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Stage interval ratio DEA models and applications

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ABSTRACT

Data envelopment analysis (DEA) is a crucial method for assessing the efficiency of decision-making units (DMUs). In practice, it is common to encounter situations where the performance of a DMU at a certain stage is evaluated, especially in policy effect evaluation. The key to stage performance evaluation is to eliminate the influence of pre-stage inputs (outputs) on stage performance. This paper proposes stage ratio DEA models that consider two types of inputs and outputs: accurate ratio data and interval ratio data. The first form considers the evaluation of stage efficiency when accurate ratios of inputs and outputs are used at the beginning and end of the stage. The second form assesses stage efficiency using interval ratio data for interval inputs and outputs. To verify the validity of the proposed models, the numerical example validates the first form of stage ratio DEA models. And the second form is applied to evaluate the sustainable efficiency of 14 energy saving and environmental protection clean enterprises (ESEPCEs). The empirical results demonstrate that the proposed models provide a more accurate assessment of stage efficiency compared to the traditional DEA-CCR model.

1. Introduction

DEA is an important non-parametric performance evaluation method for evaluating the relative efficiency of multi-input and multi-output production systems (Ripoll-Zarraga & Lozano, 2020; Topcu & Triantis, 2022). Since its first introduction in 1978 (Charnes et al., 1978), DEA has been quickly recognized as a powerful efficiency analysis tool (Dehnokhalaji et al., 2022; Omrani et al., 2022; Wanke et al., 2023). This approach can achieve two goals of efficiency evaluation: understanding past efficiency and planning to become more efficient in the future (Cordero et al., 2021; Li et al., 2018; Tone, 2001).

The traditional DEA models assume that the inputs–outputs are deterministic. However, in practice, inputs–outputs have many uncertainties because some data may be known only within specified bounds and ordinal relations. The problem of imprecise data has attracted the attention of many scholars. Cooper et al. (1999) proposed the imprecise DEA (IDEA) model, which allows for situations where there is both imprecise and exactly known data in which the IDEA models are transformed into linear programming problems. Therefore, the interval DEA model, as one of the IDEA models, has also attracted the attention of many scholars and conducted a lot of research.

Interval DEA is generated to solve the problem of interval inputs–outputs. Despotis and Smirlis (2002) proposed an approach that treated interval DEA as a peculiar case of accurate DEA. They defined lower and upper bounds for interval efficiency scores and further discriminated DMUs into fully efficient, efficient, and inefficient units. After identifying efficient and inefficient DMUs using interval DEA, one can wonder how sensitive these identifications are to data variances. Jahanshahloo et al. (2004) focused on the sensitivity and stability of interval DEA. Arabmaldar et al. (2021) formulated a new robust worst-practice frontier interval DEA model. An et al. (2022) proposed an interval DEA model based on slacks-based measures. This model addressed uncertainty based on the estimated farthest and closest distances of the DMU to the projected point on the Pareto efficient frontier. The above literature suggested a unified production frontier and corrected the distance measure efficiency scores of the former.

After discussing the frontier and stability issues of the interval DEA model, some scholars have expanded the interval DEA model from the perspective of interval inputs–outputs. For example, Arana-Jiménez et al. (2021) proposed a novel integer interval DEA model under fuzzy integer intervals. Younesi et al. (2023) proposed an inverse DEA model based on the non-radial slacks model to solve the input–output problem under integer and continuous interval data with uncertainty. Santos-Arteaga et al. (2023) considered changes in efficiency to be determined through increases or decreases based on potential inputs and outputs and the number of DMUs. As for the interval input–output in the form of a ratio, there are two main forms. One is that inputs–outputs are expressed in terms of ratios. Entani et al. (2002) proposed two new

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models: general and multiplicative non-parametric ratio models for DEA problems with interval data. The ratios of the two models refer to the form of indicators of inputs and outputs. Amin and Hajjami (2016) used the interval DEA model to select high-quality stocks by analyzing financial ratio indicators. The other form is that inputs–outputs are expressed in terms of interval scale data. Nasrabadi et al. (2019) presented a model for efficiency analysis that incorporates interval scale data in addition to ratio scale data. They considered the input–output expressed in interval ratios in DEA, which solved the problem of not being suitable or difficult to model with accurate values. Then, Nasrabadi et al. (2022) further examined the robustness of efficiency scores within the context of DEA with interval-scale data. The inputs–outputs of the above models are almost developed from accurate values to interval values. Similarly, this paper also generalizes from accurate ratio inputs–outputs to interval ratio inputs–outputs. But the form of ratio used in this paper is different from the above literature. The ratio of inputs (outputs) in this paper refers to the ratio between the end and the beginning of the evaluated stage.

The above DEA models discuss the overall performance based on a group of DMUs. However, understanding performance at a certain stage is more common in practice. For example, the effectiveness evaluation of "double first-class" construction in Chinese universities is the stage performance evaluation of the construction cycle. Stage performance evaluation is a conclusive evaluation of the DMUs' stage performance. The purpose of stage performance evaluation is to help the DMUs recognize their own strengths and weaknesses in the current stage and correct their present stage problems for better performance in the next stage. Therefore, stage evaluation methods have attracted the attention of many scholars.

DEA-Malmquist index models are often used to evaluate changes in stage performance. Fare et al. (1994) proposed the DEA-Malmquist index models to study productivity change in Swedish hospitals during the period from 1970–1985. Based on this, some scholars have further extended the DEA-Malmquist model according to different issues. Dorri and Rostamy-Malkhalifeh (2017) developed the DEA-Malmquist index model, aiming to determine the progress of the DMUs under evaluation in the presence of ratio data. Ding et al. (2020) extended Malmquist index based on cooperative game network DEA to evaluate industrial circular economy performance and its dynamic evaluation over 2012–2017. The interval DEA-Malmquist index has also attracted the attention of scholars. Huang et al. (2021) proposed a novel global Malmquist–Luenberger index with the interval slacks-based measure to evaluate the provincial green total factor productivity in China from 2000–2018. Bansal (2023) built an interval sequential Malmquist–Luenberger index model to measure productivity change intervals for 21 banks from 2011–2018. Although the DEA-Malmquist model evaluates changes in stage performance, the evaluation results are actually affected by the performance of the previous stage, thus affecting the true situation of the evaluated stage. But the existing DEA-Malmquist index models cannot solve this stage evaluation problem. Therefore, one of the tasks of this paper is to eliminate the influence of the performance of the previous stage and put forward the corresponding stage performance evaluation models.

The purpose of this paper is to study stage performance evaluation. As discussed above, stage efficiency obtained using existing models includes the impact of previous stage performance. Therefore, this paper uses the stage ratio data of input (output) as the input–output data for stage performance evaluation to eliminate the influence of pre-stage input–output on stage performance. Meanwhile, this paper proposes stage ratio DEA models that consider two types of inputs–outputs: accurate ratio data and interval ratio data. Numerical example and practical application verify the effectiveness of the proposed models, respectively. The specific work of this paper is as follows:

- Using the ratio data of inputs and outputs of the evaluated stage, i.e., (stage end)/(stage beginning), can eliminate the influence of the previous stage's efficiency.
- This paper first proposes stage ratio DEA models under accurate ratio data of inputs–outputs and gives its theorems.
- Based on the proposed stage accurate ratio DEA model, the accurate ratio inputs–outputs are further expanded to the interval ratio inputs–outputs, and the stage interval ratio DEA model is proposed.
- To validate the proposed stage ratio DEA models, a numerical example is provided to validate the first form of the stage ratio DEA model. The second form is conducted to evaluate the sustainable efficiency of 14 ESEPCEs in China during the green transition stage.

The remainder of this paper unfolds as follows. Section 2 introduces the relevant preliminaries. Section 3 describes stage ratio DEA models under two kinds of inputs–outputs and gives the related definitions and theorems. Section 4 applies a numerical example and application to verify the performance of two kinds of stage ratio DEA models. Finally, Section 5 provides the concluding remarks.

2. Preliminaries

This section reviews the traditional DEA-CCR model, interval DEA model, interval data and their arithmetic.

2.1. DEA-CCR model

Assume that we have n DMUs ($DMU_j, j = 1, \dots, n$), each associates with m inputs $X_j = \left(x_{1j}, x_{2j}, \dots, x_{mj}\right)^T$ and s outputs $Y_j = \left(y_{1j}, y_{2j}, \dots, y_{sj}\right)^T, j = 1, \dots, n$. The production possibility set T can be represented in an algebraic form:

$$T = \left\{ (X, Y) |\, X \geq \sum_{j=1}^{n} \lambda_j X_j, Y \leq \sum_{j=1}^{n} \lambda_j Y_j, \lambda_j > 0, j = 1, \dots, n \right\} \tag{1}$$

Charnes et al. (1978) proposed the following DEA-CCR model to measure the relative efficiency of DMUs:

$$\begin{aligned} &\max\ \theta_0 = \sum_{r=1}^{s} \mu_r y_{r0} \\ &s.t. \\ &\begin{cases} \sum_{r=1}^{s} \mu_r y_{rj} - \sum_{i=1}^{m} \omega_i x_{ij} \leq 0, j = 1, \dots, n, \\ \sum_{i=1}^{m} \omega_i x_{i0} = 1, \\ \mu_r, \omega_i \geq 0, r = 1, \dots, s; i = 1, \dots, m. \end{cases} \end{aligned} \tag{2}$$

where the subscript 0 indicates the DMU under evaluation, $\omega_i (i = 1, \dots, m)$ and $\mu_r (r = 1, \dots, s)$ are decision variables.

Definition 2.1. If the optimization model (2) has optimal solutions μ_r^*, ω_i^*, such that the efficiency $\theta_0^* = \sum_{r=1}^{s} \mu_r^* y_{r0} = 1$, and $\mu_r^* > 0, \omega_i^* > 0$, then DMU_{j_0} is defined as DEA efficient.

2.2. Interval DEA model

2.2.1. Interval data and their arithmetic

The statistical treatment of interval data is developed by considering them as elements that belong to the space $K_c(R) = \{[a, b] \,|\, a \leq b, a, b \in R\}$. Each compact interval $A \in K_c(R)$ can be expressed using its [inf, sup] representation, ie. $A = [\inf A, \sup A]$, where $\inf A \leq \sup A$.

To manage the intervals, natural arithmetic is defined for $K_c(R)$ using the Minkowski addition $A + B = \{a + b : a \in A, b \in B\}$ and the product using scalars $\lambda A = \{\lambda a : a \in A\}$, for any $A, B \in K_c(R)$ and $\lambda \in R$. The space $(K_c(R), +, \cdot)$ is not linear but semi-linear due to the lack of symmetric elements, which are based on addition. The operations for two intervals $A = [a^-, a^+]$ and $B = [b^-, b^+]$ in $K_c(R)$ are as follows:

- Addition: $A + B = [a^- + b^-, a^+ + b^+]$
- Subtraction: $A - B = A + (-B) = [a^- - b^+, a^+ - b^-]$
- Multiplication: For $\lambda \in R$,

$$\lambda B = \begin{cases} [\lambda b^-, \lambda b^+], \lambda \geq 0, \\ [\lambda b^+, \lambda b^-], \lambda < 0. \end{cases}$$

$$A \times B = \big[a^-b^- \wedge a^-b^+ \wedge a^+b^- \wedge a^+b^+, \quad a^-b^- \vee a^-b^+ \vee a^+b^- \vee a^+b^+ \big].$$

- Division: $A \div B = A \times \frac{1}{B} = \left[\frac{a^-}{b^+}, \frac{a^+}{b^-}\right]$, where $A, B \in K_c(R^+)$, $\frac{1}{B} = \left[\frac{1}{b^+}, \frac{1}{b^-}\right]$.

Definition 2.2 (*Ranking Intervals*)**.** Given any two intervals $A = [a^-, a^+]$, $B = [b^-, b^+]$, if $a^- < b^-$ or $a^- = b^-$, $a^+ \leq b^+$, then $A \tilde{\lesssim} B$.

2.2.2. Interval DEA model

Assume that we have n DMUs ($DMU_j, j = 1, \ldots, n$), each associates with m interval-valued inputs $X_j = (x_{1j}, x_{2j}, \cdots, x_{mj})^T$ and s interval-valued outputs $Y_j = (y_{1j}, y_{2j}, \ldots, y_{sj})^T, j = 1, \ldots, n$, where $x_{ij} = [x_{ij}^L, x_{ij}^U]$, $y_{rj} = [y_{rj}^L, y_{rj}^U]$ and the lower and upper bounds are positive and finite values. Therefore, the traditional interval DEA model (Despotis & Smirlis, 2002) for measuring left endpoint of interval-valued efficiency is as follows:

$$\begin{aligned} &\theta_0^L = \max \sum_{r=1}^{s} \mu_r y_{r0}^L \\ &s.t. \qquad\qquad\qquad\qquad\qquad\qquad (3) \\ &\begin{cases} \sum_{i=1}^{m} \omega_i x_{i0}^U = 1, \\ \sum_{r=1}^{s} \mu_r y_{r0}^L - \sum_{i=1}^{m} \omega_i x_{i0}^U \leq 0, \\ \sum_{r=1}^{s} \mu_r y_{rj}^U - \sum_{i=1}^{m} \omega_i x_{ij}^L \leq 0, j = 1, \ldots, n; j \neq 0, \\ \omega_i, \mu_r \geq 0, \forall i, r. \end{cases} \end{aligned}$$

Correspondingly, the model for measuring the right endpoint of interval-valued efficiency is as follows:

$$\begin{aligned} &\theta_0^U = \max \sum_{r=1}^{s} \mu_r y_{r0}^U \\ &s.t. \qquad\qquad\qquad\qquad\qquad\qquad (4) \\ &\begin{cases} \sum_{i=1}^{m} \omega_i x_{i0}^L = 1, \\ \sum_{r=1}^{s} \mu_r y_{r0}^U - \sum_{i=1}^{m} \omega_i x_{i0}^L \leq 0, \\ \sum_{r=1}^{s} \mu_r y_{rj}^L - \sum_{i=1}^{m} \omega_i x_{ij}^U \leq 0, j = 1, \ldots, n; j \neq 0, \\ \omega_i, \mu_r \geq 0, \forall i, r. \end{cases} \end{aligned}$$

3. Model formulation

For the convenience of later description, suppose t_0 is the beginning point of the evaluated stage, l is the length of this stage, and we define the period from t_0 to $t_0 + l$ as the stage t_0. This section proposes two forms of stage ratio DEA models to evaluate the efficiency of stage t_0 and give related definitions and theorems.

3.1. Stage ratio DEA model with accurate ratio inputs–outputs

$S = \left\{ \left(X_j^t, Y_j^t\right) | i = 1, \ldots, n; t = 1, \ldots, T \right\}$ is a panel production possibility set, where $X_j^t = \left(x_{1j}^t, x_{2j}^t, \ldots, x_{mj}^t\right)^T$, $Y_j^t = \left(y_{1j}^t, y_{2j}^t, \ldots, y_{sj}^t\right)^T$, $X_j^t > 0, Y_j^t > 0$. To better assess the efficiency of stage t_0, we use ratio data for each input–output, $\frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}}, \frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}} (t_0 \leq t_0 + l \leq T)$, in stage ratio DEA model. If $x_{ij}^{t_0}, y_{rj}^{t_0} = 0$, the non-Archimedean quantity $\varepsilon > 0$ is introduced. The ratio of inputs–outputs uses $\frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}+\varepsilon}, \frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}+\varepsilon}$. For simplicity, we assume the $x_{ij}^{t_0} > 0, y_{rj}^{t_0} > 0$ in the following.

The production possibility set S^{t_0} of stage t_0 is:

$$S^{t_0} = \left\{ (X, Y) | \sum_{j=1}^{n} \lambda_j \frac{x_j^{t_0+l}}{x_j^{t_0}} \leq X, \sum_{j=1}^{n} \lambda_j \frac{y_j^{t_0+l}}{y_j^{t_0}} \geq Y, \lambda_j \geq 0, j = 1, \ldots, n \right\}$$

It is easy to prove that the S^{t_0} satisfies the basic axioms of the production possibility set.

The efficiency of evaluated DMU_0 in stage t_0 can be evaluated by the following optimization model:

$$\begin{aligned} &\max \ \theta_0(x, y) = \sum_{r=1}^{s} \mu_r \frac{y_{r0}^{t_0+l}}{y_{r0}^{t_0}} \\ &s.t. \qquad\qquad\qquad\qquad\qquad\qquad (5) \\ &\begin{cases} \sum_{r=1}^{s} \mu_r \frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}} - \sum_{i=1}^{m} \omega_i \frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}} \leq 0, j = 1, \ldots, n, \\ \sum_{i=1}^{m} \omega_i \frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}} = 1, \\ \mu_r, \omega_i \geq 0, r = 1, \ldots, s; i = 1, \ldots, m. \end{cases} \end{aligned}$$

where t_0, $t_0 + l$ are the beginning and end periods of stage t_0, respectively, and l is the stage length. $\frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}}, \frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}}$ are the ratios of ith inputs and rth outputs of DMU_j in stage t_0, respectively, and μ_r, ω_i are weight vectors.

Using Lagrange multiplier method, the dual form of model (5) can be obtained as follows:

$$\begin{aligned} &\min \theta_0 \\ &s.t. \qquad\qquad\qquad\qquad\qquad\qquad (6) \\ &\begin{cases} \sum_{j=1}^{n} \lambda_j \frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}} \leq \theta_0 \frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}}, i = 1, \ldots, m, \\ \sum_{j=1}^{n} \lambda_j \frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}} \geq \frac{y_{r0}^{t_0+l}}{y_{r0}^{t_0}}, r = 1, \ldots, s, \\ \lambda_j \geq 0, j = 1, \ldots, n. \end{cases} \end{aligned}$$

Models (5) and (6) calculate efficiency values $\theta_0 \in [0, 1]$. When $\theta_0 = 1$, the DMU is efficient in the stage t_0; otherwise, it is inefficient. The above models use inputs (outputs) ratio data from the beginning (t_0) and end (t_0+l) of the stage t_0 to study the stage efficiency of DMU_0.

Theorem 3.1. *Model (5) is always feasible.*

Proof. According to the conditions in model (5), there exists a certain k, such that $\frac{y_{kj}^{t_0+l}}{y_{kj}^{t_0}} > 0, \frac{x_{kj}^{t_0+l}}{x_{kj}^{t_0}} > 0$.

It may be assumed $k=1$, let $\hat{\omega}=\left(\hat{\omega}_1,\ldots,\hat{\omega}_m\right)=\left(\frac{x_{1j}^{t_0}}{x_{1j}^{t_0+l}},0,\ldots,0\right)$, $\hat{\mu}=\left(\hat{\mu}_1,\ldots,\hat{\mu}_s\right)=\left(\frac{y_{1j}^{t_0}}{y_{1j}^{t_0+l}},0,\ldots,0\right)$, where $\hat{\omega}_1=\frac{x_{1j}^{t_0}}{x_{1j}^{t_0+l}}$, $\hat{\mu}_1=\min\limits_{1\le j\le n}\frac{y_{1j}^{t_0}}{y_{1j}^{t_0+l}}$, it easily to see $\hat{\omega}$, $\hat{\mu}$ satisfy

$$\begin{cases}\sum_{r=1}^{s}\hat{\mu}_r\frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}}-\sum_{i=1}^{m}\hat{\omega}_i\frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}}\le 0,\\ \sum_{i=1}^{m}\hat{\omega}_i\frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}}=1,\\ \hat{\omega}_i,\hat{\mu}_r\ge 0.\end{cases}$$

Therefore, the feasible solution for model (5) is then obtained, which holds for all constrains. □

Theorem 3.2. *The validity of model (5) is not related to the dimensional selection of input and output, nor to the multiple growth of input and output corresponding to DMU.*

Proof. There is a multiple change between different dimensions, so assume that the ith input is $\alpha_i x_{i1},\ldots,\alpha_i x_{in}, i=1,2,\ldots,m$; ($\alpha_i$ is the proportion of input increase or decrease, $\alpha_i\neq 0$) in the new dimension. Similarly, the rth output is $\beta_r y_{r1},\ldots,\beta_r y_{rn}, r=1,2,\ldots,s$; ($\beta_r$ is the proportion of output increase or decrease, $\beta_r\neq 0$) in the new dimension. Then the stage ratio DEA model under the new dimension is:

$$\max\ \bar{\theta}_0=\sum_{r=1}^{s}\mu_r\frac{\beta_r y_{r0}^{t_0+l}}{\beta_r y_{r0}^{t_0}}$$
$$s.t.\qquad (7)$$
$$\begin{cases}\sum_{r=1}^{s}\mu_r\frac{\beta_r y_{rj}^{t_0+l}}{\beta_r y_{rj}^{t_0}}-\sum_{i=1}^{m}\omega_i\frac{\alpha_i x_{ij}^{t_0+l}}{\alpha_i x_{ij}^{t_0}}\le 0, j=1,\ldots,n,\\ \sum_{i=1}^{m}\omega_i\frac{\alpha_i x_{i0}^{t_0+l}}{\alpha_i x_{i0}^{t_0}}=1,\\ \mu_r,\omega_i\ge 0, r=1,\ldots,s; i=1,\ldots,m.\end{cases}$$

If DMU is efficient before using the new dimensions, that is, there are optimal solutions $\mu_0,\omega_0\ge 0$ for model (5), such that $\theta_0=1$, then it can be verified that μ_0,ω_0 are also optimal solutions for model (7), and $\bar{\theta}_0=1$. Therefore, DMU is also efficient in the new dimension. Similarly, if the DMU is efficient under the new dimensions, it can be similarly proven that it is efficient in original dimensions. □

3.2. Stage ratio DEA model with interval ratio inputs–outputs

Suppose that a quantity with a superscript '^' indicates that it has an interval value, and that characters without superscripts are represented as real values. $S=\left\{\left(\hat{X}_j^t,\hat{Y}_j^t\right)|j=1,\ldots,n;t=1,\ldots,T\right\}$ is a panel interval production possibility set, where $\hat{X}_j^t=\left(\hat{x}_{1j}^t,\ldots,\hat{x}_{mj}^t\right)^T$ with $\hat{x}_{ij}^t=\left[\left(x_{ij}^t\right)^L,\left(x_{ij}^t\right)^U\right], i=1,\ldots,m$; $\hat{Y}_j^t=\left(\hat{y}_{1j}^t,\ldots,\hat{y}_{sj}^t\right)^T$ with $\hat{y}_{rj}^t=\left[\left(y_{rj}^t\right)^L,\left(y_{rj}^t\right)^U\right], r=1,\ldots,s$. In stage t_0, stage interval ratio DEA model is as follows:

$$\max\ \hat{\theta}_0(\hat{x},\hat{y})=\sum_{r=1}^{s}\mu_r\frac{\hat{y}_{r0}^{t_0+l}}{\hat{y}_{r0}^{t_0}}$$
$$s.t.\qquad (8)$$
$$\begin{cases}\sum_{r=1}^{s}\mu_r\frac{\hat{y}_{rj}^{t_0+l}}{\hat{y}_{rj}^{t_0}}-\sum_{i=1}^{m}\omega_i\frac{\hat{x}_{ij}^{t_0+l}}{\hat{x}_{ij}^{t_0}}\le 0, j=1,\ldots,n,\\ \sum_{i=1}^{m}\omega_i\frac{\hat{x}_{i0}^{t_0+l}}{\hat{x}_{i0}^{t_0}}=1,\\ \mu_r,\omega_i\ge 0, r=1,\ldots,s; i=1,\ldots,m.\end{cases}$$

For interval optimization problem (8), its objective function value is an interval. The stage interval ratio DEA model (8) is developed to generate upper and lower bounds for the interval-valued efficiency of each DMU:

$$\theta_0^L=\min_{\substack{\left(x_{ij}^{t_0}\right)^L\le x_{ij}^{t_0}\le\left(x_{ij}^{t_0}\right)^U\\ \left(y_{rj}^{t_0}\right)^L\le y_{rj}^{t_0}\le\left(y_{rj}^{t_0}\right)^U\\ \left(x_{ij}^{t_0+l}\right)^L\le x_{ij}^{t_0+l}\le\left(x_{ij}^{t_0+l}\right)^U\\ \left(y_{rj}^{t_0+l}\right)^L\le y_{rj}^{t_0+l}\le\left(y_{rj}^{t_0+l}\right)^U\\ \forall i,r,j}}\begin{cases}\max\sum_{r=1}^{s}\mu_r\frac{y_{r0}^{t_0+l}}{y_{r0}^{t_0}}\\ s.t.\\ \sum_{i=1}^{m}\omega_i\frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}}=1,\\ \sum_{r=1}^{s}\mu_r\frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}}-\sum_{i=1}^{m}\omega_i\frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}}\le 0, j=1,\ldots,n,\\ \mu_r,\omega_i\ge 0, r=1,\ldots,s; i=1,\ldots,m.\end{cases}\qquad (9)$$

$$\theta_0^U=\max_{\substack{\left(x_{ij}^{t_0}\right)^L\le x_{ij}^{t_0}\le\left(x_{ij}^{t_0}\right)^U\\ \left(y_{rj}^{t_0}\right)^L\le y_{rj}^{t_0}\le\left(y_{rj}^{t_0}\right)^U\\ \left(x_{ij}^{t_0+l}\right)^L\le x_{ij}^{t_0+l}\le\left(x_{ij}^{t_0+l}\right)^U\\ \left(y_{rj}^{t_0+l}\right)^L\le y_{rj}^{t_0+l}\le\left(y_{rj}^{t_0+l}\right)^U\\ \forall i,r,j}}\begin{cases}\max\sum_{r=1}^{s}\mu_r\frac{y_{r0}^{t_0+l}}{y_{r0}^{t_0}}\\ s.t.\\ \sum_{i=1}^{m}\omega_i\frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}}=1,\\ \sum_{r=1}^{s}\mu_r\frac{y_{rj}^{t_0+l}}{\hat{y}_{rj}^{t_0}}-\sum_{i=1}^{m}\omega_i\frac{\hat{x}_{ij}^{t_0+l}}{x_{ij}^{t_0}}\le 0, j=1,\ldots,n,\\ \mu_r,\omega_i\ge 0, r=1,\ldots,s; i=1,\ldots,m.\end{cases}\qquad (10)$$

The above models are converted to the following equivalent linear programming models:

$$\theta_0^L=\max\sum_{r=1}^{s}u_r\frac{\left(y_{r0}^{t_0+l}\right)^L}{\left(y_{r0}^{t_0}\right)^U}$$
$$s.t.\qquad (11)$$
$$\begin{cases}\sum_{i=1}^{m}v_i\frac{\left(x_{i0}^{t_0+l}\right)^U}{\left(x_{i0}^{t_0}\right)^L}=1,\\ \sum_{r=1}^{s}u_r\frac{\left(y_{r0}^{t_0+l}\right)^L}{\left(y_{r0}^{t_0}\right)^U}-\sum_{i=1}^{m}v_i\frac{\left(x_{i0}^{t_0+l}\right)^U}{\left(x_{i0}^{t_0}\right)^L}\le 0,\\ \sum_{r=1}^{s}u_r\frac{\left(y_{rj}^{t_0+l}\right)^U}{\left(y_{rj}^{t_0}\right)^L}-\sum_{i=1}^{m}v_i\frac{\left(x_{ij}^{t_0+l}\right)^L}{\left(x_{ij}^{t_0}\right)^U}\le 0, j=1,\ldots,n, j\neq 0,\\ u_r,v_i\ge 0,\forall r,i.\end{cases}$$

$$\theta_0^U=\max\sum_{r=1}^{s}u_r\frac{\left(y_{r0}^{t_0+l}\right)^U}{\left(y_{r0}^{t_0}\right)^L}$$
$$s.t.\qquad (12)$$

$$\begin{cases} \sum_{i=1}^{m} v_i \frac{\left(x_{i0}^{t_0+l}\right)^L}{\left(x_{i0}^{t_0}\right)^U} = 1, \\ \sum_{r=1}^{s} u_r \frac{\left(y_{r0}^{t_0+l}\right)^U}{\left(y_{r0}^{t_0}\right)^L} - \sum_{i=1}^{m} v_i \frac{\left(x_{i0}^{t_0+l}\right)^L}{\left(x_{i0}^{t_0}\right)^U} \le 0, \\ \sum_{r=1}^{s} u_r \frac{\left(y_{rj}^{t_0+l}\right)^L}{\left(y_{rj}^{t_0}\right)^U} - \sum_{i=1}^{m} v_i \frac{\left(x_{ij}^{t_0+l}\right)^U}{\left(x_{ij}^{t_0}\right)^L} \le 0, j = 1, \dots, n, j \ne 0 \\ u_r, v_i \ge 0, \forall r, i. \end{cases}$$

By repeating the solution of the above linear programming models for each DMU, we can obtain the overall stage interval-valued efficiency of n DMUs, denoted by efficiency intervals $\left[\theta_0^L, \theta_0^U\right]$ $(j = 1, \dots, n)$. Next theorem provides the dual of models (11) and (12).

Theorem 3.3. *The dual of models (11) and (12) are as follows:*

$$\theta_0^L = \min\ \theta$$
$$s.t. \tag{13}$$
$$\begin{cases} \sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{\left(x_{ij}^{t_0+l}\right)^U}{\left(x_{ij}^{t_0}\right)^L} + \left(\lambda_0 - \theta\right) \frac{\left(x_{i0}^{t_0+l}\right)^L}{\left(x_{i0}^{t_0}\right)^U} \le 0, \\ \sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{\left(y_{rj}^{t_0+l}\right)^L}{\left(y_{rj}^{t_0}\right)^U} + \left(\lambda_0 - 1\right) \frac{\left(y_{r0}^{t_0+l}\right)^U}{\left(y_{r0}^{t_0}\right)^L} \ge 0, \\ \lambda_j \ge 0, r = 1, \dots, s, i = 1, \dots, m. \end{cases}$$

$$\theta_0^U = \min\ \theta$$
$$s.t. \tag{14}$$
$$\begin{cases} \sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{\left(x_{ij}^{t_0+l}\right)^L}{\left(x_{ij}^{t_0}\right)^U} + \left(\lambda_0 - \theta\right) \frac{\left(x_{i0}^{t_0+l}\right)^U}{\left(x_{i0}^{t_0}\right)^L} \le 0, \\ \sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{\left(y_{rj}^{t_0+l}\right)^U}{\left(y_{rj}^{t_0}\right)^L} + \left(\lambda_0 - 1\right) \frac{\left(y_{r0}^{t_0+l}\right)^L}{\left(y_{r0}^{t_0}\right)^U} \ge 0, \\ \lambda_j \ge 0, r = 1, \dots, s, i = 1, \dots, m. \end{cases}$$

Proof. Model (11) is deduced by model (9). The dual model of model (9) is as follows:

$$\theta_0^L = \min_{\substack{\left(X_{ij}^{t_0}\right)^L \le x_{ij}^{t_0} \le \left(X_{ij}^{t_0}\right)^U \\ \left(Y_{rj}^{t_0}\right)^L \le y_{rj}^{t_0} \le \left(Y_{rj}^{t_0}\right)^U \\ \left(X_{ij}^{t_0+l}\right)^L \le x_{ij}^{t_0+l} \le \left(X_{ij}^{t_0+l}\right)^U \\ \left(Y_{rj}^{t_0+l}\right)^L \le y_{rj}^{t_0+l} \le \left(Y_{rj}^{t_0+l}\right)^U \\ \forall i,r,j}} \begin{cases} \min \theta \\ s.t. \\ \sum_{j=1}^{n} \lambda_j \frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}} \le \theta \frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}}, \\ \sum_{j=1}^{n} \lambda_j \frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}} \ge \frac{y_{r0}^{t_0+l}}{y_{r0}^{t_0}}, \\ \lambda_j \ge 0, r = 1, \dots, s; i = 1, \dots, m. \end{cases}$$

Since $\lambda_0 \frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}} \le \sum_{j=1}^{n} \lambda_j \frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}} \le \theta \frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}}$, where $\frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}} \ge 0, \lambda_0 \ge 0$, so $\lambda_0 - \theta \le 0$. And $0 \le \theta \le 1$, $\lambda_0 - 1 \le 0$.

Therefore, we can get $\sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}} + \left(\lambda_0 - \theta\right) \frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}} \le 0$ from $\sum_{j=1}^{n} \lambda_j \frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}} \le \theta \frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}}$. Meanwhile, we can also get $\sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}} + \left(\lambda_0 - 1\right) \frac{y_{r0}^{t_0+l}}{y_{r0}^{t_0}} \ge 0$ from $\sum_{j=1}^{n} \lambda_j \frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}} \ge \frac{y_{r0}^{t_0+l}}{y_{r0}^{t_0}}$.

So we can derive the following inequalities:

$$\begin{aligned} &\sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{\left(x_{ij}^{t_0+l}\right)^L}{\left(x_{ij}^{t_0}\right)^U} + \left(\lambda_0 - \theta\right) \frac{\left(x_{i0}^{t_0+l}\right)^U}{\left(x_{i0}^{t_0}\right)^L} \\ &\le \sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}} + \left(\lambda_0 - \theta\right) \frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}} \\ &\le \sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{\left(x_{ij}^{t_0+l}\right)^U}{\left(x_{ij}^{t_0}\right)^L} + \left(\lambda_0 - \theta\right) \frac{\left(x_{i0}^{t_0+l}\right)^L}{\left(x_{i0}^{t_0}\right)^U}, \end{aligned} \tag{15}$$

$$\begin{aligned} &\sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{\left(y_{rj}^{t_0+l}\right)^L}{\left(y_{rj}^{t_0}\right)^U} + \left(\lambda_0 - 1\right) \frac{\left(y_{r0}^{t_0+l}\right)^U}{\left(y_{r0}^{t_0}\right)^L} \\ &\le \sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}} + \left(\lambda_0 - 1\right) \frac{y_{r0}^{t_0+l}}{y_{r0}^{t_0}} \\ &\le \sum_{\substack{j=1\\ j\ne 0}}^{n} \lambda_j \frac{\left(y_{rj}^{t_0+l}\right)^U}{\left(y_{rj}^{t_0}\right)^L} + \left(\lambda_0 - 1\right) \frac{\left(y_{r0}^{t_0+l}\right)^L}{\left(y_{r0}^{t_0}\right)^U}. \end{aligned} \tag{16}$$

Therefore, model (13) can be obtained through inequalities (15) and (16).

Similarly, Model (12) is deduced by model (10). The dual model of model (10) is as follows:

$$\theta_0^U = \max_{\substack{\left(X_{ij}^{t_0}\right)^L \le x_{ij}^{t_0} \le \left(X_{ij}^{t_0}\right)^U \\ \left(Y_{rj}^{t_0}\right)^L \le y_{rj}^{t_0} \le \left(Y_{rj}^{t_0}\right)^U \\ \left(X_{ij}^{t_0+l}\right)^L \le x_{ij}^{t_0+l} \le \left(X_{ij}^{t_0+l}\right)^U \\ \left(Y_{rj}^{t_0+l}\right)^L \le y_{rj}^{t_0+l} \le \left(Y_{rj}^{t_0+l}\right)^U \\ \forall i,r,j}} \begin{cases} \max \theta \\ s.t. \\ \sum_{j=1}^{n} \lambda_j \frac{x_{ij}^{t_0+l}}{x_{ij}^{t_0}} \le \theta \frac{x_{i0}^{t_0+l}}{x_{i0}^{t_0}}, \\ \sum_{j=1}^{n} \lambda_j \frac{y_{rj}^{t_0+l}}{y_{rj}^{t_0}} \ge \frac{y_{r0}^{t_0+l}}{y_{r0}^{t_0}}, \\ \lambda_j \ge 0, r = 1, \dots, s; i = 1, \dots, m. \end{cases}$$

Model (14) can be obtained through inequalities (15) and (16). □

Theorem 3.4. *If θ_0^{L*} and θ_0^{U*} are the optimal values of models (11) and (12), then $\theta_0^{L*} \le \theta_0^{U*} \le 1$.*

Proof. θ_0^{L*} and θ_0^{U*} are the optimal values of the above optimization models (11) and (12). It is clear that $\theta_0^{L*} \le 1$, $\theta_0^{U*} \le 1$ and $\theta_0^{L*} \le \theta_0^{U*}$ from models (8) and (9). □

Theorem 3.5. *Models (11) and (12) are feasible.*

Proof. According to the conditions in model (11), there exists a certain k, such that $\frac{\left(y_{kj}^{t_0+l}\right)^U}{\left(y_{kj}^{t_0}\right)^L} \ge \frac{\left(y_{kj}^{t_0+l}\right)^L}{\left(y_{kj}^{t_0}\right)^U} > 0$, $\frac{\left(x_{kj}^{t_0+l}\right)^U}{\left(x_{kj}^{t_0}\right)^L} \ge \frac{\left(x_{kj}^{t_0+l}\right)^L}{\left(x_{kj}^{t_0}\right)^U} > 0$.

It may be assumed $k = 1$, let $\hat{\upsilon} = (\hat{\upsilon}_1, 0, \dots, 0), \hat{u} = (\hat{u}_1, 0, \dots, 0)$, where $\hat{\upsilon}_1 = \frac{\left(x_{10}^{t_0}\right)^U}{\left(x_{10}^{t_0+l}\right)^L}$, $\hat{u}_1 = \min_{1\le j\le n} \frac{\left(y_{1j}^{t_0}\right)^L}{\left(y_{1j}^{t_0+l}\right)^U}$, it easy to see $\hat{\upsilon}, \hat{u}$ satisfy

$$\begin{cases} \sum_{i=1}^{m} \hat{\upsilon}_i \frac{\left(x_{i0}^{t_0+l}\right)^L}{\left(x_{i0}^{t_0}\right)^U} = 1, \\ \sum_{r=1}^{s} \hat{u}_r \frac{\left(y_{r0}^{t_0+l}\right)^U}{\left(y_{r0}^{t_0}\right)^L} - \sum_{i=1}^{m} \hat{\upsilon}_i \frac{\left(x_{i0}^{t_0+l}\right)^L}{\left(x_{i0}^{t_0}\right)^U} \le 0, \\ \sum_{r=1}^{s} \hat{u}_r \frac{\left(y_{rj}^{t_0+l}\right)^L}{\left(y_{rj}^{t_0}\right)^U} - \sum_{i=1}^{m} \hat{\upsilon}_i \frac{\left(x_{ij}^{t_0+l}\right)^U}{\left(x_{ij}^{t_0}\right)^L} \le 0, j = 1, \dots, n, j \ne 0. \\ \hat{u}_r, \hat{\upsilon}_i \ge 0, \forall r, i. \end{cases}$$

Therefore, it can be seen that $\hat{\upsilon}$ and $\hat{u}$ are feasible solutions of the model (11). Similarly, model (12) has feasible solutions. □

Definition 3.1. A DMU is classified as strongly efficient if the lower bound of its efficiency interval is greater than or equal to 1 *i.e.*, $E_1 = \left\{ DMU_j : \theta_j^L \ge 1 \right\}$.

Definition 3.2. A DMU is classified as efficient if the lower bound of its efficiency interval is less than 1 and the upper bound is greater than or equal to 1 *i.e.*, $E_2 = \left\{ DMU_j : \theta_j^L < 1, \theta_j^U \ge 1 \right\}$.

Definition 3.3. If the upper bound of the efficiency interval of a DMU is less than 1, then that DMU is classified as inefficient *i.e.*, $E_3 = \left\{ DMU_j : \theta_j^U < 1 \right\}$.

4. Case study

In this section, the proposed two forms of stage ratio DEA models are validated separately using two cases. The numerical example validates the stage ratio DEA model with accurate ratio inputs–outputs. And the stage interval ratio DEA model is conducted to evaluate the sustainable efficiency of 14 ESEPCEs in China during the green transition stage.

4.1. Numerical example

Suppose that 15 enterprises implement a business plan within the enterprise in 2015, and the plan ends in 2018. After the end of the plan, managers want to know the performance of the enterprise during the implementation period (2015–2018). Each enterprise selected six inputs and six outputs related to this plan. Input–output data come from annual reports of enterprises. The efficiency results calculated using the stage ratio DEA model (5) are shown in Table 1 and are compared with the DEA-Malmquist index model.

The DEA-Malmquist index model is a dynamic model developed based on the DEA model to evaluate efficiency changes over different periods. When the DEA-Malmquist index is greater than 1, it means that the development from stage t_0 to $t_0 + l$ is growing; when the DEA-Malmquist index is equal to 1, it means that the development from stage t_0 to t_0+l is unchanged; and when the DEA-Malmquist index is less than 1, it means that the development from stage t_0 to $t_0 + l$ is decreasing. The stage ratio DEA model under accurate inputs–outputs in this paper is also proposed to reflect stage performance more accurately.

As can be seen from Table 1, the stage efficiencies calculated by the DEA-Malmquist index model and the proposed stage ratio DEA model are mostly consistent. When the DEA-Malmquist index is greater than 1, the stage efficiency calculated by the proposed stage ratio DEA model is equal to 1 (efficient); when the DEA-Malmquist index is less than 1, the stage efficiency of the stage ratio DEA model is less than 1 (inefficient). The effectiveness of our proposed model has been verified.

Table 1
Efficiency results for the DEA-Malmquist index and stage ratio DEA models of accurate inputs–outputs.

DMU	DEA-Malmquist	Stage ratio DEA model
1	1.5577	1.0000
2	1.0023	1.0000
3	0.8767	0.7421
4	0.8808	1.0000
5	0.5340	0.7671
6	1.5144	1.0000
7	1.9767	1.0000
8	1.5483	1.0000
9	1.0281	1.0000
10	0.6313	1.0000
11	1.7430	1.0000
12	1.9675	1.0000
13	0.8964	1.0000
14	1.0755	0.8744
15	3.7131	1.0000

However, among the 15 DMUs, there are also several with inconsistent results, such as DMU4, DMU10, and DMU13. Our experiments allow for such small errors, partly due to the quality of the input–output data itself. If the difference between the input and output data is large, the efficiency results will be different. This sensitivity to data quality is inherent to DEA and emphasizes the need for reliable data collection and preprocessing methods in performance evaluation. Another part of the reason is that the results calculated by the DEA-Malmquist index model include pre-stage influence, while our proposed model is the result after eliminating the pre-stage influence. The purpose of this paper is to eliminate the impact of pre-stage and evaluate the stage performance of the evaluated stage more accurately.

In summary, the effectiveness of the stage ratio DEA model for assessing stage performance is verified by comparing it with the DEA-Malmquist index model. In practice, using the DEA-Malmquist index model to calculate stage performance requires a large amount of data and calculations, which is time-consuming and labor-intensive. The stage ratio DEA model in this paper can not only reflect stage performance but also save time and cost.

4.2. Application example

The proposed stage interval ratio DEA model is used to evaluate the sustainable efficiency of 14 ESEPCEs to understand the business situation of the enterprises at each stage. The input–output interval ratio data of 14 ESEPCEs from 2012–2021 is selected and divided into three stages to evaluate their efficiency. The reason is that since the green transformation policy is implemented in three stages. This paper also examines the sustainable efficiency of ESEPCEs from each of the three stages. In this paper, stage 2012–2015 is defined as the pre-implementation of the policy; stage 2015–2018 is defined as the beginning of the policy implementation; and stage 2018–2021 is defined as the post-implementation of the policy. This example is carried out based on this background. The Interval DEA-Malmquist index model serves as a valuable analytical tool for assessing the dynamic efficiency of enterprises. Therefore, the proposed stage interval ratio DEA model is compared with the interval DEA-Malmquist index model and the traditional interval DEA model, which not only verifies the effectiveness of the proposed model but also better evaluates the sustainable efficiency of ESEPCEs.

Table 2
Inputs-outputs description table.

Classification	Indicator name	Indicator description
Inputs	Administrative expenses	Expenses incurred in management
	R&D expenses	Expenses incurred in R&D
	Inventory	Goods are sold by enterprises
	Number of employees	staffing levels
	Accounts receivable turnover	Operational capacity indicator
	Total asset turnover	Operational capacity indicator
Outputs	Total operating income	The income from the main business
	Return on total assets	Profitability indicator
	Return on net assets	Profitability indicator
	Net profit	The greater the net income, the better the business
	Employee compensation	Employee compensation
	Total tax payments	The higher the income, the higher the tax

4.2.1. Selection of indicators

Enterprises invest heavily in manpower, materials, and capital to sustain their business operations. Managers look at the profitability of an enterprise, taxes, salaries, and other factors to determine how well it is growing. Therefore, the inputs–outputs in Table 2 are selected for analysis in this paper.

4.2.2. Analysis of the results

Table 3 shows the interval sustainable efficiency and ranking of 14 ESEPCEs in each stage of the green transformation using the proposed stage interval ratio DEA model.

From the overall perspective of three stages, the sustainable efficiency of each ESEPCE in the pre-green transition policy implementation stage is the best among the three stages. The reason is that the main business of each enterprise before the implementation of the policy was mainly fossil energy, high demand, product diversification, and broad market prospects. Among them, Goldwind Technology, Wall Nuclear Material, and Guotou Power rank the top 3 in terms of sustainable efficiency, while the other enterprises rank lower but have higher overall sustainable efficiency compared to the other two stages. The second is the stage of 2015–2018 when the green transformation policy starts to be implemented. Unlike the previous stage, the overall sustainable efficiency of each enterprise is low. Detailed analysis reveals three main reasons for this: the high cost of green transformation; the severity of resource dependence; and the slow transformation speed. At this stage, as the policy has just started to be implemented, many obstacles make the sustainable efficiency of all enterprises low. Similarly, by the middle stage of the policy implementation process in stage 2018–2021, the sustainable efficiency of enterprises has not greatly improved. The sustainable efficiency of all enterprises is not optimistic. However, some enterprises are growing rapidly. Like other policy implementation effects, the majority of enterprises are affected by the initial green transformation policy, and the process of adjustment is slow. This slow adjustment process affects their sustainable efficiency. However, it will get better in the later stages.

On the other hand, the results in Table 3 show that the stage interval ratio DEA model can more directly reflect the sustainable efficiency of the enterprises at the evaluated stage compared to the traditional interval DEA model and interval DEA-Malmquist index model. Specifically, it can be discussed from two perspectives: stage efficient and stage inefficient sustainable efficiency values calculated by the stage interval ratio DEA model.

The stage efficiency reflected by the interval dynamic index calculated using the interval DEA-Malmquist index model is mostly consistent to the results obtained applying the stage interval ratio DEA model. However, there are also several with inconsistent results. Our experiments allow for such small errors, partly due to the quality of the input–output data itself. If the difference between the input and output data is large, the efficiency values will be different. Another part of the reason is that the results calculated by the interval DEA-Malmquist index model include pre-stage influence. Meanwhile, when the proposed interval sustainable efficiency of the stage is efficient, it indicates that the enterprise's trend is increasing in that stage; and the traditional interval DEA model manifests itself in three forms as follows:

- The interval efficiency value at the beginning of the stage is inefficient, while at the end of the stage it is efficient.
- The interval efficiency value at the beginning of the stage is efficient and the end of the stage is also efficient.
- The interval efficiency values at the beginning and end of the stage are inefficient, but the interval efficiency value at the end of the stage is closer to the efficient frontier surface.

The first form (Fig. 1) shows that at the beginning of the evaluated stage, the enterprise's revenue is not satisfactory. But at the end of the stage, the enterprise's efficiency becomes better and the enterprise's revenue increases significantly due to proper management and smooth capital flow. As shown in Table 3, stage 2012–2015 Huayi Electric, Sunlight Power; stage 2018–2021 Tianshun Wind Energy, Taisheng Wind Energy, Tongwei Shares. A detailed investigation of the reasons found that the international financial market repeatedly fluctuated significantly in 2012. Additionally, the wind power industry continued to face a persistent recession, characterized by intense competition driven by low prices. However, with the country's increasing focus on green development, the business situation gradually improved, reaching a more favorable position by 2015. While in 2018, Tianshun Wind Energy lost its high-tech enterprise qualification, Taisheng Wind Energy faced capital constraints in expanding its offshore wind power business, and Tongwei Shares experienced inefficiency due to the expansion of their photovoltaic business. And other reasons led to three enterprises business inefficiency in 2018. But by 2021, the problem had gradually resolved, and the operation was in good shape.

The interval DEA-Malmquist index model is used to calculate the stage interval dynamic index of these enterprises in the above form 1, which is also from $M < 1$ to $M > 1$. The interval stage efficiency is efficient, indicating that the development of these enterprises in the evaluated stage shows positive. For example, Huayi Electric's M index from stage 2012–2015 is (0.5628, 2.1283), and Sunlight Power's interval M index is (0.5948, 1.9373). Similarly, the interval M index of Tianshun Wind Energy, Taisheng Wind Energy, and Tongwei shares in the stage 2018–2021 also shows that these three enterprises are efficient in this stage. These results affirm that these aforementioned enterprises have exhibited positive development trajectories within this stage, showcasing their ability to maximize productivity and resource utilization.

The second form is often encountered in practice, where the business performance of the enterprise is efficient from the beginning to the end of the stage, indicating a better sustainable development of the enterprise in that stage. As shown in Table 3, stage 2012–2015 Linyang Energy, Goldwind Technology, Wall Nuclear Material, Guotou Power, Chuantou Energy, Guodian Nanrui; stage 2015–2018 Linyang Energy, Wall Nuclear Material, Guotou Power, Sunlight Power, Guodian Nanrui; stage 2018–2021 Huayi Electric, Sunlight Power. These

Table 3
Sustainable interval efficiency.

Enterprises	Traditional interval DEA model				Interval DEA-Malmquist model			Stage interval ratio DEA model					
	2012	2015	2018	2021	2012–2015	2015–2018	2018–2021	2012–2015	Rank	2015–2018	Rank	2018–2021	Rank
Lingyang Energy	(0.2749, 1.0000)	(0.1333, 1.0000)	(0.2083, 1.0000)	(0.2264, 1.0000)	(0.4412, 3.3010)	(0.5508, 3.1214)	(0.6966, 0.7911)	(0.0266, 1.0000)	7	(0.0154, 1.0000)	6	(0.0491, 0.3847)	12
Goldwind Technology	(0.1458, 1.0000)	(0.2627, 1.0000)	(0.3621, 1.0000)	(0.4573, 1.0000)	(0.4953, 1.7285)	(0.7690, 0.9344)	(0.4145, 0.7311)	(0.1353, 1.0000)	1	(0.0199, 0.6624)	7	(0.0616, 0.5630)	10
Tianshun Wind Energy	(0.1951, 1.0000)	(0.2699, 1.0000)	(0.1950, 0.8967)	(0.2846, 1.0000)	(0.5284, 0.6853)	(0.7647, 0.8691)	(0.9940, 2.1949)	(0.0609, 0.7837)	10	(0.0309, 0.2606)	12	(0.0320, 1.0000)	4
Taisheng Wind Energy	(0.1225, 0.5304)	(0.1345, 0.7418)	(0.0880, 0.7199)	(0.3313, 1.0000)	(1.6247, 1.8459)	(0.5792, 0.6328)	(2.4595, 3.4378)	(0.0850, 0.8325)	9	(0.0355, 0.3906)	10	(0.1396, 1.0000)	2
Huayi Electric	(0.1207, 0.3868)	(0.1677, 1.0000)	(0.0992, 1.0000)	(0.1930, 1.0000)	(0.5628, 2.1283)	(0.4354, 0.9388)	(1.8822, 7.1790)	(0.0732, 1.0000)	5	(0.0322, 0.3039)	11	(0.2474, 1.0000)	1
Wall Nuclear Material	(0.1304, 1.0000)	(0.1421, 1.0000)	(0.1616, 1.0000)	(0.4025, 1.0000)	(1.4880, 4.0032)	(0.3219, 1.7736)	(0.5834, 1.8006)	(0.0917, 1.0000)	2	(0.0548, 1.0000)	2	(0.0621, 0.5076)	11
Jiangsu Shentong	(0.1502, 1.0000)	(0.0831, 0.3952)	(0.2230, 0.8286)	(0.3501, 0.7653)	(0.3102, 0.6762)	(0.9972, 2.3689)	(0.8472, 2.1833)	(0.0278, 0.2374)	14	(0.0606, 0.5379)	8	(0.0776, 0.6701)	9
Juritic Material	(0.1887, 0.4942)	(0.1621, 0.3692)	(0.1686, 0.6186)	(0.4246, 0.7896)	(0.4456, 0.9121)	(1.4641, 2.2140)	(1.1292, 1.9289)	(0.0662, 0.3359)	12	(0.0933, 1.0000)	1	(0.0611, 0.8598)	6
Shanghai Electric	(0.6776, 1.0000)	(0.4803, 1.0000)	(0.7505, 1.0000)	(0.6993, 1.0000)	(1.0812, 4.2756)	(0.3007, 1.1126)	(1.1197, 1.4161)	(0.0650, 0.2841)	13	(0.0531, 0.2156)	13	(0.0410, 0.1672)	14
Guotou Power	(0.8030, 1.0000)	(0.6650, 1.0000)	(1.0000, 1.0000)	(0.6338, 0.8766)	(0.7620, 1.4644)	(0.6432, 1.0539)	(0.3559, 0.6624)	(0.0889, 1.0000)	3	(0.0523, 1.0000)	4	(0.2907, 0.3101)	13
Chuantou Energy	(0.7349, 1.0000)	(0.4587, 1.0000)	(0.3549, 1.0000)	(0.4130, 1.0000)	(1.6362, 2.4448)	(0.6696, 0.9833)	(0.5819, 1.3591)	(0.0529, 1.0000)	6	(0.0299, 0.1776)	14	(0.0214, 0.8008)	7
Sunlight Power	(0.0819, 0.4250)	(0.2484, 1.0000)	(0.1735, 1.0000)	(0.1550, 1.0000)	(0.5948, 1.9373)	(0.6324, 3.1183)	(0.3830, 1.5449)	(0.0844, 1.0000)	4	(0.0214, 1.0000)	5	(0.0092, 1.0000)	5
Tongwei shares	(0.2203, 0.8540)	(0.1386, 0.9934)	(0.4311, 0.8577)	(0.2515, 1.0000)	(0.5615, 0.7535)	(1.0727, 1.1538)	(1.0695, 2.0030)	(0.0417, 0.6325)	11	(0.0478, 0.4153)	9	(0.0493, 1.0000)	3
Guodian Nanrui	(0.2261, 1.0000)	(0.1277, 1.0000)	(0.2650, 1.0000)	(0.1871, 1.0000)	(0.4991, 1.4830)	(0.9146, 2.5357)	(0.7733, 0.9116)	(0.0095, 1.0000)	8	(0.0544, 1.0000)	3	(0.0141, 0.6950)	8

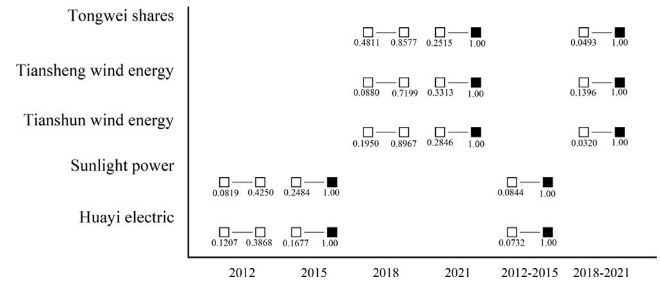

Fig. 1. Schematic diagram of interval stage efficiency efficient form 1.

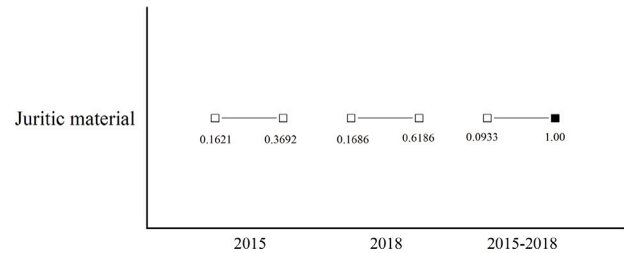

Fig. 2. Schematic diagram of interval stage efficiency efficient form 3.

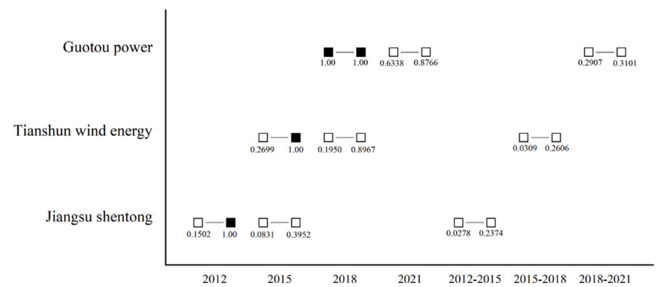

Fig. 3. Schematic diagram of interval stage efficiency inefficient form 1.

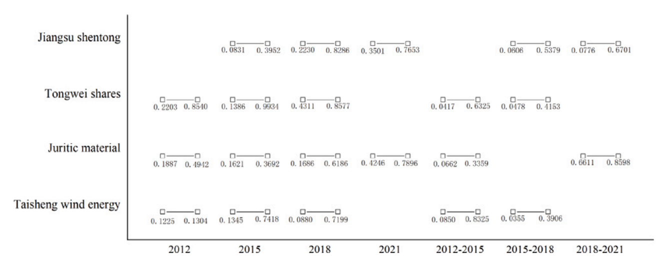

Fig. 4. Schematic diagram of interval stage efficiency inefficient form 2.

enterprises have been operating sustainably from the beginning to the end of each stage, with steady growth in performance. Huayi Electric, the top-ranked enterprise for the stage 2018–2021, is ranked 11th at the beginning of policy implementation, but as the enterprise adjusts its business structure and downsizes unnecessary operations, revenue increases, and sustainable corporate efficiency improves with the help of the policy.

When the stage is efficient, the interval M index calculated using the interval DEA-Malmquist model is also the same as the second form described by the traditional interval DEA model above, that is, from M > 1 (beginning) to M > 1 (end). This form is also one of the stage efficient forms, indicating that the development is better at the beginning of this stage, and then the development is also in the form of positive growth to the end of the stage. In Table 3, Wall Nuclear Material in the stage 2012–2015; Juritic Material and Tongwei shares in the stage 2015–2018; and Goldwind Technology, Juritic Material, Shanghai Electric and other enterprises in the stage 2018–2021 all conform to this form. This shows that these enterprises already have a solid economic foundation at the beginning of the stage, and continue to maintain a positive growth trend until the end of the stage in subsequent development.

The third form (Fig. 2) has a positive trend in sustainable efficiency in this stage, although both the beginning and end sustainable efficiency values are inefficient, indicating that the adjustment of influencing factors is favorable to sustainable efficiency but not optimal. As in the case of Juritic Material in Table 3, this enterprise have inefficient interval sustainable efficiency in 2015 or 2018, but the interval efficiency in 2018 is closer to the effective frontier surface, so the interval sustainable efficiency in the stage 2015–2018 is efficient. With the help of the policy, the management of Juritic Material improved from 12th place in stage 2012–2015 to first place in stage 2015–2018. The reason is that the enterprise starts to engage in the clean energy business before the implementation of the 2012–2015 green transformation policy, but the income is not significant. With the release of the policy, the country provides more resources and funds for the enterprises implementing green transformation, Juritic Material follows the pace, and the development of the enterprises becomes significantly better.

Using the interval DEA-Malmquist index model to calculate the interval M value of Juritic Material in the stage 2015–2018 is (1.4641, 2.2140). It shows that the development of the enterprise in the stage 2015–2018 is positive. At the same time, it is also efficient to calculate the interval efficiency by applying the stage interval ratio DEA model. Therefore, comparisons with the traditional interval DEA model and the interval DEA-Malmquist index model have verified the effectiveness of the proposed model.

Conversely, when the stage efficiency value is inefficient, it indicates that the enterprise's trend is decreasing at that stage, and the traditional interval DEA model also manifests itself in three forms as follows:

- The interval efficiency value at the beginning of the stage is efficient and the end of the stage is inefficient.
- The interval efficiency value at the beginning of the stage is inefficient, and the end of the stage is inefficient.
- The interval efficiency value at the beginning of the stage is efficient, and the end of the stage is efficient.

The first form (Fig. 3) indicates that the enterprise has a good operating turnover at the beginning of the stage. And later on, due to internal problems or external environmental influences, the enterprise ends the stage with lower operating income and poorer operating performance. As shown in Table 3, stage 2012–2015 for Jiangsu Shentong; stage 2015–2018 for Tianshun Wind Energy; and stage 2018–2021 for Guotou Power are verified. Three enterprises have good initial operating results at the beginning of each stage, but later Jiangsu Shentong's revenue decreases due to the decline in product prices caused by the market downturn in the metallurgical industry; Tianshun Wind Energy's corporate debt increases due to the expansion of an unrelated car rental business in 2017–2018; and Guotou Power's revenue decreases during 2020–2021 due to the financial crisis and the epidemic.

From a dynamic perspective, as shown in Table 3, the interval M index calculated by the interval DEA-Malmquist index model for the enterprises described by the traditional interval DEA model is inefficient. On the other hand, the calculation results of the above-mentioned enterprises using the interval DEA-Malmquist index model and the stage interval ratio DEA model reflect the same stage inefficiency, which proves the effectiveness of the proposed model.

The second form (Fig. 4) is common in practice and indicates that the business is in a worse state from the beginning of the stage to the end of the business. For example, stage 2012–2015 Taisheng Wind

Energy, Juritic Material, Tongwei Shares; stage 2015–2018 Taisheng Wind Energy, Jiangsu Shentong, Tongwei Shares; stage 2018–2021 Jiangsu Shentong, Juritic Material. The performance of these enterprises at the beginning of each stage is not satisfactory, and the situation does not improve at the end of the stage due to mismanagement and other reasons. The first-ranked Juritic Material in the last stage of the enterprise's internal management led to problems in business operations and a decline in sustainable efficiency.

Compared with the interval DEA-Malmquist index model, when the stage is inefficient, the interval M index calculated using the interval DEA-Malmquist model is also the same as the first form described by the traditional interval DEA model above, that is, from M < 1 (beginning) to M < 1 (end). This form is also one of the stage inefficient forms, indicating a decline in development from the beginning to the end of the stage. As shown in Table 3, the interval M index, calculated using the interval DEA-Malmquist index model for enterprises described by the traditional interval DEA model, is also inefficient. This indicates that the development of these enterprises is not ideal during the evaluated stage. And the results calculated using the interval DEA-Malmquist index model and the stage interval ratio DEA model reflect the same stage inefficient case, which proves the effectiveness of the proposed model.

The third form suggests that the effectiveness of the end of stage period may be due to the fact that the performance at the beginning of the stage is so good that even though the development in that stage is reduced, the lesser degree of decline does not affect the inter-firm zone sustainable efficiency effectiveness at the end of the stage period. Examples include Tianshun Wind Energy and Shanghai Electric in the stage 2012–2015; Goldwind Technology, Huayi Electric, Shanghai Electric, and Chuantou Energy in the stage 2015–2018; and Linyang Energy, Goldwind Technology, Wall Nuclear Material, Shanghai Electric, Chuantou Energy, and Guodian Nanrui in the stage 2018–2021. These enterprises have good corporate performance at the beginning of the stage and still efficient sustainable efficiency at the end of the stage, but inefficient sustainable efficiency for the whole stage, indicating that corporate sustainability is declining for the whole stage period, but the decline is low and the impact is not significant.

From a dynamic perspective, the interval M index calculated by the interval DEA-Malmquist index model for the enterprises described above by the traditional interval DEA model is also inefficient, proving the analysis results of the traditional interval DEA model. For example, Tianshun Wind Energy in the stage 2012–2015; Goldwind Technology, Huayi Electric, Shanghai Electric, Chuantou Energy, etc. in the stage 2015–2018. On the other hand, the calculation results of the above-mentioned enterprises using the interval DEA-Malmquist index model and the stage interval ratio DEA model reflect the same stage inefficient case. This consistency also demonstrates the robustness of the stage ratio DEA model and strengthens the case for adopting this approach in dynamic efficiency assessments, particularly when dealing with enterprises operating in fluctuating environments.

5. Conclusions

In general, stage evaluation becomes particularly crucial when managers try to eliminate the upfront impact of this stage and more directly check the implementation over a specific period. At the same time, when conducting stage evaluation, ratio data can reflect the characteristics of the stage more directly compared to real-valued data. From a data point of view, if the ratio of the (stage end)/(stage beginning) is larger, it means that the stage is developing in a growing trend; on the contrary, it is developing in a decreasing trend.

Taking the above into account, we proposed two forms of stage ratio DEA models to examine stage performance. It not only discusses the stage ratio DEA models of input–output under deterministic conditions but also extends them to interval conditions. The numerical example validates the stage ratio DEA model with accurate inputs and outputs. And the stage interval ratio DEA model is also used to evaluate the sustainable efficiency of 14 ESEPCEs under three stages (pre-implementation stage, beginning implementation stage, and in-implementation stage) in the context of green transition policy. As an industry that directly provides green ecological products and services, examining the business situation of ESEPCEs at this stage can provide a reference for managers to develop business plans for the next stage.

The above analysis found that the proposed two forms of stage ratio DEA models more directly respond to stage performance. The sustainable efficiency analysis of 14 ESEPCEs using the proposed models found that when the policy of green transition is not started, the enterprises are more in the business of fossil energy. And the enterprises are better run and have higher revenues. When the policy is first implemented, most enterprises have a large number of problems due to the internal transformation of the enterprise, resulting in low overall sustainable efficiency. By the time we reach the mid-implementation stage, the overall sustainable efficiency is still low, although there has been a significant improvement compared to the beginning. However, there are still many enterprises, and due to the resources and funds given by the policy, the sustainable efficiency of the enterprises improves, such as Tianshun Wind Energy, Taisheng Wind Energy, Huayi Electric, Chuantou Energy, and Tongwei Shares. But some enterprises are affected by the transformation due to their own management problems, with declining revenues and lower sustainable efficiency, such as Linyang Energy, Wall Nuclear Material, Juritic Material, Guotou Power, and Guodian Nanrui.

The stage ratio DEA model proposed in this paper eliminates the influence of the previous stage and can better evaluate the stage efficiency. However, when inputs (outputs) are zero, it is still difficult to process using the proposed models. Future research on stage DEA models can be expanded to consider stage DEA models under undesirable inputs (outputs). Meanwhile, stage DEA efficiency evaluation and prediction methods under large panel production possibility sets or stochastic environments are also worth studying.

CRediT authorship contribution statement

Bo-wen Wei: Writing – manuscript preparation, Methodology, Conceptualization. **Yi-yi Ma:** Data curation, Software, Formal analysis. **Ai-bing Ji:** Supervision, Funding acquisition, Project administration.

Data availability

The authors do not have permission to share data.

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Declaration of competing interest

We declare that we have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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