^*}{(\mathrm{SAA})^*},$$

where $()^*$ represents the optimal time. The MSCP model exhibits a PODR of 14.4% and the MLP model shows a PODR of 18%. This indicates that the MSCP model incurs a mere 14.4% increase in relative cost when compared to the SAA model. In essence, the MSCP model provides the optimal location-routeing scheme that can handle the uncertainties in demand and transportation costs with only a 14% increase in relative time.

5.6. Managerial insights

The proposed DRO model can address PELL problem along with bi-level and multi-period and simultaneously cope with related uncertainty. Managerial implications for decision makers are derived.

Combining the emergency logistics location-routeing problem with hierarchical decision-making relationships and uncertainties in demand and transportation costs is the first advantage of our developed model. By applying the DRO model, the ability of managers to decide on the location of DCs and the transportation of relief items will be improved.

The proposed DRO model is less conservative than the RO model while allowing for the controlled constraint violation. Decision makers can use it to efficiently design relief networks by utilising available distribution information. Compared to the SO model, our DRO model can hedge against demand and transportation costs ambiguity by achieving optimal decision making with relative lower costs. Decision makers can employ the DRO model for problems with partially known distribution information.

In the bi-level PELL model with ambiguous chance constraints, the ambiguity set size and tolerance can influence the optimal decisions. The ambiguity set at different scales generates different optimal values to immunise against distribution ambiguity. If the decision maker wants the optimal decision to avoid the effect of large uncertainty, then larger scale ambiguity set should be defined. Emergency managers should balance total time and efficiency, since a greater tolerance level can decrease the total time, but increase the risk that

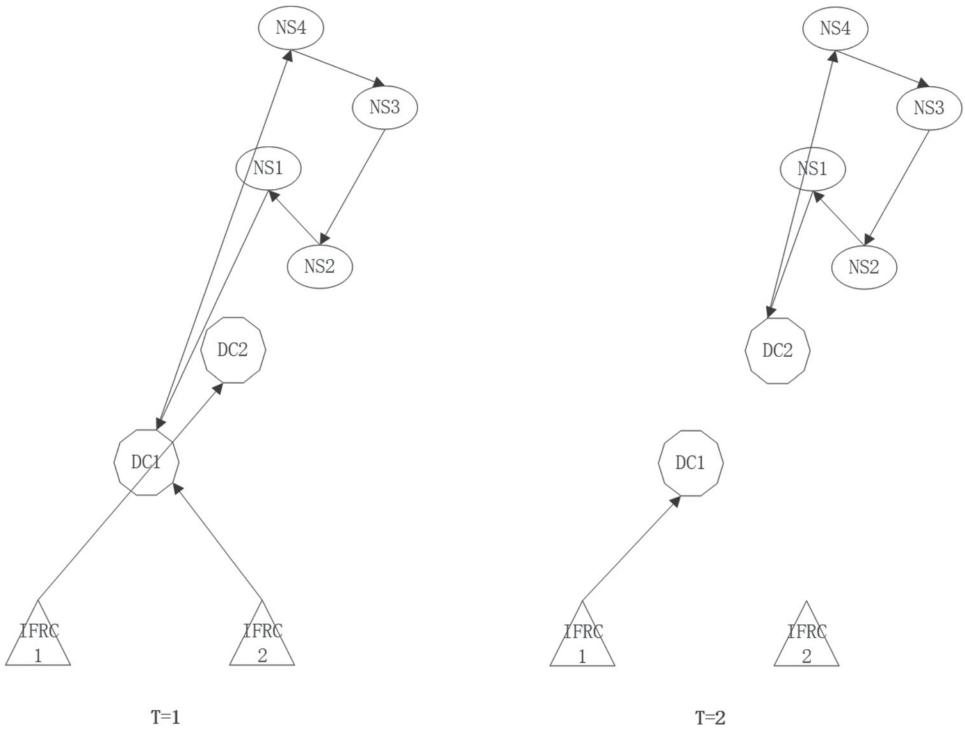


Figure 10. Material assignment and vehicle routeing under SAA model.

RIs will not be transferred within a funding level and that needs will be met. Thus, the decision maker can choose a reasonable tolerance level according to risk preference and then makes the optimal decisions.

6. Conclusions

This study presented a hierarchical decision-making model to solve the PELL problem, where the DMO aimed to reduce transportation time and the IFRC wanted to minimise supply risk. Under the assumption that the first-order moments of the random demand and transportation cost were the only known partial distribution information, a distributionally robust bi-level programming model with ambiguous chance constraints was developed. In addition, the moments were estimated by using the pivot variable method based on the available limited historical data. RCA formulations, i.e. MSCP and MLP models, were accordingly derived through optimisation theory. Furthermore, the bi-level models were converted to single-level models through KKT condition, which can be resolved directly using commercial optimisation solver.

The case experiment illustrated the applicability of the presented distributionally robust bi-level optimisation method in a situation where the distributions of stochastic parameters were ambiguous. Compared with the RO model, the advantages of using DRO were further demonstrated. The computational results demonstrated that our approach achieves better performance by using partial information about probability distributions rather than just supporting information from uncertain demand and transportation cost. In addition, the sensitivity analysis yielded managerial implications that were useful for decision makers in emergency logistics network design.

The scientific value of the article lies in the construction of ambiguity set using exponential moment information of demand and transportation costs and the application of the DRO method to solve emergency management problems. The applicability is reflected in the following aspects. First, the distributionally robust PELL model can solve the emergency management problem by providing the optimal distribution and path schemes. Second, partial information about the probability distribution of random parameters in a real-world problem can be characterised by ambiguity set. The management problem can be modelled by DRO method when only partial distribution information of the stochastic parameters is available. In addition, the key findings are summarised below.

- The proposed distributionally robust bi-level PELL model can provide managers with optimal allocation schemes as well as routeing planning schemes.
- Compared with the RO model, the proposed DRO model can reduce the conservativeness by using partial distribution information of uncertain parameters. The distributions of random parameters are difficult to obtain, and SO models typically result in computationally intractable problems. In contrast, the DRO model can be transformed into a computationally tractable system using dual theory.
- Sensitivity analysis of tolerance level shows that the higher the tolerance level, the lower the optimal transportation time. Sensitivity analysis of the ambiguity set size reveals that the larger the ambiguity set size, the more conservative the model.

Future research in this field will encompass the following topics. First, the current study utilises information about the distribution of random parameters. In future research, subjective uncertainty related to uncertain parameters will be taken into account, and robust fuzzy optimisation methods will be employed to handle this type of uncertainty (Pei, Li, and Liu 2022). Second, the current model only considers the emergency location-routeing and supply problem. Future research may expand to incorporate post-disaster ripple effects and risk reduction into the emergency relief problem.

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Disclosure statement

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Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article, further reasonable inquiries can be directed to the corresponding author.

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Appendices

Appendix 1. Proofs of Theorems 4.1–4.4

Proof of Theorem 4.1: Based on the Theorem 2.4.4 from Ben-Tal, El Ghaoui, and Nemirovski (2009), the ambiguous chance constraints

$$\mathbb{P}_{\zeta_l}\left\{CO + CP + \sum_{i,j,r,t} TC^1_{ijr}(\zeta)Y^1_{ijrt} + \sum_{j,k,r,t} TC^2_{jkr}(\zeta)X^1_{jkrt} + \sum_{m,n\in[C]\cup\{0\}}\sum_{k,v,t} TC^3(\zeta)d_{mn}Q^3_{mnkvt} + CH \le B\right\} \ge 1-\epsilon, \quad \forall\, \mathbb{P}_{\zeta_l} \in \mathscr{P}_l,$$

can be approximated by the following system.

$$TC^0 + \sum_k F_k LOC_k + \sum_{i,j,r} C_{ir}Y^1_{ijr} + \sum_{j,r,t} HR_{jr}X^3_{jrt} + \sum_{k,r,t} HD_{kr}X^4_{krt} - B = u^0 + z^0,$$

$$TC_l = u_l + z_l, \quad \forall l \in [L],$$

$$u^0 + \sum_{l\in[L]} |u_l| \le 0,$$

$$z^0 + \sum_{l\in[L]} \max[\mu_l^- z_l, \mu_l^+ z_l] + \sqrt{2\ln(1/\epsilon)}\sqrt{\sum_{l\in[L]}(\sigma_l)^2(z_l)^2} \le 0,$$

which is the robust counterpart of the uncertain inequality

$$\sum_k F_k + \sum_{i,j,r} C_{ir}Y_{ijr}^1 + \sum_{i,j,r} T_{ijr}^1(\boldsymbol{\zeta})Y_{ijr}^1 + \sum_{j,k,r} T_{jkr}^2(\boldsymbol{\zeta})X_{jkr}^1 +$$
$$\sum_{m,n\in[C]\cup\{0\}}\sum_{k,v} T^3(\boldsymbol{\zeta})d_{mn}Q_{mnkr}^3 \sum_{j,r,t} HR_{jr}X_{jrt}^3 + \sum_{k,r,t} HD_{kr}X_{krt}^4 \le B$$

with respect to the following perturbation set:

$$\mathcal{Z} = \left\{\boldsymbol{\zeta} : \exists \boldsymbol{a} : \mu_l^- \le \zeta_l - a_l \le \mu_l^+, -1 \le \zeta_l \le 1, \sqrt{\sum_{l\in[L]}(a_l)^2/(\sigma_l)^2} \le \sqrt{2\ln(1/\epsilon)}, \quad \forall\, l \in [L]\right\}.$$

The proof of Theorem 4.1 is complete. ■

Proof of Theorem 4.2: Based on the Theorem 2.4.4 from Ben-Tal, El Ghaoui, and Nemirovski (2009), the following system

$$D_{mrt}^0 - \sum_v X_{mrvt}^2 = u_{mrt}^0 + z_{mrt}^0, \quad \forall m, r, t,$$

$$D_{mrte} = u_{mrte} + z_{mrte}, \quad \forall e \in [E], m, r, t,$$

$$u_{mrt}^0 + \sum_{e\in[E]} |u_{mrte}| \le 0, \quad \forall m \in [C], r \in [R], t \in [T],$$

$$z_{mrt}^0 + \sum_{e\in[E]} \max[\mu_e^- z_{mrte}, \mu_e^+ z_{mrte}] + \sqrt{2\ln(1/\epsilon)}\sqrt{\sum_{e\in[E]}(\sigma_e)^2(z_{mrte})^2} \le 0, \quad \forall m, r, t,$$

is a safe approximation to constraints

$$\mathbb{P}_{\eta_{mrte}}\left\{\sum_v X_{mrvt}^2 \ge D_{mrt}(\boldsymbol{\eta}_{mrt})\right\} \ge 1 - \epsilon_{mrt}, \quad \forall m, r, t, \quad \forall\, \mathbb{P}_{\eta_{mrte}} \in \mathscr{P}_{mrte}.$$

Furthermore, the system is the robust counterpart of the uncertain inequality $\sum_{v\in[V]} X_{mrvt}^2 \ge D_{mrt}(\boldsymbol{\eta}_{mrt})$ with respect to the following perturbation set:

$$\mathcal{Z}_{mrt} = \left\{\boldsymbol{\eta}_{mrt} | \exists \boldsymbol{a}_{mrt} : \mu_{mrte}^- \le \eta_{mrte} - a_{mrte} \le \mu_{mrte}^+, -1 \le \eta_{mrte} \le 1, \sqrt{\sum_{e\in[E]}(a_{mrte})^2/(\sigma_{mrte})^2} \right.$$
$$\left. \le \sqrt{2\ln(1/\epsilon)}, \quad \forall\, e, m, r, t\right\},$$

which completes the proof of Theorem 4.2. ■

Proof of Theorem 4.3: Let $(\boldsymbol{Y}_{ijr}^1, \boldsymbol{X}_{jkr}^1, \boldsymbol{Q}_{mnkr}^3, \boldsymbol{\kappa}, \boldsymbol{\nu})$ be feasible for (34). Then we have

$$||\sigma\nu||_2^2 = \sum_l (\sigma_l)^2(\nu_l)^2 \le \sum_l |\sigma_l\nu_l|||\sigma\nu||_\infty = ||\sigma\nu||_\infty \sum_l |\sigma_l\nu_l| \le ||\sigma\nu||_\infty * \sqrt{L}||\sigma\nu||_2,$$

where the last $\le$ uses the properties of Cauchy inequality. Thus, $||\sigma\nu||_2 \le \sqrt{L}||\sigma\nu||_\infty$. Since $(\boldsymbol{Y}_{ijr}^1, \boldsymbol{X}_{jkr}^1, \boldsymbol{Q}_{mnkr}^3, \boldsymbol{\kappa}, \boldsymbol{\nu})$ satisfies (34), we have

$$\nu^0 + \sum_{l\in[L]} \max[\mu_l^- \nu_l, \mu_l^+ \nu_l]$$

$$+\sqrt{2\ln(1/\epsilon)}\sqrt{\sum_{l\in[L]}(\sigma_l)^2(\nu_l)^2} \le 0,$$

which combines with (34) implying that $\boldsymbol{Y}^1_{ijr}, \boldsymbol{X}^1_{jkr}, \boldsymbol{Q}^3_{mnkr}, \boldsymbol{\kappa}, \boldsymbol{\nu}$ satisfies (30) with $\sqrt{2\ln(1/\epsilon)}$. Now we can apply Theorem 4.1 to obtain the following conclusion, i.e. in the case of $\bar{P}.1$ and $\bar{P}.2$, $\boldsymbol{Y}^1_{ijr}, \boldsymbol{X}^1_{jkr}, \boldsymbol{Q}^3_{mnkr}$ satisfies

$$CO + CP + \sum_{i,k,r,t} TC^1_{ijr}(\boldsymbol{\zeta})Y^1_{ijrt} + \sum_{j,k,r,t} TC^2_{jkr}(\boldsymbol{\zeta})X^1_{jkrt} + \sum_{m,n\in[C]\cup\{0\}}\ \sum_{k,v,t} TC^3(\boldsymbol{\zeta})d_{mn}Q^3_{mnkvt} + CH \le B$$

with probability $\ge 1 - \exp\{-\ln(1/\epsilon)\}$.

Moreover, system (34) is the robust counterpart of the uncertain inequality

$$\sum_k F_k + \sum_{i,j,r} C_{ir}Y^1_{ijr} + \sum_{i,j,r} T^1_{ijr}(\boldsymbol{\zeta})Y^1_{ijr} + \sum_{j,k,r} T^2_{jkr}(\boldsymbol{\zeta})X^1_{jkr} +$$
$$\sum_{m,n\in[C]\cup\{0\}}\ \sum_{k,v} T^3(\boldsymbol{\zeta})d_{mn}Q^3_{mnkr}\sum_{j,r,t} HR_{jr}X^3_{jrt} + \sum_{k,r,t} HD_{kr}X^4_{krt} \le B$$

with respect to the following perturbation set:

$$\hat{\mathcal{Z}} = \left\{\boldsymbol{\zeta}\,|\exists \boldsymbol{a} : \mu^-_l \le \zeta_l - a_l \le \mu^+_l, -1 \le \zeta_l \le 1, \sum_{l\in[L]} |a_l/\sigma_l| \le \sqrt{L}\sqrt{2\ln(1/\epsilon)}, \quad \forall\, l \in [L]\right\}.$$

The proof of Theorem 4.3 is complete. ■

Proof of Theorem 4.4: Let $(\boldsymbol{X}^2_{mrvt}, \boldsymbol{\kappa}_{mrt}, \boldsymbol{\nu}_{mrt})$ be feasible for (36). Then we have

$$||\sigma_{mrt}\nu_{mrt}||_2^2 = \sum_e (\sigma_{mrte})^2(\nu_{mrte})^2 \le \sum_e |\sigma_{mrte}\nu_{mrte}|\,||\sigma_{mrt}\nu_{mrt}||_\infty$$
$$= ||\sigma_{mrt}\nu_{mrt}||_\infty \sum_e |\sigma_{mrte}\nu_{mrte}| \le ||\sigma_{mrt}\nu_{mrt}||_\infty * \sqrt{E}||\sigma_{mrt}\nu_{mrt}||_2,$$

where the last $\le$ is determined by the Cauchy inequality. Thus, $||\sigma_{mrt}\nu_{mrt}||_2 \le \sqrt{E}||\sigma_{mrt}\nu_{mrt}||_\infty$. Since $(\boldsymbol{X}^2_{mrvt}, \boldsymbol{\kappa}_{mrt}, \boldsymbol{\nu}_{mrt})$ satisfy (36), we have the following implication,

$$\nu^0_{mrt} + \sum_{e\in[E]} \max[\mu^-_{mrte}\nu_{mrte}, \mu^+_{mrte}\nu_{mrte}]$$
$$+\sqrt{2\ln(1/\epsilon)}\sqrt{\sum_{e\in[E]}(\sigma_{mrte})^2(\nu_{mrte})^2} \le 0, \quad \forall\, m \in [C],\ r \in [R],\ t \in [T],$$

which combines with (36) implying that $\boldsymbol{X}^2_{mrvt}, \boldsymbol{\kappa}_{mrt}, \boldsymbol{\nu}_{mrt}$ satisfy (32) with $\sqrt{2\ln(1/\epsilon)}$. By applying Theorem 4.2, we can obtain the conclusion below, i.e. in the case of $P.1$ and $P.2$, $\boldsymbol{X}^2_{mrvt}$ satisfy

$$\sum_v X^2_{mrvt} \ge D_{mrt}(\boldsymbol{\eta}_{mrt}), \quad \forall\, m, r, t$$

with probability $\ge 1 - \exp\{-\ln(1/\epsilon)\}$.

Furthermore, the system (36) is the robust counterpart of the uncertain inequality $\sum_{v\in[V]} X^2_{mrvt} \ge D_{mrt}(\boldsymbol{\eta}_{mrt})$ with respect to the following perturbation set:

$$\hat{\mathcal{Z}}_{mrt} = \Big\{\boldsymbol{\eta}_{mrt}|\exists \boldsymbol{a}_{mrt} : \mu^-_{mrte} \le \eta_{mrte} - a_{mrte} \le \mu^+_{mrte}, -1 \le \eta_{mrte}$$
$$\le 1, \sum_{e\in[E]} |a_{mrte}/\sigma_{mrte}| \le \sqrt{E}\sqrt{2\ln(1/\epsilon)}, \forall e, m, r, t\Big\},$$

which completes the proof of Theorem 4.4. ■

Appendix 2. The process of AHP

The process of AHP includes the following four steps:

Step 1: The relationship among various factors is analysed and the recursive hierarchy of the system is established. As shown in Figure A1, the problem needs to be decided at three levels, with the top level being the objective O, selecting the right supplier, the bottom level being the plan P, i.e. three suppliers S1, S2, S3, and the middle level being the criterion C, including four indicators of quality, delivery time, supply flexibility, and reputation.

Step 2: A two-by-two comparison of the significance of every element of the same layer relative to the criteria of the previous layer is performed and a two-by-two judgment matrix (JM) is constructed. The data in the JM generally adopts the nine-point scale method, which is detailed in Table A1. The JM O-C between the target level and the criterion level is shown in Table A1, and the JM C-P between the criterion level and the plan level is presented in Table A2. The data in the JM generally adopts the nine-point scale method.

Step 3: Based on the JM, we calculate the relative weights of the elements being compared with respect to that criterion and perform a consistency test. The consistency index (CI) is first calculated,

$$CI = \frac{\lambda_{\max} - n}{n - 1},$$

where $\lambda_{\max}$ is the maximum eigenvalue of the JM. Eigenvalues can be calculated by Matlab software. We can find the corresponding average random CI. Finally, the consistency ratio (CR) is calculated,

$$\mathrm{CR} = \frac{\mathrm{CI}}{\text{Random CI}}.$$

If CR ¡ 0.1, the consistency of the JM can be considered acceptable; otherwise, the consistency of the JM needs to be corrected.

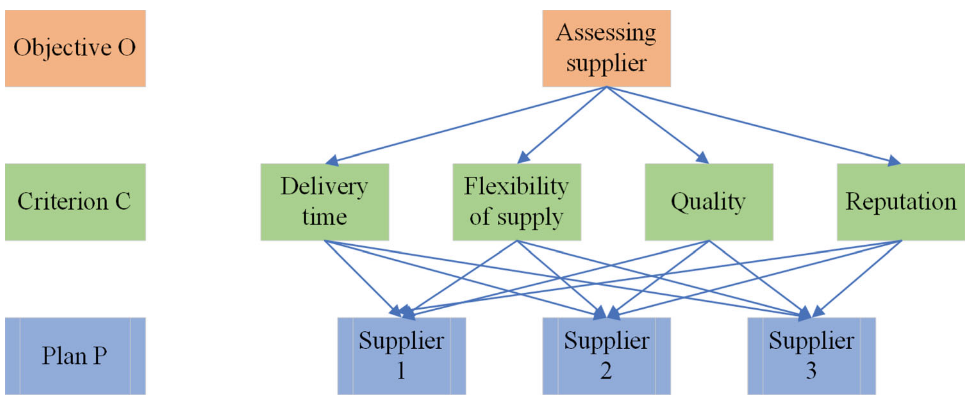

Figure A1. Hierarchy chart for evaluating suppliers.

Table A1. The nine-point scale and its definition.

Scale b_{ij}	The definitions
1	Indicates that factor *i* is equally important compared to factor *j*;
3	Indicates that factor *i* is slightly more important than factor *j*;
5	Indicates that factor *i* is significantly more important than *j* compared to factor *j*;
7	Indicates that factor *i* is more strongly important than *j* compared to factor *j*;
9	Indicates that factor *i* is extremely more important than *j* compared to factor *j*;
2,4,6,8	The scale value of the importance of factor *i* compared to factor *j* is between the above two adjacent levels;
Inverse of scale value	Inverse comparison of factor *i* with factor *j*: $b_{ji} = 1/b_{ij}$.

Table A1. Judgement matrix O-C.

O	C1	C2	C3	C4
C1	1	3	6	7
C2	1/3	1	3	6
C3	1/6	1/3	1	3
C4	1/7	1/6	1/3	1

Table A2. Judgement matrix C-P.

C1	S1	S2	C2	S1	S2
S1	1	3	S1	1	1/4
S2	1/3	1	S1	4	1
C3	S1	S2	C4	S1	S2
S1	1	2	S1	1	3
S2	1/2	1	S2	1/3	1

Table A3. Weight matrix.

	Indicator weight	S1	S2
C1	0.5758	0.75	0.25
C2	0.2641	0.2	0.8
C3	0.1083	0.6667	0.3333
C4	0.0518	0.75	0.25

Step 4: The score based on Table A3 is calculated and ranked. First, the maximum eigenvalue of the consistency matrix and its eigenvector are found. Second, the eigenvectors are normalised to obtain the weights.

Appendix 3. The RO model used in model comparison

$$
\begin{aligned}
\min\quad & (3)\\
\text{s.t.}\quad & \sum_{k} F_k LOC_k + \sum_{i,j,r} C_{ir} Y^1_{ijr} + \sum_{j,r,t} HR_{jr} X^3_{jrt} + \sum_{k,r,t} HD_{kr} X^4_{krt} + \sum_{i,j,r} TC^1_{ijr}(\zeta) Y^1_{ijr}\\
& + \sum_{j,k,r} TC^2_{jkr}(\zeta) X^1_{jkr} + \sum_{m,n\in[C]\cup\{0\}} \sum_{k,v} TC^3(\zeta) d_{mn} Q^3_{mnkr} \le B, \quad \forall\, \zeta \in [-1,1]^L,\\
& D_{mrt}(\eta_{mrt}) \le \sum_{v} X^2_{mrvt}, \quad \forall\, m,r,t, \quad \forall\, \eta_{mrt} \in [-1,1]^E,\\
& \gamma^1_{ijrt}, \gamma^2_{jrt}, \gamma^3, \gamma^4_{irt}, \gamma^5_{jt} \in \{0,1\},\\
& (4)-(20),\ (24)-(28),\ (39)-(41),\ (49)-(58).
\end{aligned}
$$