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Designing a resilient supply chain network under ambiguous information and disruption risk

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ABSTRACT

In case of supply chain disruption following severe disasters, many supply chains tend to collapse and take a long time to recover. Resilient supply chain network design (RSCND) is an important research problem in supply chain management, which means that the supply chain can maintain continuous supply and quickly restore the supply capability in part destruction. Based on the limited distribution information of uncertain demand, a two-stage distributionally robust optimization (DRO) model with ambiguous chance constraint (ACC) is proposed to solve the RSCND problem under demand uncertainty and disruption scenario to provide decision support for planning the supply chain network. Finally, to verify the effectiveness and practicability of the proposed DRO model, we apply the method to a real case study in Wuhan, China, about designing a resilient RSC network to withstand disruption. By comparison and sensitivity analysis in numerical experiments, some management insights of industry decision-makers are obtained.

1. Introduction

With the frequent occurrence of global destructive emergencies, the supply chain network faces more and more risks in production operations. Supply chain network design is an important part of supply chain management, and resilience is a key element of the supply chain network, which is the ability to protect the supply chain and quickly recover from the adverse effects of disruption events (Sazvar et al., 2021). Disruptions caused by disasters, while relatively rare, can have devastating long-term consequences, and the recovery process may not be quick. In recent years, since COVID-19 has caused severe impacts on the supply chains around the world, it is necessary to optimize the operation of the manufacturing industry supply chain network timely. Today, the retail supply chain (RSC) management industry is facing more financial expenses and challenges than ever before Drofenik et al. (2023). For example, cooperation between European and some Middle Eastern countries ceased in 2020 during COVID-19, and the companies face challenges in ensuring sustainability across the supply chain Yılmaz et al. (2021). Then, from a RSC perspective, single or more stages of a RSC are greatly influenced by its design complexity and the difficulty of ripple effects (Ivanov and Keskin, 2023).

The supply chain disruption problem that appeared in this epidemic exposes the short board of enterprise's resilient supply chain construction, which also reveals the shortcomings of existing research on the RSCND problem (Sawik, 2022). A challenging issue in supply chain disruption is the uncertainty of product demand.

In most traditional related decision-making environments, the important parameters in the supply chain network, such as the demands of retailers were considered to be deterministic (De and Giri, 2020). However, in actual decision-making process, the demand of some supply chains is uncertain and changes with the severity of the disruption scenario. Especially in the environment of supply chain disruption, the demand for daily necessities is also uncertain due to the panic of residents caused by disasters. Accordingly, research on the RSCND based on demand uncertainty and disruption scenario has significant theoretical importance and practical implications, and it is becoming a popular issue in modern operation management. The alleged uncertainty refers to those uncertain parameters that are difficult to be described with probability or frequency, especially in the case of rare data or the absence of data. Even based on big data, the true distribution function of demand may not be known. This uncertainty poses many challenges to the establishment of the model and the search for the optimal solution for the supply chain network design problem. As a result, RSC needs effective optimization techniques to improve its performance.

In the real world, decision-makers may face a worse scenario, that is, the actual probability distribution of uncertain parameters is unknown, or at most can only obtain partial information about the probability distribution. This may lead to a serious inaccurate solution for decision-maker. In addition, facility or transportation disruptions,

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especially some caused by natural disasters, are highly impossible or difficult to predict. For this reason, the pre-disaster phase and post-disaster phase will be considered separately. Therefore, we adopt a two-stage DRO method to model the problem. The brief innovations of this study that distinguish our paper from the existing literature are as follows:

- Considering the uncertainty of product demand under different disruption scenarios.
- Designing a resilient RSC network under two types of uncertainty by a two-stage DRO method.
- The proposed two-stage DRO model with ACC is transformed into MISOCP and MILP, respectively.
- Studying a real case via the new resilient RSC network design method.

To facilitate the understanding of this study, the remainder of this article is shown as follows. Section 2 is dedicated to the literature review, and introduces the contributions of the article. Section 3 describes the demand uncertainty with an ambiguity set and proposes a two-stage distributionally robust RSCND model. Section 4 derives the safe approximations of ACC in the proposed model. In Section 5, a practical application is introduced, and the effectiveness of our optimization method is illustrated via experimental results. Section 6 presents some managerial observations. The concluding section discusses the theoretical and practical significance of the research results.

2. Literature review

This study takes three independent but complementary flows of literature including demand uncertainty, the DRO method, and managing the supply chain under disruptions. Then, we provide an overview of the literature in these flows.

2.1. Demand uncertainty

In the pre-disaster phase and the post-disaster phase of the supply chain, many uncertain factors directly affect the designing of a resilient supply chain network. The most common uncertainty factor in supply chain research is uncertain demand Tabandeh et al. (2022). In the pre-disaster phase, Alizadeh and Karimi (2023) proposed a bi-objective mixed-integer linear model, which took the resilience measures as the optimization tool, used the adjustable possibilistic programming, chance-constrained programming, scenario-based programming, and p-robust optimization method to deal with the uncertainty before the disruption. Clavijo-Buritica et al. (2022) studied the sustainability and resilience of the agri-food supply chain, used the simulation of destructive events and the mathematical programming to find resilient designs to solve problems, the application of the framework in the Colombian coffee supply chain was assessed. Ni et al. (2022) studied a systematic supply chain resilience evaluation method to deal with the uncertainty of demand, production and inventory, presented different scenarios in the pre-disaster phase and put forward inventory strategy to build model. In the post-disaster phase, Salehi et al. (2022) proposed a MILP model to design a resilient and sustainable biomass supply chain network and used robust possibility programming methods to deal with the bioenergy demand uncertainty. Alikhani et al. (2021) depicted demand uncertainty in supply chain networks through different scenarios and used scenario simplification methods to simulate the original uncertainty distribution. Jalal et al. (2023) studied the location-transportation problem under demand uncertainty, and used robust counterpart method to solve it, and proposed a solution method based on repair and optimization heuristics. Other uncertainties, such as variable costs, supply, and transportation times have also been considered in some literature. The relevant data of the uncertain parameters in the literature is directly given (Tabandeh et al., 2022), or some professional and different company questionnaires (Sturm et al., 2023) and distribution information obtained through different methods is extracted from official websites. In the RSCND problem, there are few literature that directly uses real data for research and verification.

Therefore, the research gap prompted us to conduct further study on the RSCND problem based on demand uncertainty and disruption scenario. This paper is different from the existing literature by using real data, which makes our method more appropriate to solve practical decision-making problems.

2.2. DRO method

Due to the demand uncertainty in the post-disaster phase, the structure and inventory allocation of RSC networks may be affected. The main challenge in addressing RSCND problems is how to deal with the inherent uncertainties.

Traditionally, the stochastic optimization method is a common method to deal with uncertainty, which requires accurate knowledge of the probability distribution of uncertain parameters (Gabrel et al., 2014). Yılmaz et al. (2021) proposed a mixed integer stochastic optimization model to design resilient RSC networks in the presence of ripple effects. Ahmadvand and Sowlati (2022) presented uncertainties in energy demand, used robust optimization methods to develop tactical supply chain optimization models for forest Gasification. Mandal et al. (2020) established a vehicle speed optimization model that minimized supply chain cost under uncertain demand through stochastic optimization method. However, the stochastic optimization method does not apply to all RSCND problems. Decision-makers are unlikely to predict the exact post-disaster conditions, which is very difficult for managers to get precise distribution functions. In recent years, great progress has been made in the study of DRO (Gong and You, 2017). Compared with traditional robust optimization methods, the DRO method has a less conservative solution (Jiang and Guan, 2015). Due to the limited information on the known probability distribution, now research literature mainly aims to provide better approximations or algorithms to deal with uncertainty. In addition, due to its computational tractability and the advantages described above, the DRO method has played an important role in various fields, such as portfolio optimization (Aldrighetti et al., 2023), hybrid vehicle routing problem (Yin and Zhao, 2022), lane reservation problem (Han et al., 2022), capacity sizing problem (Xie et al., 2023), multi-item inventory allocation problem (Ren and Bidkhori, 2022) and location and sizing problem (Yuan et al., 2023). Yin and Zhao (2022) proposed a DRO method that utilized the central limit theorem to build ambiguity sets to solve nonlinear hybrid vehicle routing problems. Xie et al. (2023) considered the capacity scale issue during the transition to a low-carbon power system. A DRO method based on the Wasserstein metric is proposed to capture uncertain renewable energy output.

At present, there is no research on the two-stage DRO model with ACC to the RSCND problem based on demand uncertainty and disruption scenario. In this paper, we propose a two-stage DRO model with ACC for a resilient RSC network problem. The aim is to optimize the total expected cost in the pre-disaster stage and post-disaster stage. The corresponding optimal RSC network satisfies ACC. That is to say, it satisfies the probability constraint under the worst-case scenario of probability distribution.

2.3. Managing supply chain under disruptions

In recent years, researchers have increasingly focused on supply chain disruption risk and operational risk (Sazvar et al., 2021). In addition to the enormous loss of life and economy, how to absorb the impact of supply chain disruption and restore the inherent ability after the damage has become an important research topic (Abimbola and Khan, 2019). Regarding the RSCND problem, scholars have carried out relevant research work from different perspectives. Foroozesh et al. (2022) designed a resilient and green supply chain network under epistemic uncertainties and disruption risks, their model used a variety of resilience strategies, including capacity utilization strategy, multiple sourcing strategy, coverage strategy and path strategy to respond quickly to disruption events and reduce impacts in the food supply chain. Yavari and Zaker (2020) adopted four resilience strategies to deal with supply chain disruption, and in order to minimize the expected total network cost and expected total carbon emissions, a resilient green closed-loop supply chain network is designed. Namdar et al. (2017) used the purchasing strategy to realize supply chain resilience in the case of disruption. The strategies used include backup suppliers, single and multiple sourcing, on-site purchasing, and so on.

As the managing supply chain under disruptions has recently become a hot topic, the disruption of the COVID-19 pandemic has also emerged as a new challenge, different from any previously seen challenges (Singh et al., 2021). In the past, the disasters that caused supply chain disruption usually had the characteristics of great damage, short duration, and low recurrence rate (Lim et al., 2013), such as tornadoes, earthquakes, extreme weather (Ni et al., 2022) etc. However, the epidemic characteristics of the COVID-19 pandemic were great damage (Cordeiro et al., 2022), long duration, and high recurrence rate (Gkiotsalitis and Cats, 2020).

In this paper, we consider the risk of disruption caused by a disaster with the same or similar characteristics as the COVID-19 pandemic when designing RSC networks under disruption scenario, while adding a safe inventory strategy and reserved capacity strategy, which is different significantly from previous studies in this point.

2.4. Research gaps and contributions

Taking into account the research of the above literature, a summary of the studies on modeling method, uncertain parameters, and solution approach is provided in Table 1. Based on this, we summarize the following three research gaps:

The first is related to the uncertainty sets. Despite more attention being paid to the research of parameter uncertainty, most are based on exact distribution. Besides, the existing literature always makes simplified assumptions for uncertainty. Secondly, about the research on the resilient supply chain, uncertainty, and DRO method, until now, few references have used the DRO method for the RSCND problem under demand uncertainty and disruption scenario. Thirdly, from the discussion above, the impact of disasters on facilities is one of the most significant problems in most countries. To be more realistic, generating and solving these problems in new cases are more beneficial studies. So far, very few papers might have used real cases to study the RSCND problem based on demand uncertainty and disruption scenario.

Therefore, this paper aims to build a two-stage DRO method to study the decision of facility level construction, inventory allocation, transportation planning, and other decisions under multiple resilience strategies to the supply chain disruption. In general, the contributions of this paper, compared to its peers, are described as:

From the theoretical perspective, this research first thoroughly expounds the reasons that supply chain resilience and demand uncertainty need to be reflected comprehensively in RSCND. From this, we propose a new two-stage DRO model with ACC, which can simultaneously deal with distribution uncertainty of demand and improve supply chain resilience. The robust counterpart approximation of the DRO model is a semi-infinite programming model belonging to the family of hard optimization problems. We transform the robust counterpart approximation of ACC into computationally tractable forms under Budget and Box-Ball perturbation sets. And then the proposed model can be transformed into a deterministic MILP model or a deterministic MISOCP model that can be solved with commercial software, respectively.

From the practical implementation perspective, we address a real case of RSC network design in Wuhan to verify the effectiveness of our model. The calculation results show that the DRO method is not only feasible, but also effective to resist the uncertainty of probability distribution.

Finally, this paper is the first one that adopts a two-stage DRO method with ACC to solve the RSCND problem based on demand uncertainty and disruption scenario. We investigate the impact of several key parameters on the optimal cost, transportation planning, and facility level construction decisions in the resilient supply chain.

3. Problem description and model formulation

3.1. RSCND problem description

Since the twenty-first century, major public health emergencies such as SARS, Ebola, and COVID-19 have erupted frequently in the world, which has brought great challenges to the supply chain (Raj et al., 2022). Especially, COVID-19 has exposed the fragility of RSC systems to a certain extent, which threatens the supply of daily necessities (Duan et al., 2020). The closure of production during the Spring Festival, the shortage of raw materials, and the increase in isolation protection demand resulted in the shortage of daily necessities[1]. In the early stage of major public health emergencies, a sufficient supply of daily necessities is particularly important for epidemic prevention and control (Shafiee et al., 2022). We need to build a complete and reliable RSC network to deal with the above disruptions. In contrast with the only slight fluctuations in consumer demand for products before the outbreak, in the early stage of the outbreak, there exist sudden great changes in demand since consumers tend to hoard several daily necessities. Since resilient supply chain network is durable inherent, ignoring the influence of decision environment uncertainty that can be reflected in some parameters may seriously affect the network design decision.

We consider the RSCND problem related to the pre-disaster and post-disaster stages. This multi-level RSC network includes suppliers, distribution centers (DC), and retail stores that provide products to customers in different regions, which is illustrated in Fig. 1. Notably, the choice of suppliers, opening of major DCs and retailers, and opening a pathway are the decisions that have been considered in the pre-disaster stage. After the disruption scenario occurs, several decision variables are determined, including the path of the vehicle and the location–allocation of facility. Also, the path and quantity of trucks transported products will be determined so that each customer's need is met.

3.2. Notations

In this section, we introduce the notations which will be used to build our RSCND model.

Sets:

Q Candidate supplier, $Q = \{q|q = 1, 2, \ldots, |Q|\}$.

W Candidate DC, $W = \{w|w = 1, 2, \ldots, |W|\}$.

E Candidate retail store, $E = \{e|e = 1, 2, \ldots, |E|\}$.

F Customer area, $F = \{f|f = 1, 2, \ldots, |F|\}$.

P Product type, $P = \{p|p = 1, 2, \ldots, |P|\}$.

X Facility level, $X = \{x|x = 1, 2, \ldots, |X|\}$.

S Post-disaster disruption scenario, $S = \{s|s = 1, 2, \ldots, |S|\}$.

[1] https://baijiahao.baidu.com/s?id=1657444920714618053&wfr=spider&for=pc

Table 1
Literature review of relevant works.

Reference	Modeling method		Uncertain parameters and variables			Solution approach		Disruption	Comparison between
	DRO	Stochastic	Supply	Demand	Inventory	CPLEX	Other	concept/model	solution methods
Schmitt and Singh (2012)			✓	✓			✓		
Lim et al. (2013)			✓						
Jabbarzadeh et al. (2016)		✓	✓	✓					
Rezapour et al. (2017)						✓			
Fattahi et al. (2017)				✓		✓			
Namdar et al. (2017)		✓	✓	✓		✓		✓	
Cavalcante et al. (2019)		✓	✓				✓	✓	
Mandal et al. (2020)				✓			✓		✓
Singh et al. (2021)				✓			✓	✓	
Hajiagha et al. (2021)	✓	✓	✓				✓	✓	
Salehi et al. (2022)	✓			✓				✓	
Feng et al. (2022)	✓	✓		✓		✓			✓
Fan and Xie (2022)	✓	✓		✓				✓	✓
This study	✓	✓	✓	✓	✓	✓		✓	✓

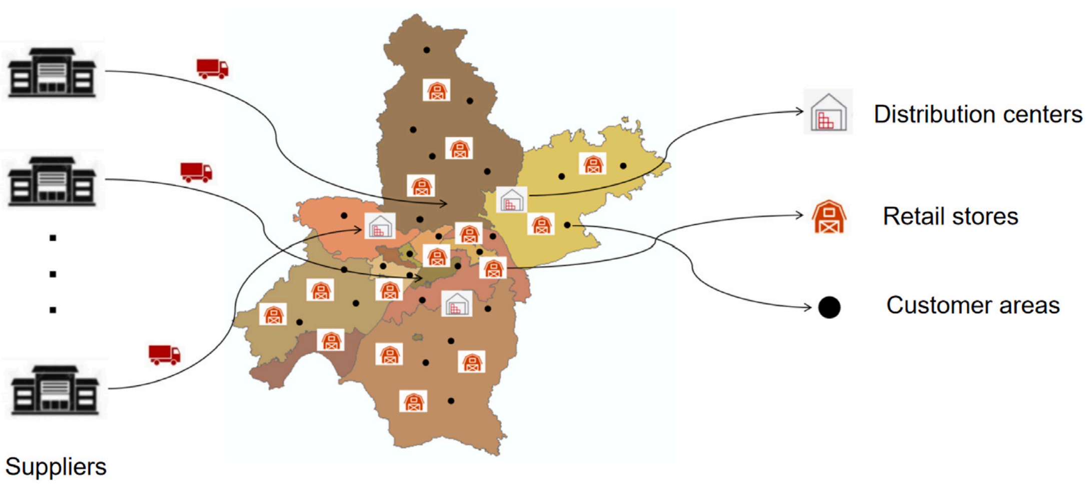


Fig. 1. The network structure.

Arc Transport pathway between node i and node j, $Arc = \{(i,j)|(i \in (Q \cup W \cup E), j \in (W \cup E \cup F))\}$.

Decision variables:

yq_q^x Binary variables, 1, if supplier q with level x is chosen; and 0, otherwise.

yw_w^x Binary variables, 1, if DC w with level x is chosen; and 0, otherwise.

ye_e^x Binary variables, 1, if retail store e with level x is chosen; and 0, otherwise.

lin_{ij}^p Binary variables, 1, if node i is connected with node j for product type p; and 0, otherwise.

t_{ij}^{ps} The quantity of products type p transported via arc (i,j) under disruption scenario s.

inv_w^p The quantity of inventory product type p in DC w.

inv'^{ps}_w The quantity of unused inventory product type p in DC w under disruption scenario s.

$\widehat{inv'}^{ps}_w$ The quantity of unused inventory product type p in DC w under disruption scenario s for colleague RSC.

ϱ_e^{ps} Unused capacity for product type p under disruption scenario s in node e.

sh_f^{ps} Shortage quantity, represents the quantity of unmet demand for product type p in each customer area f under disruption scenario s.

$\widehat{sh}_f^{ps}$ Shortage quantity, represents the quantity of unmet demand for product type p in each customer area f from colleague RSC under disruption scenario s.

Tp_w^{ps} Transshipment quantity, represents the quantity of additional product type p purchased from colleague RSC's DC w under disruption scenario s, when the quantity of product type p transported in the supply chain is insufficient to support customer demand.

$\widehat{Tp}_w^{ps}$ Transshipment quantity, represents the quantity of additional product type p purchased from our RSC's DC w when the supplier's supply capacity of colleague RSC under disruption scenario s is insufficient.

Deterministic parameters:

cq_q^x Fixed cost of the selected supplier node q with facility level x.

cw_w^x Fixed cost of the selected DC w with facility level x.

ce_e^x Fixed cost of the selected retail store e with facility level x.

arc_{ij} Fixed cost of the selected path between node i and node j, $i \in (Q \cup W \cup E), j \in (W \cup E \cup F)$.

cap_w^p Capacity of DC w for product type p.

m_w^p Unit purchasing cost of product type p in DC w.

n_j^p Minimum cover number of node j for product p.

T_{ij}^p Maximum quantity of shipped product type p in arc (i,j).

tr_{ij}^p Cost of transported unit product type p in the arc (i,j).

g_w^p Inventory cost of holding unit product type p in DC w.

h_w^p Inventory cost of processing unit product type p in DC w.

pef_f^p Penalty cost of unmet demand for product type p in customer area f.

pee_e^p Penalty cost of idle capacity in node e for product type p.

$\mathfrak{S}_w^p$ Purchasing cost of unit product type p between retailers in DC w.

sal_w^p Salvage value of an unused product type p in DC w.

mq'^p_q Purchasing cost of unit product type p in supplier q in post-disaster phase.

mw'^p_w Purchasing cost of unit product type p in DC w in post-disaster phase.

M Proportion parameter, represents the proportion relationship between the product quantity flowing into and out of DC. (It measures the multiple that the product quantity flowing into DC should be greater than the product quantity flowing out of DC on the basis of the balance of the inflow and outflow at DC via constraint (6).)

ϖ Proportion parameter, represents the proportion relationship between the quantity of shortage and the transshipment quantity of product from colleague RSC under disruption scenario s. (When the transported products are not sufficient to support customer demand, the products purchased from the colleague RSC should be greater than a certain proportion ϖ of the shortage quantity.)

$\hbar$ Proportion parameter, represents the proportion relationship between the product quantity delivered by suppliers and the variation of inventory product quantity. (It can be adjusted according to the preferences of different enterprises in the supply chain, for example, increasing the transportation quantity of supplier or consuming the existing inventory as soon as possible.)

ρ^s The probability that disruption scenario s happens.

Uncertain parameters:

cq'^{xs}_q Destruction cost in supplier q with facility level x under disruption scenario s.

cw'^{xs}_w Destruction cost in DC w with facility level x under disruption scenario s.

ce'^{xs}_e Destruction cost of retail store e with facility level x under disruption scenario s.

d_f^{ps} The demand in customer area f for product type p under disruption scenario s.

Ξ_{ij}^{ps} Maximum quantity of shipped product type p on arc (i,j) under disruption scenario s.

$\eth^{ps}$ The actual sale quantity of product type p in RSC under disruption scenario s.

cap'^{pxs}_i Remaining capacity in node i with facility level x of product type p under disruption scenario s.

3.3. A two-stage DRO model

3.3.1. Post-disaster stage

Constraints

Constraint (1) confirms that the transported (i.e. purchased) products and the required shortage meet the actual demand for each customer area f under scenario s.

$$\sum_{(e,f)\in Arc} t_{ef}^{ps} + sh_f^{ps} \ge d_f^{ps}, \qquad \forall p \in P, s\in S, f\in F. \tag{1}$$

Under scenario s, due to the demand uncertainty, constraint (1) does not always hold with probability 1. Let

$$d_f^{ps} = d_f^0 + \zeta_f \widetilde{d_f^{ps}},$$

where d_f^0 is nominal value of demand, $\widetilde{d_f^{ps}}$ symbolizes basic shift, and $\zeta = [\zeta_1, \zeta_2, \dots, \zeta_F]$ is random variable.

In general, it is difficult for decision-makers to get an accurate probability distribution function of uncertain demand through historical data, that is, the distribution function is not precisely known, it belongs to an ambiguity set that satisfies certain conditions. This section assumes that only the following distribution information of ζ is obtained: the support set, the mean value, and the components of ζ are independent of each other. Based on the above partial distribution information of ζ, the following ambiguity set is constructed:

$$\begin{aligned}\mathscr{A} = \{&A \mid \zeta \sim A, E[\zeta_j]=0 \ \&\ |\zeta_j| \le 1,\\ &j=1,\dots,F \ \&\ \{\zeta_j\}_{j=1}^F \mathit{are\ independent\ random\ variables}\}.\end{aligned} \tag{2}$$

The ambiguity set includes some known distribution information: supported on $[-1,1]$, the mean value is zero, and the random variables are independent of each other.

Then, the ACC related to demand takes the following form:

$$\begin{aligned} inf_{A\in\mathscr{A}} Prob_{\zeta_f\sim A}\{ \sum_{(e,f)\in Arc} t_{ef}^{ps} + sh_f^{ps} \ge d_f^{ps} = d_f^0 + \zeta_f \widetilde{d_f^{ps}} \} \ge 1-\epsilon, \\ \forall p\in P, f\in F, s\in S. \end{aligned} \tag{3}$$

Constraint (3) ensures that, under scenario s, the required daily necessities from a retailer node to the consumer area should meet the demand of consumers with a certain probability $1-\epsilon$ for any distribution A. That is to say, the worst-case probability is at least $1-\epsilon \in (0,1)$.

Constraints (4)–(6) control service levels in the post-disaster stage. Constraint (4) indicates that the total sales are the sum of the inventory that has been used and the quantity of products issued by the supplier.

$$\sum_{w\in W} inv_w^p - \sum_{w\in W} inv'^{ps}_w + \sum_{(q,j)\in Arc} t_{qj}^{ps} = \eth^{ps}, \qquad \forall p\in P, s\in S, j\in (W\cup E). \tag{4}$$

Constraint (5) controls the proportion between the quantity of remaining inventory and the quantity of transported products from the supplier in the decision-making process. The parameter $\hbar$ is used to reflect the proportion relationship between the quantity of the remaining inventory and the quantity of the transported products from the supplier.

$$\sum_{(q,j)\in Arc} t_{qj}^{ps} \ge \hbar \sum_{w\in W} (inv_w^p - inv'^{ps}_w), \qquad \forall p\in P, s\in S, j\in (W\cup E). \tag{5}$$

Constraint (6) represents the proportion relationship between the quantity of products that are transported into the DC and the quantity of products that are shipped out of the DC. The proportion parameter M makes it easy for decision-makers to control the quantity of products stored in DC. This proportion parameter can measure the multiple that the quantity of products entering the DC should be larger than the quantity of products flowing out of the DC.

$$\sum_{q\in Q} t_{qw}^{ps} \ge M \sum_{e\in E} t_{we}^{ps}, \qquad \forall p\in P, s\in S, w\in W. \tag{6}$$

Constraint (7) is the flow balance constraint, which ensures the inflow and outflow balance of DC.

$$\sum_{(q,w)\in Arc} t_{qw}^{ps}+inv_{w}^{p}-inv'^{ps}_{w}+Tp_{w}^{ps}=\sum_{(w,e)\in Arc} t_{we}^{ps}, \qquad \forall w\in W, s\in S, p\in P. \tag{7}$$

Constraint (8) is capacity limit constraint, which means that the quantity of transported products cannot exceed the capacity of the corresponding node itself.

$$\sum_{(w,e)\in Arc} t_{we}^{ps}\le \sum_{x\in X} cap'^{pxs}_{w} yw_{w}^{x}, \qquad \forall w\in W, s\in S, p\in P, e\in E. \tag{8}$$

It is similar to constraints (7)–(8), we have the following flow balance constraint (9) for each retailer store and capacity limit constraints (10)–(12) for other links.

$$\sum_{(i,e)\in Arc} t_{ie}^{ps}=\sum_{(e,f)\in Arc} t_{ef}^{ps}, \qquad \forall p\in P, s\in S, e\in E, i\in (Q\cup W). \tag{9}$$

$$\sum_{(q,j)\in Arc} t_{qj}^{ps}\le \sum_{x\in X} cap'^{pxs}_{q} yq_{q}^{x}, \qquad \forall p\in P, s\in S, q\in Q, j\in (W\cup E). \tag{10}$$

$$\sum_{(e,f)\in Arc} t_{ef}^{ps}\le \sum_{x\in X} cap'^{pxs}_{e} ye_{e}^{x}, \qquad \forall p\in P, s\in S, f\in F, e\in E. \tag{11}$$

$$\sum_{(i,e)\in Arc} t_{ie}^{ps}+\varrho_{e}^{ps}\le \sum_{x\in X} cap'^{pxs}_{e} ye_{e}^{x}, \qquad \forall p\in P, s\in S, e\in E, i\in (Q\cup W). \tag{12}$$

Supply chain transportation commonsensible constraints are required. They include three types: transportation quantity cannot exceed the maximum limit, DC transshipment quantity are not more than the purchase quantity, the relationship between the quantity of shortage and the quantity of transported products, and so on. The proportion parameter ϖ is used to control the quantity of products purchased from the colleague RSC, which measures the product shortage proportion that the decision-maker can accept to reduce the high cost of products from the colleague RSC.

$$t_{ij}^{ps}\le lin_{ij}^{p} T_{ij}^{p}, \qquad \forall p\in P, s\in S, i\in (Q\cup W\cup E), j\in (W\cup E\cup F). \tag{13}$$

$$t_{ij}^{ps}\le \Xi_{ij}^{ps}, \qquad \forall p\in P, s\in S, (i,j)\in Arc. \tag{14}$$

$$Tp_{w}^{ps}\le inv'^{ps}_{w}, \qquad \forall w\in W, s\in S, p\in P. \tag{15}$$

$$\widehat{Tp}_{w}^{ps}\le \widehat{inv'}^{ps}_{w}, \qquad \forall w\in W, s\in S, p\in P. \tag{16}$$

$$\sum_{w\in W} Tp_{w}^{ps}\le \sum_{f\in F} \widehat{sh}_{f}^{ps}, \qquad \forall s\in S, p\in P. \tag{17}$$

$$\sum_{w\in W} Tp_{w}^{ps}\ge \varpi \sum_{f\in F} sh_{f}^{ps}, \qquad \forall p\in P, s\in S. \tag{18}$$

Constraint (19) is non-negativity constraint.

$$t_{ij}^{ps}, inv'^{ps}_{w}, \widehat{inv'}^{ps}_{w}, sh_{f}^{ps}, \widehat{sh}_{f}^{ps}, Tp_{w}^{ps}, \widehat{Tp}_{w}^{ps}, \varrho_{e}^{ps}\ge 0, \quad \forall p\in P, s\in S, w\in W, f\in F, e\in E. \tag{19}$$

Remark 1. Referring to Alikhani et al. (2021), transshipment quantity and shortage quantity are also used in our model. To adjust the product transportation structure and the cost of the supply chain, we introduce three new types of proportion parameters into our RSCND model. The value of the proportion parameter can be set flexibly according to the actual situation of enterprises. Under the preset value, our method can find the robust optimal resilient supply chain network for the decision-maker.

The objective function

Before setting up the objective function, we define the total cost of the post-disaster stage under disruption scenario s, which includes five terms. The first term is the destruction cost of open facilities under disruption scenario s. When the supply chain is disrupted, each node may be damaged to varying degrees, then destruction cost occur. Destruction cost OC^{s} is one part of the total costs and represents the total destruction cost of the open suppliers, DCs and retail stores under scenario s. The expression is

$$OC^{s}=\sum_{q\in Q}\sum_{x\in X} cq'^{xs}_{q} yq_{q}^{x}+\sum_{w\in W}\sum_{x\in X} cw'^{xs}_{w} yw_{w}^{x}+\sum_{e\in E}\sum_{x\in X} ce'^{xs}_{e} ye_{e}^{x}.$$

The first item is the destruction cost of the established supplier facilities with various facility level x under scenario s. Similarly, the second item is the destruction cost for the established DC facilities, and the third item is the destruction cost for the established retail store facilities.

The second term is holding cost and processing cost in DC:

$$OD^{s}=\sum_{w\in W}\sum_{p\in P} g_{w}^{p} inv'^{ps}_{w}+\sum_{w\in W}\sum_{(w,j)\in Arc}\sum_{p\in P} h_{w}^{p} t_{wj}^{ps}.$$

The third term is transportation cost:

$$OT^{s}=\sum_{(i,j)\in Arc}\sum_{p\in P} tr_{ij}^{p} t_{ij}^{ps}.$$

The fourth term is penalty cost:

$$OP^{s}=\sum_{f\in F}\sum_{p\in P} pef_{f}^{p} sh_{f}^{ps}+\sum_{e\in E}\sum_{p\in P} pee_{e}^{p}\varrho_{e}^{ps}.$$

The fifth term is the cost of purchasing products from different channels after disruption:

$$OPP^{s}=\sum_{q\in Q}\sum_{(q,w)\in Arc}\sum_{p\in P} mq'^{p}_{q} t_{qw}^{ps}+\sum_{q\in Q}\sum_{(q,e)\in Arc}\sum_{p\in P} mq'^{p}_{q} t_{qe}^{ps} +\sum_{w\in W}\sum_{p\in P}(mw'^{p}_{w}+\mathfrak{I}_{w}^{p})Tp_{w}^{ps}+\sum_{w\in W}\sum_{p\in P}(m_{w}^{p}-sal_{w}^{p})inv'^{ps}_{w}.$$

According to the above notations, the total expected cost in the post-disaster stage is as follows:

$$Cost'=\sum_{s\in S}\rho^{s}(OC^{s}+OD^{s}+OT^{s}+OP^{s}+OPP^{s}).$$

3.3.2. Pre-disaster stage

Constraints

Constraint (20) ensures that DC node $w\in W$ of the supply chain is linked by at least n_{e}^{p} upper nodes during the transportation of product p. This is a method based on multi-set coverage strategy.

$$\sum_{w\in W} lin_{we}^{p}\ge n_{e}^{p}, \qquad \forall p\in P, e\in E. \tag{20}$$

Constraint (21) ensures that each established node selects at most one facility level.

$$\sum_{x\in X} yw_{w}^{x}\le 1, \qquad \forall w\in W. \tag{21}$$

Constraint (22) means that two nodes of the connecting path should first be open nodes. To be precise, the flow between overlay nodes w and e means that both nodes w and e should be selected.

$$lin_{we}^{p}\le \sum_{x\in X} ye_{e}^{x}, \qquad \forall p\in P, e\in E, w\in W. \tag{22}$$

$$lin_{we}^{p}\le \sum_{x\in X} yw_{w}^{x}, \qquad \forall p\in P, e\in E, w\in W. \tag{23}$$

Constraint (24) means the capacity limit of inventory holding and processing of DC.

$$inv_{w}^{p}\le cap_{w}^{p}\times \sum_{x\in X} yw_{w}^{x}, \qquad \forall p\in P, w\in W. \tag{24}$$

Constraints (25)–(26) are binary and non-negativity constraint for decisions, respectively.

$$yq_q^x, yw_w^x, ye_e^x, lin_{ij}^p \in \{0,1\}, \quad \forall p \in P, w \in W, x \in X, q \in Q, e \in E, i \in Q \cup W \cup E, j \in W \cup E \cup F. \tag{25}$$

$$inv_w^p \geq 0, \qquad \forall p \in P, w \in W. \tag{26}$$

For any consumer area, selected retail store, and selected DC, the following constraints (27)–(29) can be explained similarly to constraint (20) just for different types of upper nodes.

$$\sum_{e \in E} lin_{ef}^p \geq n_f^p, \qquad \forall p \in P, f \in F. \tag{27}$$

$$\sum_{q \in Q} lin_{qe}^p \geq n_e^p, \qquad \forall p \in P, e \in E. \tag{28}$$

$$\sum_{q \in Q} lin_{qw}^p \geq n_w^p, \qquad \forall p \in P, w \in W. \tag{29}$$

The following constraints (30)–(31) are similar to constraint (21) for any selected supplier and retail store.

$$\sum_{x \in X} yq_q^x \leq 1, \qquad \forall q \in Q. \tag{30}$$

$$\sum_{x \in X} ye_e^x \leq 1, \qquad \forall e \in E. \tag{31}$$

For any flow in other connecting paths, similar to constraints (22)–(23), we have the following constraints (32)–(36).

$$lin_{ef}^p \leq \sum_{x \in X} ye_e^x, \qquad \forall p \in P, e \in E, f \in F. \tag{32}$$

$$lin_{qe}^p \leq \sum_{x \in X} ye_e^x, \qquad \forall p \in P, e \in E, q \in Q. \tag{33}$$

$$lin_{qe}^p \leq \sum_{x \in X} yq_q^x, \qquad \forall p \in P, e \in E, q \in Q. \tag{34}$$

$$lin_{qw}^p \leq \sum_{x \in X} yw_w^x, \qquad \forall p \in P, q \in Q, w \in W. \tag{35}$$

$$lin_{qw}^p \leq \sum_{x \in X} yq_q^x, \qquad \forall p \in P, q \in Q, w \in W. \tag{36}$$

Remark 2. Referring to Jabbarzadeh et al. (2016), facility level refers to the facility strength if the node facility is established, higher level means higher facility strength while lower level means lower facility strength. The facility level is a resilience strategy, meaning that the disruption risks are reduced by establishing protective measures (such as installing structural reinforcement, retaining standby emergency equipment, preventive monitoring, and so on).

The objective function

The total cost in the pre-disaster stage is described as function (37), the first item is the cost of the selected supplier, the second item is the construction cost of DC, the third item is the construction cost of retail stores, the fourth item is the cost of opening up the path, and the fifth item is the inventory cost.

$$\begin{aligned} Cost = & \sum_{q \in Q} \sum_{x \in X} cq_q^x yq_q^x + \sum_{w \in W} \sum_{x \in X} cw_w^x yw_w^x + \sum_{e \in E} \sum_{x \in X} ce_e^x ye_e^x \\ & + \sum_{(i,j) \in Arc} \sum_{p \in P} arc_{ij} lin_{ij}^p + \sum_{w \in W} \sum_{p \in P} m_w^p inv_w^p. \end{aligned} \tag{37}$$

3.3.3. DRO model with ACC

During the operation and management of enterprises, decision-makers usually hope to increase profits by reducing costs. Under different disruption scenarios, some decisions are different. Consequently the costs are scenario-dependent. We optimize the total cost in the pre-disaster stage and post-disaster stage via the expected value criterion. In view of the probability distribution information of the uncertain demand is partly known, we adopt the DRO method to resist the uncertainty of distribution. Our aim is to find a resilient RSC network that optimizes the cost under the worst-case scenario of probability distribution about uncertain demand. Therefore, when the distribution information of the uncertain parameter d is partially known under scenario s, we present the following two-stage DRO model with ACC

$$\begin{aligned} & \min Cost + Cost' \\ & s.t. \ (3)\text{–}(36). \end{aligned} \tag{38}$$

In model (38), demand d is uncertain, so the model has finite decision variables and infinite constraints. In fact, handling constraint (3) is not an easy assignment directly. Due to the existence of the demand uncertainty, constraint (3) is semi-infinite, which increases the complexity of the model. To get the optimal solution, it is vital to address constraint (3). One common way is to solve model (38) with a computationally tractable approximation form of ACC (3). In the following section, we will derive robust counterpart approximation for ACC (3).

Remark 3. The ambiguity set $\mathscr{A}$ is composed of all probability distributions which satisfy the constraint conditions. The support set $\mathcal{Z} = \{d | (d-d_0)^T \Gamma_0^{-1}(d-d_0) \leq \Omega^2, -l \leq d \leq l\}$ indicates that the demand locates in a ball with radius Ω centered on d_0. And the distribution of each element in $\mathscr{A}$ has the same mean and variance. For example, $\Gamma_0 = I, d = (d_1, d_2), d_0 = (0,0)$, and $\Omega = 1$, I is an identity matrix, and $l = 1$. Compliance with the above constraints is available $d_1^2 + d_2^2 \leq 1, -1 \leq d_1 \leq 1$ and $-1 \leq d_2 \leq 1$. Support set of the solution is $[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}] \times [-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}]$.

4. Tractable form of distributionally robust resilient RSC optimization model

As we know, the safe approximations of chance constraints depend to a large extent on the selection of the perturbation set. With the purpose to obtain a computationally tractable form of resilient RSC optimization model (38), ACC (3) needs to be transformed into an explicit finite convex constraints system. From the partial distribution information in the known ambiguity set (2), ζ is a bounded random variable. Following the idea of Ben-Tal et al. (2009) and Bertsimas and Sim (2004) to reduce the conservativeness, we choose the budget set and the intersection of the box set and ball set as two perturbation sets. We give two safe approximations of ACC under these two different perturbation sets. See Theorems 1–2 in Appendix for detailed approximation systems.

Suppose the random variable ζ in ACC (3) belongs to ambiguity set (2). Then, we further derive the probability level at which the corresponding approximate feasible solution satisfies (3). See Propositions 1–3 in Appendix for detailed results.

Therefore, under perturbation set $Z_{Box-Ball}$, based on Proposition 2, distributionally robust resilient RSC optimization model (38) with the Box-Ball robust counterpart approximation (A.1) of ACC (3) is the following MISOCP model:

$$\begin{aligned} & \min Cost + Cost' \\ & s.t. \ (4)\text{–}(36), (A.1). \end{aligned} \tag{39}$$

Under perturbation set Z_{Budget}, based on Proposition 3, distributionally robust resilient RSC optimization model (38) with the Budget robust counterpart approximation (A.4) of ACC (3) is the following MILP model:

$$\min Cost + Cost' \quad s.t.\ (4)\text{–}(36), (A.4). \tag{40}$$

In conclusion, when the known partial distribution information is supported on [−1, 1] and the mean value is zero, under perturbation set $Z_{Box-Ball}$ and budget perturbation set Z_{Budget}, ACC (3) is converted to convex constraint systems (A.1) and (A.4), respectively. Accordingly, the former safe tractable approximation model (39) is referred to as the Box-Ball-DRO model, while the latter model (40) is called the Budget-DRO model. Hence, we can approximate the original DRO model (38) by (39) and (40) under the intersection of the box and ball $Z_{Box-Ball}$, and the budget perturbation sets Z_{Budget}, respectively.

In this section, we have transformed the DRO model into the approximate MISOCP model (39) or MILP model (40) through robust counterpart approximation, respectively. In the next section, to verify the applicability and effectiveness of our distributionally robust resilient RSC optimization model, numerical experiments will be used. The proposed DRO model with uncertain demand will be compared to a stochastic model with deterministic demand, which is shown as model (41).

$$\min Cost + Cost' \quad s.t.\ (1), (4)\text{–}(36). \tag{41}$$

5. A case study: Retail supply chain in Wuhan

In this section, we use a real RSC network and conduct some numerical experiments from three aspects to verify our model. First, the experimental results of the stochastic model (41), the Box-Ball-DRO model (39), and the Budget-DRO model (40) are given. Second, from the perspective of cost optimization, the above three models are analyzed and compared. Finally, sensitivity analysis is applied to verify the efficiency of the DRO model. All numerical experiments are conducted with CPLEX 12.6.3 optimization software, installed on a Ryzen 7 5800H with Radeon Graphics 3.20 GHz PC, running under Windows 10 (64-bit) 16 GB of memory.

For the stated parameters, the cost unit is CNY (Chinese Yuan), the product is in kilograms, the distance is in kilometers, and the quantity of product transported is calculated by product weight. Also, the distance between any pair of potential locations is estimated using Baidu Maps. Let node coverage number $n_j^p = 2$, the uncertain parameter d fluctuates by 10%.

5.1. Problem background and data description

Take the RSC network in Wuhan, Hubei Province, China, as an example, and its main enterprises are three chain supermarkets. In 2020, when the global COVID-19 incident broke out, Wuhan entered a state of emergency campaign. The supermarket inventory continued to decrease, and local fruit and vegetable supply was disrupted.[2] To make the living necessity market in Wuhan running stable, the necessary vegetables and fruits for residents were taken as products to be transported via the supply chain network in the instance. There were various uncertain factors in the scheduling of Wuhan emergency food. It was difficult to estimate the exact data about food demand in the affected area.

On February 2, 2020, the troops stationed in Hubei cooperated with the three major companies of WuShang, ZhongBai, and ZhongShang to transport vegetables and fruits from the major DCs in Wuhan to these three chain supermarkets.[3] Due to the high cost of building a brand new DC, renting the local DCs is only considered. Managers are concerned about which warehouse should be rented, as well as the best distribution solution from each leased warehouse to each supermarket. In Wuhan, the three major supermarkets are mainly distributed in Jiang'an District, Jianghan District, Qiaokou District, Hanyang District, Wuchang District, and Qingshan District, as shown in Fig. 2. In the above considered 6 residential areas, 128 supermarkets are used as retail stores. In addition, 3 DCs and 4 suppliers are considered. The locations of suppliers, DCs, and retail stores are obtained from Baidu Maps.[4] Specifically, the partial parameter values used to obtain the case study results are shown in Tables 2 and 3. We let $X = \{x|x = 1, 2, 3, 4\}$, where $x = 1$ implies a facility with the lowest disaster resistance level, the lowest construction cost, and the probability that the corresponding facility is damaged due to various disasters is the highest. Compared with $x = 1$, $x = 2$ means higher facility's resistance level and construction cost, and lower probability that the corresponding facility is damaged due to various disasters. Compared with $x = 2$, $x = 3$ means higher facility's resistance level and construction cost, and lower probability that the corresponding facility is damaged due to various disasters. $x = 4$ implies that the facility is with the highest disaster resistance level, the construction cost is the highest, and the probability that the corresponding facility is damaged due to various disasters is the lowest. The disruptive events are divided into two levels from high to low according to the degree of destruction, urgency, and development trend, namely $S = \{1, 2\}$. We apply the proposed RSCND method to optimize the supply chain network. Destructive events can also be divided into more levels according to their severity and actual needs, as different scenarios.

5.2. Computational results under the DRO model

This section will conduct some experiments by adjusting tolerance parameter ϵ. From Propositions 2 and 3, that is, $\epsilon = 1 - exp\{-\frac{\Omega^2}{2}\}$, $\Omega \geq 0$ and $\epsilon = 1 - exp\{-\frac{\gamma^2}{2F}\}$, when parameter ϵ changes its value, the values of the corresponding parameters Ω and γ will change. We consider two instances. In the first one, the probabilities of two disruption scenarios are 0.7 and 0.3, respectively. In the second one, the probabilities are 0.9 and 0.1, respectively. For those numerical experiments, Table 4 shows the computational results of this model. In instance 1, from the Box-Ball-DRO model, the optimal cost is 367,457,446.9 when $\epsilon = 0.09$, $\Omega = 2.20$, and from the Budget-DRO model, the optimal cost is 367,506,650.3 when $\epsilon = 0.09$, $\gamma = 5.38$. In instance 2, from the Box-Ball-DRO model, the optimal cost is 392,080,134.5 when $\epsilon = 0.09$, $\Omega = 2.20$, and from the Budget-DRO model, the optimal cost provided by the Budget-DRO model is 392,170,276.8 when $\epsilon = 0.09$, $\gamma = 5.38$.

From Table 4, the inventory quantity in the third DC is always zero in either instance, which indicates that the retail stores in the vicinity of this DC do not purchase the products from this DC. This is caused by the resilience strategy direct-to-store delivery (DSD). The DSD strategy used means that the supplier's products can be distributed directly to each retail store, which saves the holding cost and processing cost of the DC to some extent, but increases the transportation cost correspondingly. From the experimental results, the retail stores purchase products directly from the suppliers, such as ZhongShang supermarket (ZhongNan store) and ZhongBai supermarket (Oriental Rhine store). Therefore, the third DC is not put into use under the DSD strategy, which shows the validity of the DSD strategy.

To analyze the impact of resilience strategy on the optimal transportation path, we first randomly select some retail stores in the central area of Wuhan, as shown in Fig. 2. Then, we carry out some

[2] https://data.stats.gov.cn/easyquery.htm?cn=E0104&zb=A0104&reg=420100&sj=202203

[3] http://www.mofcom.gov.cn/article/ae/ai/202002/20200202933649.shtml

[4] https://map.baidu.com/@12728273.29,3561757.28,13z

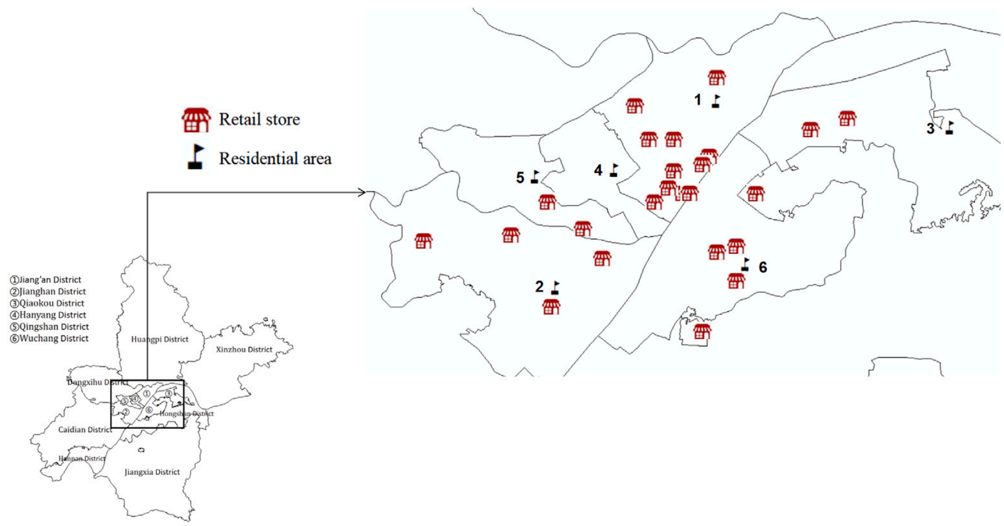


Fig. 2. A selection of retail stores in Wuhan.

Table 2
The values of parameters cq'^{xs}_q, cw'^{xs}_w, ce'^{xs}_e, and cap'^{pxs}_w under different facility level.

	ce'^{x1}_e	ce'^{x2}_e	cq'^{x1}_1	cq'^{x2}_1	cq'^{x1}_2	cq'^{x2}_2	cq'^{x1}_3	cq'^{x2}_3	cq'^{x1}_4	cq'^{x2}_4
$x = 1$	457,470	457,830	381,900	375,216.75	395,220	397,433.232	595,290	595,944.819	425,160	472,522.824
$x = 2$	304,980	305,220	254,600	250,144.5	263,480	264,955.488	396,860	397,296.546	283,440	315,015.216
$x = 3$	152,490	152,610	127,300	125,072.25	131,740	132,477.744	198,430	198,648.273	141,720	157,507.608
$x = 4$	0	0	0	0	0	0	0	0	0	0
	cap'^{px1}_w	cap'^{px2}_w	cap'^{px1}_e	cap'^{px2}_e	cw'^{x1}_1	cw'^{x2}_1	cw'^{x1}_2	cw'^{x2}_2	cw'^{x1}_3	cw'^{x2}_3
$x = 1$	500,000	0	4,000	2,500	1,328,490	1,328,490	846,990	845,280	892,935	893,880
$x = 2$	800,000	500,000	7,500	5,000	885,660	885,660	564,660	563,520	595,290	595,920
$x = 3$	1,000,000	900,000	10,000	12,000	442,830	442,830	282,330	281,760	297,645	297,960
$x = 4$	1,500,000	1,200,000	15,000	15,000	0	0	0	0	0	0
	cq^x_1	cq^x_2	cq^x_3	cq^x_4	cw^x_1	cw^x_2	cw^x_3	ce^x_e		
$x = 1$	1,273,000	1,317,400	1,984,300	1,417,200	4,428,300	2,823,300	2,976,450	1,524,900		
$x = 2$	1,400,300	1,449,140	2,182,730	1,558,920	4,871,130	3,105,630	3,274,095	1,677,390		
$x = 3$	1,540,330	1,594,054	2,401,003	1,714,812	5,358,243	3416193	3,601,504.5	1,845,129		
$x = 4$	1,694,363	1,753,459.4	2,641,103.3	1,886,293.2	5,894,067.3	3,757,812.3	3,961,654.95	2,029,641.9		

Table 3
The value of parameters that are independent of the facility level.

M	ϖ	$\hbar$	cap^p_w	g^p_w	h^p_w	pef^p_f	pee^p_e
0.5	0.9	0.5	1,000,000	0.1	1	50	15,249
$\mathfrak{J}^p_w$	sal^p_w	$\eth^{p1}$	$\eth^{p2}$	m^p_w	mq'^p_q	mw'^p_w	
9	4	509,804	1,094,000	6.31	5.56	8	

experiments about the Box-Ball-DRO model and the Budget-DRO model when $\epsilon = 0.05$. The experimental results are shown in Fig. 3.

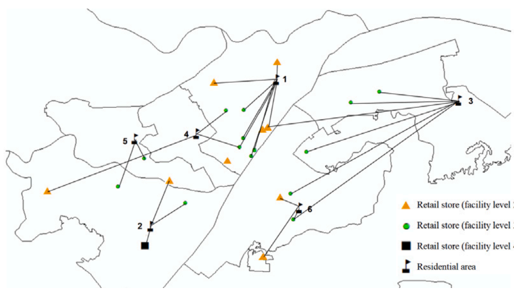
Fig. 3 shows the paths of daily fruit and vegetable supply for some retail stores and residential areas. From Fig. 3, most retail stores use the proximity principle to transport products. But there is one retail store that supplies products to multiple residential areas because multi-set coverage strategy is effective. During the transportation of the product, it increases supply chain resilience to cover each node by at least two upper nodes. At the same time, from the experimental results, we can notice that the facility level of the retail stores changes accordingly when the transportation paths change, which shows the effectiveness of the facility defense strategy. For example, Fig. 3(a) shows that, even if the retail store is close to residential area 6, it transports products to meet the large demand of residential area 3. However, compared with Fig. 3(a), in Fig. 3(c) the facility levels of retail stores are improved to supply products to residential area 3, and retail stores supply products to residential area 3 and residential area 6 at the same time to meet the demand of residents. The path distributions provided by the Box-Ball-DRO model and the Budget-DRO model in instance 2 are similar to that in Fig. 3(a) and Fig. 3(c), which are shown in Figs. 3(b) and 3(d). This shows that the Box-Ball-DRO model and the Budget-DRO model provide different optimal paths for transported products.

In summary, no matter using the ambiguity set under the Box-Ball perturbation set or the ambiguity set under the Budget perturbation set, the supply chain network given by the distributionally robust model can not only maintain the supply capacity, but also quickly restore its operation capacity through resilience strategies.

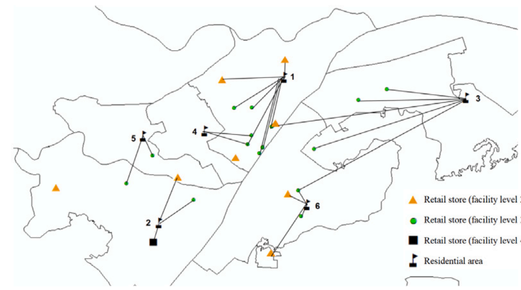
5.3. Comparison with stochastic model

This subsection compares the Box-Ball-DRO model (39) and the Budget-DRO model (40) to the stochastic model (41), respectively.

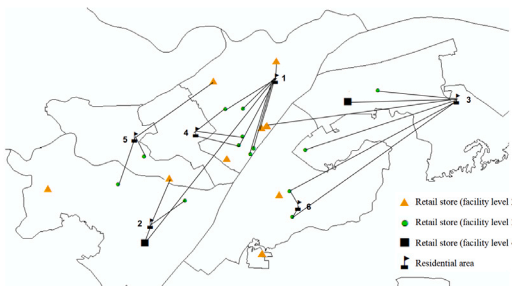
Let $\epsilon = 0.05$. The quantity of retail stores at different defense levels is shown in Table 5. From Table 5, facility defenses are established for all 128 retail stores via the stochastic model (41), including 38 nodes with facility defense level 2 are established, 87 nodes with facility defense level 3 are established, and 3 nodes with facility defense level 4 are established in instance 1, 35 nodes with facility defense level 2 are established, 84 nodes with facility defense level 3 are established, and 9 nodes with facility defense level 4 are established in instance 2.



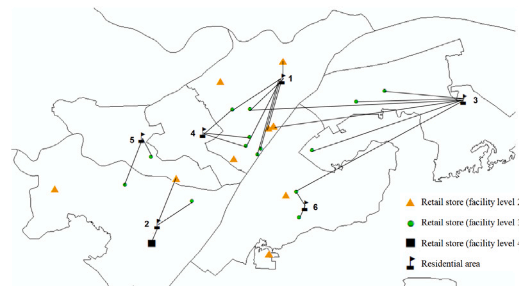
(a) The path distribution provided by Box-Ball-DRO model in instance 1



(b) The path distribution provided by Box-Ball-DRO model in instance 2



(c) The path distribution provided by Budget-DRO model in instance 1



(d) The path distribution provided by Budget-DRO model in instance 2

Fig. 3. The path distribution provided by DRO model.

Table 4
Computational results of the Box-Ball-DRO model and Budget-DRO model.

Model	ϵ	Ω	γ	Instance	DC (Product quantity (unit: KG))			Total cost
					DC 1	DC 2	DC 3	(unit: CNY)
	0.05	2.45	–	I1	55,000	119,500	0	367,746,430.2
			–	I2	84,221	157,500	0	392,472,402.5
Box-Ball-DRO model	0.06	2.37	–	I1	53,000	117,000	0	367,658,079.1
			–	I2	85,500	157,500	0	392,365,601.5
	0.07	2.31	–	I1	52,500	118,500	0	367,584,109.3
			–	I2	84,358	157,500	0	392,262,067.1
	0.08	2.25	–	I1	52,954	119,500	0	367,509,318.0
			–	I2	83,000	157,500	0	392,169,938.7
	0.09	2.20	–	I1	52,787	117,000	0	367,457,446.9
			–	I2	84,552	157,500	0	392,080,134.5
	0.05	–	6.00	I1	52,866	118,500	0	367,749,475.2
		–		I2	83,000	157,500	0	392,457,791.7
Budget-DRO model	0.06	–	5.81	I1	51,503	118,500	0	367,661,731.5
		–		I2	83,000	157,500	0	392,371,394.9
	0.07	–	5.65	I1	52,500	116,000	0	367,608,137.7
		–		I2	84,508	157,500	0	392,294,813.8
	0.08	–	5.51	I1	57,500	116,000	0	367,575,787.6
		–		I2	81,845	157,500	0	392,228,498.8
	0.09	–	5.38	I1	51,500	118,500	0	367,506,650.3
		–		I2	80,500	157,500	0	392,170,276.8

[1] Instance 1 is expressed as I1.
[2] Instance 2 is expressed as I2.

Table 5
Comparison of resilient strategies from three models.

Resilience strategy \ Model	The stochastic model		Box-Ball-DRO model		Budget-DRO model	
	I1	I2	I1	I2	I1	I2
Facility defense level 1	0	0	0	0	0	0
Facility defense level 2	38	35	41	35	39	34
Facility defense level 3	87	84	85	87	87	89
Facility defense level 4	3	9	2	6	2	5
Multi-set coverage strategy 1	365,011,609.3808	388,663,256.3868	367,746,430.1548	392,472,402.5058	367,749,475.2308	392,457,791.6848
Multi-set coverage strategy 2	366,605,710.8728	390,094,814.4368	369,119,727.8708	393,545,638.2438	369,128,641.1628	393,517,368.761803
Multi-set coverage strategy 3	368,970,264.9884	392,416,659.7006	371,454,074.9842	395,628,319.8736	371,425,299.7174	395,650,299.1536
Multi-set coverage strategy 4	368,970,264.9884	392,416,659.7006	371,436,497.1172	395,648,542.2536	371,433,369.7174	395,627,674.7476

[1] Instance 1 is expressed as I1.
[2] instance 2 is expressed as I2.

Table 6
The cost of the stochastic and two robust counterparts.

Cost list (unit: CNY)	The stochastic model		Box-Ball-DRO model		Budget-DRO model	
	I1	I2	I1	I2	I1	I2
Fixed cost (Cost)	246,156,410	248,052,295	245,450,543	247,447,699	245,766,022	247,422,938
Destruction cost (OC)	54,350,445.5	51,903,773.1	55,570,845.5	52,819,073.1	54,960,645.5	52,819,073.1
Inventory cost (OD)	591,233.2	748,232	561,884.2	698,429.3	555,425.2	695,863.2
Transportation cost (OT)	272,807,892	268,090,966	282,695,835	278,101,321	282,673,666	278,137,578
Penalty cost (OP)	0	0	100	50	0	0
Acquisition cost (OPP)	7,435,251.24	7,180,003.94	7,540,566.78	7,341,205.63	7,550,198.86	7,345,156.88
The total costs	365,011,609.3808	388,663,256.3868	367,746,430.1548	392,472,402.5058	367,749,475.2308	392,457,791.6848

[1] Instance 1 is expressed as I1.
[2] Instance 2 is expressed as I2.

Compared with the stochastic model, the DRO model establishes fewer high-level facilities in the same instance, which indicates that the distributionally robust model does not only simply raise the facility level to resist supply chain disruptions, but also uses different resilience strategies to improve the robustness and other performance of the supply chain.

For our numerical experiments, the fixed, destruction, inventory, transportation, penalty, and acquisition costs are collected in Table 6. The optimal total cost of the stochastic model is approximately 365,011,609.4 CNY in instance 1, including fixed cost of 246,156,410 CNY, destruction cost of 54,350,445.5 CNY, inventory cost of 591,233.2 CNY, transportation cost of 272,807,892 CNY and acquisition cost of 7,435,251.24 CNY. Similarly, the optimal total cost of the stochastic model is approximately 388,663,256.4 CNY in instance 2, including fixed cost of 248,052,295 CNY, destruction cost of 51,903,773.1 CNY, inventory cost of 748,232 CNY, transportation cost of 268,090,966 CNY and acquisition cost of 7,180,003.94 CNY.

The transportation cost given by the DRO model is higher than that given by the stochastic model, where the transportation delay is the main obstacle to product distribution. From the comparison of the above results, we find that the DRO model provides higher costs when faced with the ambiguous distribution of uncertain demand. This is reasonable and consistent with the worst-case-oriented criterion.

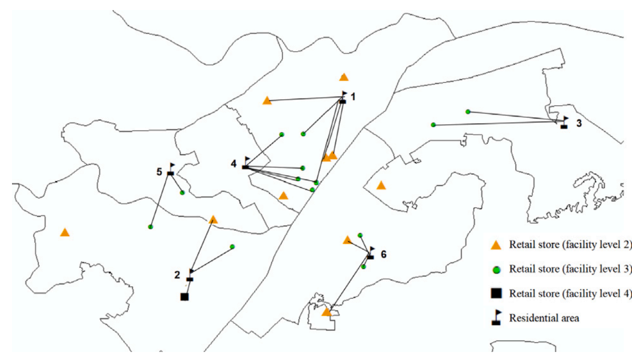
The stochastic model and DRO model provide different transportation paths. Fig. 4 shows the paths from the selected retail stores to residential areas for the daily supply of fruits and vegetables, which is determined by the stochastic model. From Fig. 3 and Fig. 4, there exist some differences in the paths for residential area 3 and residential area 4. In the same condition, the DRO model focuses more on transported products to the remote residential area 3, and opens up more paths for the retail stores. The stochastic model determines more transportation paths for residential area 4. This shows that the DRO model obtains a relatively robust solution in terms of demand perturbation.

The above comparative experiments show that there are significant differences in the optimal decisions of the DRO model and the stochastic model. In comparison to the stochastic model, the total cost determined via the DRO model is higher. The price of distributionally robust (PDR) is introduced to further characterize the degree of difference between the stochastic model and the DRO model with ACC. We define PDR as follows:

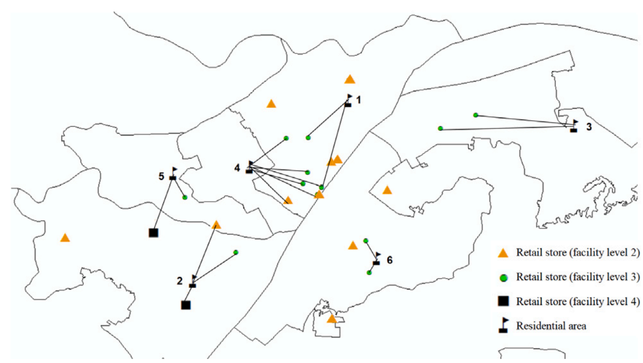
$$PDR = \frac{robust\ value^{*} - stochastic\ value^{*}}{stochastic\ value^{*}} \times 100\%,$$

where $(\cdot)^{*}$ represents the optimal cost.

In the comparison between the stochastic model (41) and the Box-Ball-DRO model (39), the calculated PDR is 0.7492% in instance 1. In other words, compared with the stochastic model (41), using the Box-Ball-DRO model (39), the cost only increases by 0.7492% in exchange for better resistance to the uncertainty of the demand distribution function. One has PDR = 0.98% in instance 2. That is to say, using the Box-Ball-DRO model (39) to resist the uncertainty of the distribution function about uncertain demand, the cost only increases by 0.98%.



(a) The path distribution provided by stochastic model in instance 1



(b) The path distribution provided by stochastic model in instance 2

Fig. 4. The path distribution provided by stochastic model.

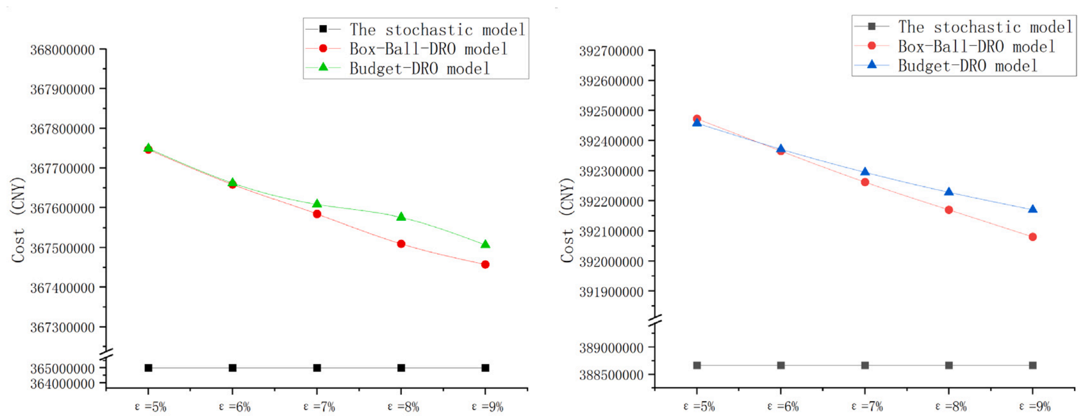
Similarly, in the comparison between the stochastic model (41) and the Budget-DRO model (40), the calculated PDR is 0.75% in instance 1. So the cost only increases by 0.75% if the Budget-DRO model (40) is used to find a robust supply chain network. One has PDR = 0.9763% in instance 2. The same as above, determining a robust supply chain network via the Budget-DRO model (40), the cost paid only increases by 0.9763%.

5.4. Sensitivity analysis

The probability level ϵ reflects the possibility that the demand is not met, and the resilience strategy controls the ability of the supply chain network to maintain a continuous supply. In this subsection, we conduct sensitivity analyses with respect to probability level ϵ and multi-set coverage strategy to explore their effects on the decisions about the supply chain network.

5.4.1. Effects of probability level ϵ on decisions

Parameters Ω and γ control the size of perturbation sets and vary with ϵ that takes the following different values: 5%, 6%, 7%, 8%, 9%.



(a) The influence of probability level ϵ on the optimal cost in instance 1 (b) The influence of probability level ϵ on the optimal cost in instance 2

Fig. 5. The influence of probability level ϵ on the optimal cost.

The following series of experiments demonstrate the effects on the optimal cost and inventory quantity. Here, instance 1 and instance 2 are all considered when we solve the problem.

As can be seen from the experimental results (see Table 4), for instance 1, corresponding to different parameter values $\epsilon = 5\%, 7\%, 9\%$, from the Box-Ball-DRO model, the optimal costs are 367,746,430.2, 367,584,109.3 and 367,457,446.9, from the Budget-DRO model, the optimal costs are 367,749,475.2, 367,608,137.7 and 367,506,650.3, respectively. From the Box-Ball-DRO model, the cost decreases with ϵ, and the optimal inventory fluctuates with ϵ. From the Budget-DRO model, the optimal cost decreases with ϵ. For instance 2, the conclusion is also similar, from the Box-Ball-DRO model, the optimal cost decreases with ϵ, and the optimal inventory fluctuates with ϵ. For the convenience of observing and comparing the influence of probability level ϵ on the optimal cost, in Fig. 5, we plot curves for ϵ vs cost for the stochastic, Box-Ball, and Budget models. Hence, to reasonably allocate inventory, the decision maker can set probability level ϵ based on personal experience and knowledge.

5.4.2. Effects of multi-set coverage strategy on decisions

To illustrate the effect of a multi-set coverage strategy on the decisions, we choose four multi-set coverage strategies for the experiment. The node coverage number of multi-set coverage strategy 1, multi-set coverage strategy 2, multi-set coverage strategy 3, and multi-set coverage strategy 4 are represented by n^p_{1j}, n^p_{2j}, n^p_{3j}, n^p_{4j}, where $n^p_{1j} \leq n^p_{2j} \leq n^p_{3j} \leq n^p_{4j}, \forall j \in \{W, E, F\}$, respectively.

Let $(n^p_{1w}, n^p_{1e}, n^p_{1f}) = (2, 2, 2)$, $(n^p_{2w}, n^p_{2e}, n^p_{2f}) = (3, 3, 3)$, $(n^p_{3w}, n^p_{3e}, n^p_{3f}) = (4, 3, 4)$, $(n^p_{4w}, n^p_{4e}, n^p_{4f}) = (4, 3, 5)$. Table 5 shows that the optimal costs of different models corresponding to different multi-set coverage strategies exhibit great differences. The multi-set coverage strategy with the larger node coverage number corresponds to more costs. The reason is that it makes the supply chain establish more paths to improve the supply chain resilience, which leads to a large increase in fixed cost. Therefore, the optimal total cost increases.

The above results mean that multi-set coverage strategy greatly impacts the optimal cost. It is extremely significant for decision-makers to clarify the suitable multi-set coverage strategy to formulate effective decisions about the supply chain network.

The sensitivity analysis results show that the optimal inventory allocation and the realization of the optimal cost are sensitive to the probability level ϵ and multi-set coverage strategy.

6. Managerial insights

Based on a comparative study and sensitivity analysis for the proposed two-stage DRO method, some managerial insights for enterprise decision-makers are extracted as follows:

Firstly, the resilient supply chain networks designed by the proposed two-stage DRO model and its corresponding stochastic model are different. This implies that the ambiguous probability distributions of uncertain demand have a significant impact on the optimal RSC network structure. In the design of a RSC network under different disruption scenarios, if it is difficult to get precise demand, even the exact distribution of random demand, through historical data, the decision-maker can accept the proposed DRO model to design a resilient RSC network to deal with different disruption scenarios and ensure the smooth operation and cost reduction of enterprises. In this case, decision-makers should not use the decision provided by the stochastic model to design the supply chain network.

Secondly, according to the sensitivity analysis, a multi-set coverage strategy with fewer nodes will lead to fewer transportation paths established in the supply chain network, which may bring more destruction cost. That is to say, a multi-set coverage strategy with more nodes is better adequate for emergency situations such as sudden increases in demand and supply chain disruption caused by natural disasters. Thus, a multi-set coverage strategy with more nodes can effectively alleviate the supply shortage caused by supply chain disruption. However, if the node coverage number is too large, it will lead to a large increase in fixed cost. Therefore, when there are multiple candidate suppliers, DCs, and retail stores, the decision-maker should select an appropriate node coverage number. For example, in the case of natural disasters with many transmission chains, high speed, and wide range, the node coverage number should be larger, whereas in the stage after the corresponding control of the disaster, the node coverage number can be smaller.

Finally, for RSCND and management problems, the decision-makers can no longer manage the supply chain network based on only the experience and intuition. The paper proposes a more scientific mathematical optimization method based on DRO to design the optimal resilient supply chain network. The proposed method can deal with both different disruption scenarios and demand uncertainty. It can fully use the historically available data on demand. Our optimization method can not only avoid the requirement of exact distribution in stochastic programming, but also provide a less conservative design scheme than that provided by robust optimization, which includes the construction of node facilities, inventory allocation, traffic path planning, and so on.

7. Conclusion

This paper proposed a novel two-stage DRO model with ACC for the RSCND problem. The main conclusions are as follows:

Due to increasingly unpredictable disruptions, adjusting the resilience supply chain network to such disruption is an important issue.

This study suggested a two-stage DRO model with ACC for designing a RSC network by considering resilience strategies under different disruption scenarios. With our DRO model, decision-makers can make wise decisions for designing the resilient RSC network under the ambiguous probability distribution of uncertain demand.

The robust counterpart approximation of the DRO model with ACC is generally a difficult optimization issue and computationally intractable. Under Box-Ball and Budget perturbation sets, the robust counterpart approximations of our DRO model, and the computationally tractable MISOCP model or MILP model were derived. Consequently, the optimal resilient supply chain network can be attained via CPLEX software.

We took the supply chain network design of three chain supermarkets in Wuhan as a case and conducted a series of comprehensive numerical experiments to prove the effectiveness and availability of our new optimization method. We also compared the DRO model with ACC and the stochastic model, and conducted sensitivity analysis. Comparative studies showed the effectiveness of our method from the optimal cost and the optimal path of RSC transportation. From the comparison and sensitivity analysis, some management insights were recommended.

This novel optimization method can provide a supply chain network that can resist the risk from ambiguous probability distributions of uncertain demand to some extent. It provides a reliable method for supply chain operation before and after the disaster. The proposed optimization method is also applicable to different companies that are committed to developing resilient RSC networks and responding aggressively to supply chain disruption.

When designing a resilient RSC network, the minimum node coverage number may vary depending on the different node. So the node coverage number can be taken as a non-negative integer variable, and the corresponding extended model can increase supply chain resilience in terms of arcs. This is an interesting expansion to this study in our future research.

CRediT authorship contribution statement

Shengjie Chen: Methodology, Writing – original draft, Software, Formal analysis. **Yanju Chen:** Methodology, Conceptualization, Supervision, Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

The data is available at https://maifile.cn/est/d2516952766555/pdf.

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Appendix A

Theorem 1. *Under scenario s, if the distribution of random variable ζ changes in ambiguity set (2), then the following convex constraints system (A.1) is a robust counterpart approximation of ACC (3).*

$$\begin{cases} \sum_{f=1}^{F} |\tau_f| + \Omega \sqrt{\sum_{f=1}^{F} \varphi_f^2} \le \sum_{(e,f)\in Arc} t_{ef}^{ps} + sh_f^{ps} - d_f^0, \\ \tau_f + \varphi_f = -\widetilde{d_f^{ps}}, \qquad \forall f \in F. \end{cases} \tag{A.1}$$

In other words, (A.1) is the robust counterpart of uncertain inequality (1) with respect to the perturbation set $Z_{Box-Ball}$, where $Z_{Box-Ball}$ is the intersection of box and ball.

Proof. The intersection of box and ball is expressed as $Z_{Box-Ball}$, specifically,

$$Z_{Box-Ball} = \{\zeta \in R^F : -1 \le \zeta_f \le 1, f \in F, \sqrt{\sum_f \zeta_f^2} \le \Omega\}, \tag{A.2}$$

where $\Omega \ge 0$ and $\sigma \ge 0$.

Constraint (1), i.e.,

$$\sum_{(e,f)\in Arc} t_{ef}^{ps} + sh_f^{ps} \ge d_f^{ps}$$

can be rewritten as

$$-\sum_{(e,f)\in Arc} t_{ef}^{ps} - sh_f^{ps} \le -d_f^0 - \zeta_f \widetilde{d_f^{ps}}.$$

The perturbation set $Z_{Box-Ball}$ has a cone representation as follows

$$Z = \{\zeta \in R^F : P_1\zeta + p_1 \in L^1, P_2\zeta + p_2 \in L^2\}, \tag{A.3}$$

where $P_1\zeta \equiv [\zeta; 0], p_1 = [0_{F\times 1}; 1]$ and $L^1 = \{(z,t) \in R^F \times R : t \ge \|z\|_\infty\}$, then $L_*^1 = \{(z,t) \in R^F \times R : t \ge \|z\|_1\}$; $P_2\zeta \equiv [\sum^{-1}\zeta; 0]$ with $\sum = Diag\{\sigma_1, \dots, \sigma_F\}$, $p_2 = [0_{F\times 1}; \Omega]$, and K^2 is a Lorentz cone of the dimension of the cone is $L+1$ (then $L_*^2 = L^2$).

Setting $y^1 = [\eta_1; \tau_1]$, $y^2 = [\eta_2; \tau_2]$, one-dimensional τ_1, τ_2 and L-dimensional η_1, η_2, (A.3) becomes the following constraints (a)–(d) in variables η, x, τ:

(a) $\tau_1 + \Omega\tau_2 + [a^0]^T x \le b^0$,
(b) $(\eta_1 + \sum^{-1}\eta_2)_f = b^f - [a^f]^T x, f = 1, \dots, F$,
(c) $\|\eta_1\|_1 \le \tau_1$ $\quad [\Leftrightarrow [\eta_1; \tau_1] \in L_*^1]$,
(d) $\|\eta_2\|_2 \le \tau_2$ $\quad [\Leftrightarrow [\eta_2; \tau_2] \in L_*^2]$.

For all feasible solutions of above system eliminating τ_1, τ_2, then get $\tau_1 \ge \overline{\tau}_1 \equiv \|\eta_1\|_1, \tau_2 \ge \overline{\tau}_2 \equiv \|\eta_2\|_2$. Displacing τ_1, τ_2 with $\overline{\tau}_1, \overline{\tau}_2$, the obtained solution is feasible still. The system $(a) - (d)$ reads

$$\sum_{f=1}^{F} |\tau_f| + \Omega \sqrt{\sum_f \varphi_f^2} \le b^0 - [a^0]^T x = \sum_{(e,f)\in Arc} t_{ef}^{ps} + sh_f^{ps} - d_f^0, \quad \square$$

$$\tau_f + \varphi_f = b^f - [a^f]^T x = -\widetilde{d_f^{ps}}, f = 1, \dots, F.$$

Theorem 2. *Under scenario s, if the distribution of random variable ζ changes in ambiguity set (2), then the following convex constraints system (A.4) is a robust counterpart approximation of ACC (3).*

$$\begin{cases} \sum_{f=1}^{F} |\lambda_f| + \gamma max_f |w_f| \le \sum_{(e,f)\in Arc} t_{ef}^{ps} + sh_f^{ps} - d_f^0, \\ \lambda_f + w_f = -\widetilde{d_f^{ps}} \qquad \forall f \in F. \end{cases} \tag{A.4}$$

In other words, (A.4) is the robust counterpart of uncertain inequality (1) with respect to the perturbation set Z_{Budget}.

Proof. Consider perturbation set

$$Z_{Budget} = \{\zeta \in R^F : \|\zeta\|_1 \leq \gamma, \|\zeta\|_\infty \leq 1\}, \tag{A.5}$$

where γ is a given uncertain budget.

Theorem 2 can be obtained with the following constraints system:

$$\sum_{f=1}^{F} |\lambda_f| + \gamma max_f |w_f| \leq b^0 - [a^0]^T x = \sum_{(e,f)\in Arc} t_{ef}^{ps} + sh_f^{ps} - d_f^0,$$

$$\lambda_f + w_f = b^f - [a^f]^T x = -\widetilde{d_f^{ps}}, f = 1, \ldots, F,$$

which is equivalent to constraints system (A.4). □

Proposition 1. *Let coefficients $z_f, f = 1, \ldots, F$, be deterministic, random variables $\zeta_f, f = 1, \ldots, F$, be independent, which have zero mean and belong to $[-1, 1]$. Then, we get*

$$Prob\{\zeta : \sum_{f=1}^{F} z_f \zeta_f \geq \Omega \sqrt{\sum_{f=1}^{F} z_f^2}\} \leq exp\{-\frac{\Omega^2}{2}\}$$

for each $\Omega \geq 0$.

As an immediate conclusion, based on ambiguity set (2), we get

$$Prob\{\sum_{f=1}^{F} [a^f]^T x - b^f \zeta_f \geq \Omega \sqrt{\sum_{f=1}^{F} ([a^f]^T x - b^f)^2}\} \leq exp\{-\frac{\Omega^2}{2}\} \quad \forall \Omega \geq 0$$

$$\Longrightarrow Prob\{\sum_{f=1}^{F} [\widetilde{d_f^{ps}}]\zeta_f \geq \Omega \sqrt{\sum_{f=1}^{F} \widetilde{d_f^{ps}}^2}\} \leq exp\{-\frac{\Omega^2}{2}\} \qquad \forall \Omega \geq 0.$$

Proposition 2. *The robust counterpart approximation of ACC (3) under perturbation set (A.2) is cone quadratic constraints (A.1). When the independent random variables $\zeta_1, \zeta_2, \ldots, \zeta_F$ belong to ambiguity set (2), then the probability that the feasible solution of cone quadratic constraints (A.1) satisfies the inequality in ACC (3) is at least $1 - exp\{-\frac{\Omega^2}{2}\}$, $\Omega \geq 0$.*

Proof. It is a known fact that cone quadratic constraints (A.1) represent the robust counterpart approximation of ACC (3) under $Z_{Box-Ball}$. Now we prove that if $\zeta_1, \zeta_2, \ldots, \zeta_F$ satisfy assumption (2) and x, τ, φ are available from (A.1), then x is solution for (3) with probability not less than $1 - exp\{-\frac{\Omega^2}{2}\}$, $\Omega \geq 0$. Indeed, when $\|\zeta\|_\infty \leq 1$, we have

$$\sum_{f=1}^{F} [[a^f]^T x - b^f]\zeta_f > b^0 - [a^0]^T x$$

$$\Longrightarrow \sum_{f=1}^{F} [\tau_f + \varphi_f]\zeta_f > b^0 - [a^0]^T x$$

$$\Longrightarrow \sum_{f=1}^{F} |\tau_f| + \sum_{f=1}^{F} \varphi_f \zeta_f > b^0 - [a^0]^T x$$

$$\Longrightarrow \sum_{f=1}^{F} |\tau_f| + \sum_{f=1}^{F} \varphi_f \zeta_f > \sum_{f=1}^{F} |\tau_f| + \Omega \sqrt{\sum_f \varphi_f^2}$$

$$\Longrightarrow \sum_{f=1}^{F} \varphi_f \zeta_f > \Omega \sqrt{\sum_f \varphi_f^2}.$$

Thus, for each distribution A that is compatible with (2), we obtain

$$Prob_{\zeta_f \sim A}\{x\ is\ feasible\ for\ (37)\}$$
$$\leq Prob_{\zeta_f \sim A}\{\sum_{f=1}^{F} \varphi_f \zeta_f > \Omega \sqrt{\sum_f \varphi_f^2}\} \leq exp\{-\frac{\Omega^2}{2}\}, \tag{A.6}$$

where the last inequality is due to Proposition 1. □

Proposition 3. *The robust counterpart approximation of ACC (3) under perturbation set (A.5) is cone quadratic constraints (A.4). When the independent random variables $\zeta_1, \zeta_2, \ldots, \zeta_F$ belong to ambiguity set (2), then the probability that the feasible solution of cone quadratic constraints (A.4) satisfies the inequality in ACC (3) is at least $1 - exp\{-\frac{\gamma^2}{2F}\}$.*

Proof. This proof is similar to that of Proposition 2. □

Thus, parameter $\frac{\gamma}{\sqrt{F}}$ in Proposition 3 plays the equal role as Ω plays in Proposition 2.

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.compchemeng.2023.108428.

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