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A robust multi-supplier multi-period inventory model with uncertain market demand and carbon emission constraint

Yuqiang Feng[a], Yankui Liu[b], Yanju Chen[a,*]

[a] College of Mathematics & Information Science, Hebei University, Baoding, Hebei 071002, China
[b] Hebei Key Laboratory of Machine Learning and Computational Intelligence, College of Mathematics & Information Science, Hebei University, Baoding, Hebei 071002, China

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ABSTRACT

The multi-period inventory management determines the optimal order quantity of products at the beginning of each period, which is an important research issue in supply chain management. In this paper, we propose a robust multi-supplier multi-period inventory model with uncertain market demand and uncertain unit carbon emission in transportation. We select product quality, ordering cost, service level, and emergency capacity as evaluation criteria for each supplier, employ analytic hierarchy process (AHP) technique to comprehensively evaluate and score each supplier. According to the evaluated score, the order weight for each supplier is obtained. In our model, we develop carbon emission constraint based on the carbon cap and carbon trading mechanism. As for uncertain market demand and uncertain unit carbon emission in transportation, we construct Box+Ball and Budget uncertainty sets to characterize various model uncertainties. The robust inventory model is a semi-infinite programming model, and cannot be solved directly. We transform the semi-infinite programming model into a mixed-integer second-order cone-programming (MISOCP) one via cone duality theory. Finally, the effectiveness of the proposed optimization method is illustrated by a case study.

1. Introduction

A complete supply chain includes three important links: product production, product transportation and product inventory. Each link cannot be ignored. Inventory management is an important part of supply chain management. To some extent, it is the key link for making a profit in supply chain operations. In this paper, we study the link of product inventory. The classical inventory theory has been well developed in inventory management literature, and its research results have been accepted by a large number of scholars. In the classical inventory model, the market demand is assumed to be deterministic and the products usually come from single-supplier. These assumptions are too restrictive. Decision-making in inventory management problem is always accompanied by demand uncertainty, especially for new products, seasonal products and fashion goods (Li & Liu, 2019). At the same time, supply disruption or shortage frequently happens if the orders come from single-supplier (Svoboda, Minner, & Yao, 2021). In addition, the classical inventory model doesn't consider the environmental cost. The above deficiencies about the classical inventory model bring many limitations to its application in practice.

In traditional inventory management, the orders often originate from single-supplier, which has two shortcomings. The first shortcoming, a single source is inflexible, the dependence on the supplier is high, and supply disruptions occur frequently. The second shortcoming, single-supplier may be unable to fill the orders especially if the customers' demand suddenly increases (Chakraborty, Chauhan, & Ouhimmou, 2020). For example, due to the outbreak of COVID-19, many suppliers have been forced to shut down. It has brought crisis and challenge to enterprises that cooperate with only single-supplier (Craven, Liu, Mysore, & Wilson, 2020). Another instance is about Philips, which is a semiconductor supplier located in Mexico, and is responsible for supplying semiconductors to some enterprises including Nokia and Ericsson. Philips' supply of semiconductors for Nokia and Ericsson was disrupted due to a sudden big fire. Because of the disruption, Ericsson lost 400 million, while Nokia suffered less because it has cooperated with a backup supplier in time (Silbermayr & Minner, 2014). The resilient supply chain can reduce the risk of supply disruption (Hosseini, Ivanov, & Dolgui, 2019; Torabi, Baghersad, & Mansouri, 2015). One way to increase supply chain resilience is to order from multi-supplier (Hosseini et al., 2019). So we consider the enterprise orders from

* Corresponding author.
E-mail addresses: yliu@hbu.edu.cn (Y. Liu), yanjuchen@hbu.edu.cn (Y. Chen).

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multi-supplier in this paper. The reasons for ordering from multi-supplier are as follows. First of all, supply disruption of products can cause great reputation loss and certain economic punishment to the enterprise. And the orders from multi-supplier can effectively avoid the occurrence of disruption. Moreover, if the enterprise cooperates with only one supplier, the enterprise must strengthen the supervision on this supplier to ensure the competitiveness of the supplier's products, which will increase the extra ordering cost of the enterprise. As a result, single order channel can easily lead to a monopoly, which may enhance the price of products. If the enterprise cooperates with multi-supplier, the monopoly phenomenon can be avoided and there is healthy competition among suppliers. Then the increase of product price and the decrease of product quality can be avoided. Therefore, it is necessary to consider orders from multi-supplier.

In recent years, a series of concepts such as "low carbon economy" and "low carbon development" have become the discussion focus in society. With the increasing emission of carbon pollutants in the world, the European Union and the United Nations have formulated a corresponding policy-carbon trading mechanism to control carbon emissions (UNFCCC, 1997). Under a series of carbon emission policies, a firm is required to reconstruct its optimal decisions in inventory management to reduce carbon emissions (Mishra, Wu, & Sarkar, 2021). The carbon emissions in the inventory problem come from transportation and storage (Hovelaque & Bironneau, 2015; Hua, Cheng, & Wang, 2009). The carbon emissions in storage mainly result from the electricity consumption of equipment in the warehouse (e.g. air conditioners). The carbon emissions in transportation are mainly due to the fuel consumption of vehicles. The fuel consumption rate is not constant, which is often affected by many factors, such as weather conditions, road conditions, load weight, and vehicle speed (Mohammadi, Torabi, & Tavakkoli-Moghaddam, 2014; Tajik, Tavakkoli-Moghaddam, Vahdani, & Mousavi, 2014; Yin, Chen, Song, & Liu, 2019). Therefore, the unit carbon emission in transportation is uncertain. Accordingly, unlike the existing literature, in which the unit carbon emission in transportation is assumed to be a certain parameter, this paper studies the inventory problem by considering the unit carbon emission in transportation as an uncertain parameter.

The market demand is often an uncertain parameter, which is difficult to be predicted in practice (Alinovi, Bottani, & Montanari, 2012; Mutha, Bansal, & Guide, 2016). Uncertain demand usually leads to an imperfect match between supply and demand, which results in surplus or shortage of inventory about components and parts (Song, Tang, Zhao, & Zhang, 2021). In this case, if the cost is calculated according to the deterministic market demand, there will be a huge error between the budget cost and the actual cost. This is a main limitation of the traditional inventory model. What's more, for uncertain market demand, the inventory manager may have no (or insufficient) data to evaluate its distribution in many cases. Therefore, how to optimize inventory under uncertain market demand is a critical issue to be solved for inventory manager, where the distribution information of uncertain parameter is unknown. The recently popular robust optimization (RO) method can be used to solve this problem. Soyster (1973) provided the first study in the RO method. His idea is to obtain an optimal solution that retains feasibility for all possible realizations of uncertain parameters within prescribed interval uncertainty set. In recent years, the RO method has been continuously developed and widely applied in various fields, such as vehicle routing problem, portfolio problem and humanitarian relief problem. In this paper, we use the RO method to optimize inventory problem with uncertain market demand and uncertain unit carbon emission in transportation.

The above considerations motivate us to propose a robust multi-supplier multi-period inventory model with uncertain market demand and uncertain unit carbon emission in transportation. The model is to determine the optimal order quantity in each period under the conditions of multi-supplier, uncertain demand, and uncertain unit carbon emission. The objective of our model is to minimize the total cost (including order cost, holding cost/penalty cost, environmental cost). Our main contributions can be summarized as follows:

- Firstly, this paper studies the orders from multi-supplier from a new viewpoint. We take product quality, ordering cost, service level, and emergency capacity as four evaluation criteria, employ analytic hierarchy process (AHP) technique to comprehensively evaluate the order weight of each supplier according to the corresponding score.
- Secondly, we help manager make decisions under uncertain demand and uncertain unit carbon emission in transportation. We characterize these two uncertain parameters in two different uncertainty sets. To the best of our knowledge, this work is the first time that the unit carbon emission is modeled as an uncertain parameter in inventory management problem.
- Thirdly, based on the uncertain market demand and uncertain unit carbon emission in transportation, we propose a new robust multi-supplier multi-period inventory model. We transform the robust model into a mixed-integer second-order cone-programming model, which can be solved by general commercial software.
- Finally, we provide a case study about Apple's DHL warehouse in Shanghai to test the effectiveness of our model. The experimental results show that our optimization method is not only feasible but can offer a robust solution to immunize the influence of the uncertainties.

The rest of this paper is organized as follows. Section 2 briefly reviews some related literature. Section 3 gives the problem statement in detail and presents a robust multi-supplier multi-period inventory model. Section 4 analyzes the proposed model, determines the order weight of each supplier and constructs the uncertainty sets. In Section 5, a case study about Apple's DHL warehouse is provided to demonstrate the effectiveness of the robust multi-supplier multi-period inventory model. Conclusion and future research are given in Section 6.

2. Literature review

This section presents the literature review and shows the research gaps of the existing studies on multi-period inventory management. We limit our literature review on the following issues: multi-supplier, uncertain demand, and environmental issue.

For inventory management, there is some literature that orders come from multi-supplier. Dada, Petruzzi, and Schwarz (2007) studied single period newsvendor problem, where multi-supplier is divided into two types: reliable and unreliable. They developed a single-product, multi-supplier newsvendor model under both demand and supply uncertainty and captured key factors for vendor-selection. Silbermayr and Minner (2014) assumed the orders can come from a set of potential suppliers, and investigated the trade-off between single and multiple suppliers, as well as keeping inventory and having a back-up supplier. Their results illustrated that ordering from multi-supplier can save cost compared with single-supplier. In terms of supplier selection, Karsak and Dursun (2015) presented an integrated fuzzy multi-criteria decision making (MCDM) approach to evaluate and select suppliers. They established relevant supplier evaluation criteria while considering the impacts of inner dependence among them to evaluate and select suppliers. Wang, Zhang, Chong, and Wang (2017) proposed a supplier selection framework, which works by integrating building information modeling and a geographic information system in the resilient supply chain. They combined the analytic hierarchy process (AHP) and grey correlation analysis (GCA) to evaluate each supplier's performance through comprehensive analysis of seventeen resilient criteria, and rank every supplier. Jadidi, Jaber, Zolfaghri, Pinto, and Firouzi (2021) studied the dynamic price and lot size problem of a newsvendor over multi-supplier, multi-period, and random demand. Their orders come from multi-supplier. Each supplier has limited supply capacity and offers all-unit discounts in each order period. A mixed-integer nonlinear programming problem was proposed to find the optimum retail price and order quantities from the suppliers in each period.

At present, most of the relevant literature on the inventory problem under uncertain conditions focuses on uncertain demand, including Liao and Deng (2018), Guo, Tian, and Liu (2017), Song, Ran, and Shang (2017), Wei, Li, and Cai (2011), Bertsimas and Thiele (2006) and Thorsen and Yao (2017). Stochastic optimization (SO) method and fuzzy optimization (FO) method are often used to solve the inventory problem under uncertain conditions. Song et al. (2017) and Liao and Deng (2018) studied the inventory problem, took the uncertain market demand as a random variable, and developed a stochastic inventory model, respectively. Guo et al. (2017) took the uncertain market demand as a triangular fuzzy variable, then transformed the fuzzy inventory model into a deterministic inventory model. In the study of Li and Liu (2019), the demand was assumed to be triangular, trapezoidal and Erlang fuzzy variable, respectively, three classes of fuzzy multi-item inventory optimization models were built. They used a risk-averse fuzzy optimization method to solve the multi-item inventory problem. Li, Fang, and Baykal-Gürsoy (2021) studied an inventory management problem with a financially constrained retailer under constantly updated demand. The demand was assumed to be stochastic. The initial demand information was updated by using a Bayesian approach. They investigated the existence and uniqueness of the optimal inventory/financing solution jointly and gave two ordering strategies. However, SO method requires that the probability distribution of uncertain parameter is known accurately. It usually needs a large amount of data to estimate the deterministic probability distribution of random variable. While, in practice, it is difficult for decision makers to estimate the true distribution (Prékopa, 2013). FO method requires decision makers to obtain the fuzzy membership function of uncertain parameter according to their personal experience, which often has great subjectivity. Robust optimization (RO) method is an attractive method for dealing with models with uncertain parameters, which has emerged recently and has been applied to solve inventory problems. Bertsimas and Thiele (2006) studied inventory problem, took into account the uncertain demand without assuming a specific distribution, and used the RO method to solve the inventory problem under uncertain demand. Guo and Liu (2018) dealt with the single-period inventory model, and considered the uncertain market demand as an uncertain parameter. They applied distributionally robust optimization (DRO) method to solve it, and compared the DRO model with the fuzzy optimization model to verify the feasibility of the DRO model. There is less literature that simultaneously considers uncertain demand and other uncertain conditions. Wei et al. (2011) studied inventory and re-manufacturing production planning problem, considered simultaneously uncertain market demand and uncertain product returns, and applied the RO method to deal with the uncertainty. Thorsen and Yao (2017) took both the order lead time and market demand as uncertain parameters and established a robust inventory model based on budget uncertainty set and central limit theorem-based uncertainty set.

Under the call of green management, more and more scholars take environmental issue into account in their inventory problem. Tiwari, Daryanto, and Wee (2018), Tao and Tang (2015) and Liao and Deng (2018) studied inventory management where the total carbon emissions consist of two parts: carbon emissions in transportation and that in storage. They added the cost of carbon emissions to the total cost, and further looked for the best ordering strategy. Hovelaque and Bironneau (2015) proposed a novel EOQ model, which took into account the link between inventory policy, total carbon emissions, and both price and dependent demands. Their experiments illustrated that environmental strategy is more significant for cheaper and green-labeled products. Bozorgi (2016) studied multi-product inventory models with carbon emissions, which mainly result from transportation and storage, and took the total carbon emissions as a target function to be minimized. When Li and Hai (2019) studied the inventory management problem with one warehouse multi-retailer, they considered the carbon emissions from the holding and replenishing inventory activities in the system. Huang, Fang, and Lin (2020) proposed an integrated inventory model for a two-echelon supply chain regarding the limited total carbon emissions, carbon taxation, and cap-and-trade to minimize the total cost (including the cost of green investment). In their model, the carbon emissions come from the processes of production, delivery, and storage. Gao, Chen, Tang, and Zhang (2020) developed a multi-period, multi-raw material periodic-review inventory management model under carbon emission constraint. Their objective is to minimize the total cost (including carbon cost). They considered that the main source of carbon emissions is the transportation process from supplier to manufacturer.

To identify the research gaps of the existing studies on inventory management problem and to clarify the innovations of this paper, we classify the related literature with five terms: period, supplier, uncertain demand, carbon emission, and optimization method in Table 1. We summarize the following three research gaps: (i) when studying inventory management problem, the existing studies are based on deterministic carbon emissions, and there is no research addressing uncertain carbon emissions; (ii) there is little research in which the orders come from multi-supplier; (iii) there are few works on the application of the RO method to solve inventory management problems with uncertain parameters. The innovations of this paper are listed below. Firstly, this paper is the first to study the inventory problem with both uncertain demand and uncertain unit carbon emission in transportation. Secondly, different from the majority of studies (the orders only come from single-supplier), the orders simultaneously come from multi-supplier in this paper. Thirdly, our inventory management problem with two uncertain parameters was formulated by the RO method. Therefore, this study departs significantly from previous studies and is a step forward in inventory management problem.

3. A robust multi-supplier multi-period inventory model

3.1. Problem description

We consider an inventory problem over a finite time horizon of T periods: the warehouse has an initial inventory level before the whole system runs. At the beginning of each period, the warehouse's orders come from multi-supplier. The manager determines the total order quantity from suppliers. We assume the lead time of each supplier is equal to 0. Our objective function is to optimize the total cost, which includes startup cost, order cost, holding cost/shortage cost, and environmental cost. At the end of the planning horizon, no demand occurs. When there are surplus products at the end of the t-th period, the products will be backlogged and holding cost will incur. If product shortage occurs in the t-th period, the order quantity q_{t+1} in the $t+1$-th period will first compensate for the shortage in the t-th period and then meet the demand in the $t+1$-th period. So, in the $t+1$-th period, if the remaining quantity is still greater than d_{t+1} after q_{t+1} has filled the shortage of the t-th period, then there are residual products. The holding cost will occur for the residual quantity. The system will stop operating at period $T+1$. The salvage value of the stock is assumed to be zero at period $T+1$. The supply level of each supplier depends on its production scale. In addition, the scales of suppliers are different, which causes the supply levels of suppliers to be different. We consider carbon constraint to our problem. The total carbon emissions include two parts: carbon emissions in transportation and that in product storage.

We give notations used in our model, description of the costs and carbon emission constraint in the following two subsections.

3.2. Notations

The sets, indices, parameters, and variables used in our model are described below.

Sets:

$[N]$: The set of supplier, indice $n \in [N] = \{1, 2, \cdots, N\}$.

Table 1
A review on relevant works in the literature.

Reference	Period		Supplier		Demand		Carbon emission		Optimization method		
	Single	Multiple	Single	Multiple	Certain	Uncertain	Certain	Uncertain	Stochastic	Fuzzy	Robust
Bertsimas and Thiele (2006)		√	√			√					√
Wei et al. (2011)		√	√			√					√
Taleizadeh et al. (2011)	√		√			√				√	
Silbermayr and Minner (2014)		√		√		√					
Guo et al. (2017)	√		√			√				√	
Thorsen and Yao (2017)		√	√			√					√
Liao and Deng (2018)	√		√			√	√		√		
Tiwari et al. (2018)		√	√		√		√				
Guo and Liu (2018)	√		√			√				√	√
Song et al. (2017)		√	√			√			√		
Li and Hai (2019)		√	√		√		√				
Li and Liu (2019)	√		√			√				√	
Huang et al. (2020)		√	√		√		√				
Gao et al. (2020)		√	√		√		√		√		
Jadidi et al. (2021)		√		√		√			√		
Li et al. (2021)	√		√			√			√		
This research		√		√		√		√			√

$[T]$: The set of period, indice $t \in [T] = \{1, 2, \cdots, T\}$.
Parameters:
s_n: Distance from the n-th supplier to the warehouse.
ϕ: Order startup cost.
ω_n: Order weight of the n-th supplier.
γ_n: Maximum supply level of the n-th supplier in each period.
d_t: Uncertain demand in the t-th period.
c_n: The unit price of the product ordered from the n-th supplier.
c_h: Holding cost per unit product.
c_p: Shortage cost per unit product.
e^l: The unit carbon emission of transporting unit product.
e^h: The unit carbon emission of storing unit product.
τ: The price of per unit carbon emission.
E^{cap}: Carbon cap (upper bound on carbon emissions).
M: Maximum inventory level.
α, β, ψ: Weight factors.
w_t: Inventory level at the beginning of the t-th period.
Decision variables:
x_t: Binary variables, $x_t = 1$ if order occurs in the t-th period, 0 otherwise.
q_t: Order quantity during the t-th period.

3.3. Model formulation

The warehouse orders from N suppliers. The order weight of the n-th supplier is ω_n. So for T periods, the total order cost is

$$\sum_{t=1}^{T}\left[\phi x_t + \sum_{n=1}^{N} c_n q_t \omega_n\right].$$

At the end of the t-th period, if there are surplus products, they will be backlogged, and holding cost will occur. If there is a shortage of products in the t-th period, the shortage cost will incur, and the quantity of shortage in the t-th period will be compensated in the $t+1$-th period. Therefore, the inventory balance constraints is

$$w_{t+1} = w_1 + \sum_{i=1}^{t}\left(q_i - d_i\right), \quad \forall t \in \left[T\right].$$

In each period, the holding cost and shortage cost don't occur simultaneously. The holding cost corresponds to the cost associated with having excess inventory (i.e. $w_1 + \sum_{i=1}^{t}(q_i - d_i) \geqslant 0$) at the end of the t-th period. The shortage cost corresponds to the cost associated with having unfilled demand (i.e. $w_1 + \sum_{i=1}^{t}(q_i - d_i) \leqslant 0$) at the end of the t-th period. So, the holding/shortage cost in the t-th period can be expressed as $\max\{c_h w_{t+1}, -c_p w_{t+1}\}$. The total holding cost/shortage cost can be expressed as

$$\sum_{t=1}^{T} \max\left\{c_h w_{t+1}, -c_p w_{t+1}\right\}.$$

The total carbon emissions include two parts: carbon emissions in transportation and that in storage. Carbon emissions in transportation are produced during vehicle transportation from multi-supplier to warehouse. Here, we adopt unified vehicles to transport the products. At the end of the t-th period, if there are surplus products, they will be stored. The storage system produces carbon emissions. Fig. 1 shows the sources of carbon emissions. Therefore, the total carbon emissions is

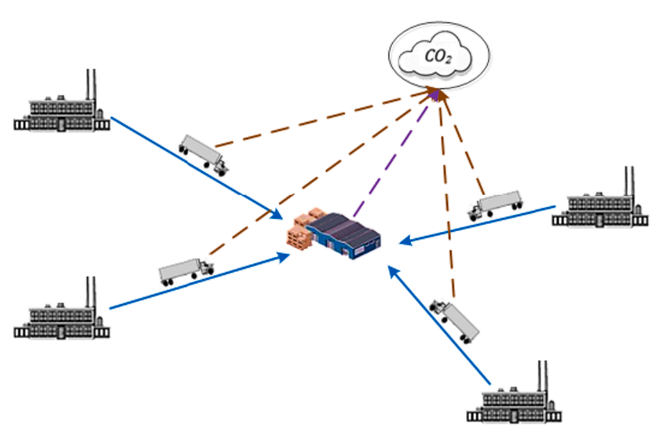

Fig. 1. The sources of carbon emissions.

$$\sum_{t=1}^{T}\left[\sum_{n=1}^{N} e^{l} s_n \omega_n q_t + \max\left\{e^{h} w_{t+1}, 0\right\}\right].$$

Based on carbon trading mechanism, the whole system has a carbon emission quota, and the company can sell the remaining carbon emissions in the carbon trading market if the total carbon emissions of the system are less than the quota. In contrast, if the total carbon emissions are greater than the quota, the company needs to buy additional carbon emissions from the carbon trading market. We use E^- to represent the amount of carbon to be sold in the carbon trading market, and E^+ to represent the amount of carbon to be bought in the carbon trading market. Therefore, the carbon emission constraint is such a "soft" constraint:

$$\sum_{t=1}^{T}\left[\sum_{n=1}^{N} e^{l} s_n \omega_n q_t + \max\left\{e^{h} w_{t+1}, 0\right\}\right] + E^- \leqslant E^{cap} + E^+.$$

The price of unit carbon emission is τ. We define the cost of buying or selling carbon emissions as an environmental cost in the carbon trading market. The environmental cost can be expressed as

$$\tau(E^+ - E^-).$$

3.4. Modified multi-supplier multi-period inventory model

In the previous section, we introduce order cost, holding cost/shortage cost, environmental cost, and carbon emission constraint. Based on these, we present a modified multi-supplier multi-period inventory model, which is as follows:

$$\begin{aligned}(A):\quad \min_{q_t}\quad & \alpha\sum_{t=1}^{T}\left[\phi x_t + \sum_{n=1}^{N} c_n q_t \omega_n\right] + \beta\sum_{t=1}^{T}\max\left\{c_h w_{t+1}, -c_p w_{t+1}\right\} + \psi[\tau(E^+ - E^-)] \\ \text{s.t.}\quad & \sum_{t=1}^{T}\left[\sum_{n=1}^{N} e^{l} s_n \omega_n q_t + \max\left\{e^{h} w_{t+1}, 0\right\}\right] + E^- \leqslant E^{cap} + E^+,\end{aligned} \tag{1}$$

$$w_{t+1} = w_1 + \sum_{i=1}^{t}\left(q_i - d_i\right), \qquad \forall t \in \left[T\right], \tag{2}$$

$$w_{t+1} \leqslant M, \qquad \forall t \in [T], \tag{3}$$

$$0 \leqslant \omega_n q_t \leqslant \gamma_n x_t, x_t \in \{0,1\}, \forall n \in [N], \forall t \in [T]. \tag{4}$$

In model (A), α,β, and ψ are the weight factors corresponding to the above three parts of cost. Decision maker can control the effect of each part by adjusting α,β, and ψ. For example, decision maker can show how much he values the environment by adjusting ψ. Larger ψ means that decision maker pays more attention to the environmental effects. The objective function is the weighted sum of the above three parts of cost. Constraint (1) denotes the "soft" constraint about the total carbon emissions. Constraint (2) represents the inventory transfer equations. Constraint (3) indicates that the remaining inventory in each period should not exceed the maximum inventory level M. Constraint (4) means that if an order occurs in the t-th period, then the order quantity from each supplier in each period should not exceed the upper limit γ_n.

Remark 1. If the orders only come from single-supplier, the carbon emissions are not considered, and the weight factors α,β, and ψ are equal to 1, then model (A) degenerates into the following model:

$$\begin{aligned}\min_{q_t}\quad & \sum_{t=1}^{T}\left\{c q_t + \phi x_t + \max\left\{c_h w_{t+1}, -c_p w_{t+1}\right\}\right\} \\ \text{s.t.}\quad & w_{t+1} = w_0 + \sum_{i=1}^{t}\left(q_i - d_i\right), \quad \forall t \in \left[T\right], \\ & w_{t+1} \leqslant M, \quad \forall t \in [T], \\ & 0 \leqslant q_t \leqslant \gamma x_t, x_t \in \{0,1\}, \quad \forall t \in [T],\end{aligned}$$

where c is the unit price of product. The objective is to minimize the total cost, which is composed of the order startup cost, order cost, and holding/shortage cost. The first constraint is the inventory transfer equations. The second constraint denotes that the remaining product inventory in each period should not exceed inventory capacity M. The third constraint means that if an order occurs in the t-th period, then the order quantity should not exceed γ. This model corresponds to the capacitated inventory model of Bertsimas and Thiele (2006). Model (A) is nonlinear because of the max function in the objective and constraint. We introduce epigraph auxiliary variables $y_t = \max\{c_h w_{t+1}, -c_p w_{t+1}\}$ and $z_t = \max\{e^h w_{t+1}, 0\}$ to linearize it. Correspondingly, model (A) is equivalent to the following linear programming:

$$\begin{aligned}(B):\quad \min_{q_t}\quad & \alpha\sum_{t=1}^{T}\left[\phi x_t + \sum_{n=1}^{N} c_n q_t \omega_n\right] + \beta\sum_{t=1}^{T} y_t + \psi[\tau(E^+ - E^-)] \\ \text{s.t.}\quad & \sum_{t=1}^{T}\left[\sum_{n=1}^{N} e^{l} s_n \omega_n q_t + z_t\right] + E^- \leqslant E^{cap} + E^+,\end{aligned} \tag{5}$$

$$y_t \geqslant c_h w_{t+1}, \quad \forall t \in [T], \tag{6}$$

$$y_t \geqslant - c_p w_{t+1}, \quad \forall t \in \left[T\right], \tag{7}$$

$$z_t \geqslant e_h w_{t+1}, \quad \forall t \in [T], \tag{8}$$

$$z_t \geqslant 0, \quad \forall t \in [T], \tag{9}$$

$$\text{Constraints}(2) - (4).$$

3.5. Robust multi-supplier multi-period inventory model

In reality, the demand is often influenced by many factors, such as the price of complementary products. Consequently, it is uncertain in many cases. In addition, carbon emissions in transportation come from fuel consumption. The unit fuel consumption of transportation vehicles is not a constant, and is often affected by many factors, such as weather conditions, road conditions, load weight, vehicle speed. So, the unit carbon emission in transportation is also uncertain. Here, we simultaneously consider the uncertain market demand and uncertain unit carbon emission in transportation.

For uncertain demand $\boldsymbol{d} = (d_t)_{t\in[T]}$ and the uncertain unit carbon emission e^l in transportation, we only know that $\boldsymbol{d} \in \Psi, e^l \in \Theta$, where Θ and Ψ are following uncertainty sets:

$$\Theta = \left\{ e^l | e^l = e_0^l + \sum_{k=1}^{K} \xi_k e_k^l,\ \boldsymbol{\xi} \in \mathcal{Z}_1 \right\}, \tag{10}$$

$$\Psi = \left\{ \boldsymbol{d} | d_t = d_t^0 + \zeta_t \widetilde{d}_t,\ \forall t \in [T], \boldsymbol{\zeta} \in \mathcal{Z}_2 \right\}. \tag{11}$$

In the above uncertainty sets, d_t^0 and e_0^l denote the nominal values, $\widetilde{d}_t$ and e_k^l are basic shifts, $\boldsymbol{\xi} = (\xi_1, \xi_2, \cdots, \xi_K)$ and $\boldsymbol{\zeta} = (\zeta_1, \zeta_2, \cdots, \zeta_T)$ are perturbation vectors, $\mathcal{Z}_1$ and $\mathcal{Z}_2$ are perturbation sets of $\boldsymbol{\xi}$ and $\boldsymbol{\zeta}$ respectively.

Based on Ψ and Θ, the robust multi-supplier multi-period inventory model is formulated as follows:

$$\begin{aligned} (C):\quad & \min_{q_t}\ \alpha \sum_{t=1}^{T} \left[\phi x_t + \sum_{n=1}^{N} c_n q_t \omega_n \right] + \beta \sum_{t=1}^{T} y_t + \psi[\tau(E^+ - E^-)] \\ & \text{s.t.} \sum_{t=1}^{T} \left[\sum_{n=1}^{N} e^l s_n \omega_n q_t + z_t \right] + E^- \leqslant E^{cap} + E^+, \quad \forall e^l \in \Theta, \end{aligned} \tag{12}$$

$$y_t \geqslant c_h \left(w_1 + \sum_{i=1}^{t} (q_i - d_i) \right), \forall \boldsymbol{d} \in \Psi, \forall t \in [T], \tag{13}$$

$$y_t \geqslant -c_p \left(w_1 + \sum_{i=1}^{t} (q_i - d_i) \right), \forall \boldsymbol{d} \in \Psi, \forall t \in [T], \tag{14}$$

$$z_t \geqslant e_h \left(w_1 + \sum_{i=1}^{t} (q_i - d_i) \right), \forall \boldsymbol{d} \in \Psi, \forall t \in [T], \tag{15}$$

$$w_1 + \sum_{i=1}^{t} \left(q_i - d_i \right) \leqslant M, \quad \forall \boldsymbol{d} \in \Psi, \forall t \in [T], \tag{16}$$

Constraints(4)and(9).

In model (C), the number of decision variables is finite. But the number of constraints is infinite because $\boldsymbol{d}$ and e^l are uncertain parameters. Therefore, model (C) is a semi-infinite programming model, which is not easy to solve directly. In the next section, we determine weights ω_n, and transform model (C) into a computable form.

4. Model analysis

In this section, we determine the order weight of each supplier and transform the semi-infinite constraints into their tractable form.

Table 2
A standard nine-point scale.

The scale	Meaning (i factor compared to j)
1	i is equally important as j
3	i is moderately more important than j
5	i is strongly more important than j
7	i is demonstrated more important than j
9	i is extremely more important than j
2, 4, 6, 8	Intermediate values
b_{ji}	When j is compared to i ($b_{ji} = 1/b_{ij}$)

Table 3
The form of judgment matrix about criteria.

	Pq	Oc	Sl	Ec
Pq	$S1_{11}$	$S1_{12}$	$S1_{13}$	$S1_{14}$
Oc	$S1_{21}$	$S1_{22}$	$S1_{23}$	$S1_{24}$
Sl	$S1_{31}$	$S1_{32}$	$S1_{33}$	$S1_{34}$
Ec	$S1_{41}$	$S1_{42}$	$S1_{43}$	$S1_{44}$

Table 4
The form of supplier's judgment matrix about Pq.

Pq	Supplier-1	Supplier-2	⋯	Supplier-n
Supplier-1	$S2_{11}$	$S2_{12}$	⋯	$S2_{14}$
Supplier-2	$S2_{21}$	$S2_{22}$	⋯	$S2_{24}$
⋯	⋯	⋯	⋯	⋯
Supplier-n	$S2_{41}$	$S2_{42}$	⋯	$S2_{nn}$

4.1. Determining order weight of supplier

The order weight of each supplier directly influences the total cost and the order quantity. In this subsection, we use the Analytic Hierarchy Process (AHP) technique to comprehensively evaluate each supplier, and calculate the order weight according to the corresponding score.

AHP takes the multi-criteria decision problem as a system and makes decisions according to the mode of decomposition, comparative judgment and synthesis. Deshmukh and Chaudhari (2011) summarized and reviewed the supplier evaluation criteria and methods. They pointed out that AHP is a modern multi-criteria decision making method, which structures the problem in the form of a hierarchy. Through AHP, all criteria are fairly compared to further determine the relative weight of each supplier. In this paper, we analyze and evaluate each supplier based on four criteria: product quality (Pq), ordering cost (Oc), service level (Sl), and emergency capacity (Ec). The process of applying AHP to evaluate each supplier can be divided into the following 5 steps.

Firstly, according to the above four criteria, we establish a hierarchical structure of supplier evaluation, which is divided into three

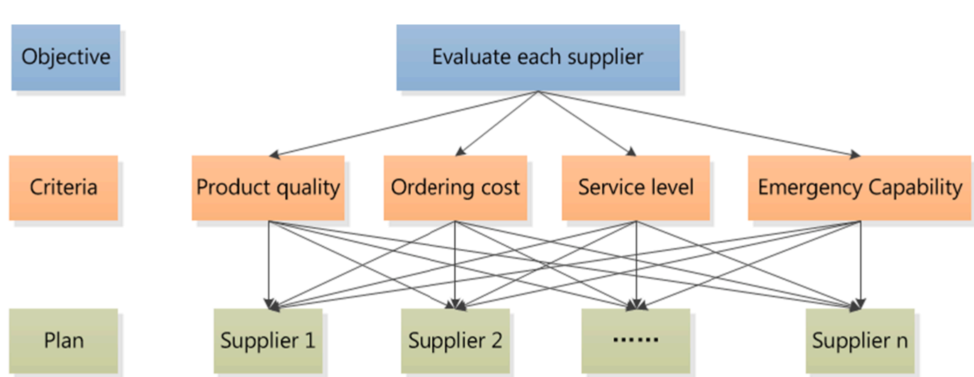

Fig. 2. The hierarchical structure of supplier evaluation.

Table 5
Standard random consistency index RI.

n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
RI	0	0	0.52	0.89	1.12	1.26	1.35	1.41	1.46	1.49	1.52	1.54	1.56	1.58	1.59

layers: objective layer, criteria layer, and plan layer. The corresponding hierarchical structure is shown in Fig. 2.

Secondly, according to Table 2 (Thomas & Doherty, 1980), the judgment matrix of criteria is established by comparing the importance of each criterion. The form of judgment matrix is shown in Table 3, where the element $S1_{ij}$ is determined by Table 2.

Thirdly, according to Table 2, we also establish the judgment matrices of suppliers by comparing suppliers in the following four criteria: Pq, Oc, Sl and Ec. The form of judgment matrix about Pq is shown in Table 4, where $S2_{ij}$ is also determined by Table 2. The judgment matrices of suppliers about the other three criteria are similar to Table 4.

Fourthly, after the judgment matrices are obtained, we calculate the eigenvalue of each judgment matrix and the consistency index CI, find the corresponding standard random consistency index RI, then calculate the consistency ratio CR. Table 5 shows the standard random consistency index RI. If CR$\leqslant$0.1, this judgment matrix is accepted. Otherwise, this judgment matrix needs to be adjusted. The formulas for CI and CR are as follows:

$$\mathrm{CI} = \frac{\lambda_{\max} - n}{n-1},$$
$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}}.$$

Finally, when all judgment matrices pass the consistency test, we calculate the weights of criteria and suppliers. The formulas for the weights of criteria are

$$\mathrm{W_{Pq}} = \frac{1}{4}\sum_{j=1}^{4}\frac{\mathrm{S1_{1j}}}{\sum_{m=1}^{4}\mathrm{S1_{mj}}},$$

$$\mathrm{W_{Oc}} = \frac{1}{4}\sum_{j=1}^{4}\frac{\mathrm{S1_{1j}}}{\sum_{m=1}^{4}\mathrm{S1_{mj}}},$$

$$\mathrm{W_{Sl}} = \frac{1}{4}\sum_{j=1}^{4}\frac{\mathrm{S1_{1j}}}{\sum_{m=1}^{4}\mathrm{S1_{mj}}},$$

$$\mathrm{W_{Ec}} = \frac{1}{4}\sum_{j=1}^{4}\frac{\mathrm{S1_{1j}}}{\sum_{m=1}^{4}\mathrm{S1_{mj}}}.$$

The formula for the weight of the n-th supplier about Pq is

$$\mathrm{Wn_{Pq}} = \frac{1}{N}\sum_{j=1}^{N}\frac{\mathrm{S2_{nj}}}{\sum_{m=1}^{N}\mathrm{S2_{mj}}}.$$

Similarly, $\mathrm{Wn_{Oc}}, \mathrm{Wn_{Sl}}$ and $\mathrm{Wn_{Ec}}$ are calculated.

Next, we need to fill in the final weight matrices with $\mathrm{W_{Pq}}, \mathrm{W_{Oc}}, \mathrm{W_{Sl}}, \mathrm{W_{Ec}}, \mathrm{Wn_{Pq}}, \mathrm{Wn_{Oc}}, \mathrm{Wn_{Sl}}$ and $\mathrm{Wn_{Ec}}$. The final weight matrix is shown in Table 6.

Table 6
The weight matrix.

	Weight	Supplier-1	Supplier-2	⋯	Supplier-n
Pq	$\mathrm{W_{Pq}}$	$\mathrm{W1_{Pq}}$	$\mathrm{W2_{Pq}}$	⋯	$\mathrm{Wn_{Pq}}$
Oc	$\mathrm{W_{Oc}}$	$\mathrm{W1_{Oc}}$	$\mathrm{W2_{Oc}}$	⋯	$\mathrm{Wn_{Oc}}$
Sl	$\mathrm{W_{Sl}}$	$\mathrm{W1_{Sl}}$	$\mathrm{W2_{Sl}}$	⋯	$\mathrm{Wn_{Sl}}$
Ec	$\mathrm{W_{Ec}}$	$\mathrm{W1_{Ec}}$	$\mathrm{W2_{Ec}}$	⋯	$\mathrm{Wn_{Ec}}$

According to Table 6, we obtain the final order weights:

$$\omega_n = \mathrm{W_{Pq}}\cdot\mathrm{Wn_{Pq}} + \mathrm{W_{Oc}}\cdot\mathrm{Wn_{Oc}} + \mathrm{W_{Sl}}\cdot\mathrm{Wn_{Sl}} + \mathrm{W_{Ec}}\cdot\mathrm{Wn_{Ec}}. \quad (17)$$

Now, we have introduced AHP technique and the calculating steps of order weight. In the next subsection, we will introduce how to transform the semi-infinite programming model into a computationally tractable form.

4.2. Tractable form of robust multi-supplier multi-period inventory model

Before using the RO to deal with the uncertain demand and uncertain unit carbon emission in transportation, we must construct suitable perturbation sets. In this paper, following the idea of Ben-Tal, El Ghaoui, and Nemirovski (2009) and Bertsimas and Sim (2004), we construct the following two perturbation sets:

$$\mathcal{Z}_1 = \mathcal{Z}_{\mathrm{Budget}} = \left\{\boldsymbol{\xi}\in\mathbb{R}^K : -1\leqslant\xi_k\leqslant 1, \sum_{k=1}^{K}\xi_k\leqslant\Gamma,\ \forall k\leqslant K\right\}, \quad (18)$$

$$\mathcal{Z}_2 = \mathcal{Z}_{\mathrm{Box+Ball}} = \left\{\boldsymbol{\zeta}\in\mathbb{R}^T : -1\leqslant\zeta_i\leq 1,\ \sqrt{\sum_{i=1}^{t}\zeta_i^2}\leqslant\Omega_t,\ i\leqslant t, t\leqslant T\right\}. \quad (19)$$

To transform the semi-infinite constraints (12)–(16), we provide the following two theorems.

Theorem 1. *Based on perturbation set $\mathcal{Z}_{\mathrm{Budget}}$, the robust counterpart of semi-infinite constraint (12) can be represented by the following system of constraints in variables $\boldsymbol{\lambda}, \boldsymbol{w}$ and $\boldsymbol{q}$, respectively.*

$$\begin{cases} \sum_{k=1}^{K}|\lambda_k| + \Gamma\max_k|w_k| + e_0^l\sum_{t=1}^{T}\left(\sum_{n=1}^{N}s_n\omega_n q_t + z_t\right) + E^- \leqslant E^{cap} + E^+, \\ \lambda_k + w_k = e_k^l\sum_{n=1}^{N}s_n\omega_n q_t, \quad \forall k\in[K]. \end{cases} \quad (20)$$

Proof 1. The proof of Theorem 1 is in Appendix A. □

Theorem 2. *Based on perturbation set $\mathcal{Z}_{\mathrm{Box+Ball}}$, the robust counterparts of semi-infinite constraints (13)-(16) can be represented by the following systems of constraints in variables $\boldsymbol{\delta}, \boldsymbol{\eta}$ and $\boldsymbol{q}$, respectively.*

$$\begin{cases} c_h w_1 - c_h\sum_{i=1}^{t}\left(d_i^0 - q_i - |\delta 1_{t,i}|\right) + c_h\Omega_t\sqrt{\sum_{i=1}^{t}(\eta 1_{t,i})^2}\leqslant y_t, \quad \forall t\in\left[T\right], \\ \delta 1_{t,i} + \eta 1_{t,i} = \widetilde{d}_i, \quad \forall t\in\left[T\right],\ i\in\left[t\right], \end{cases} \quad (21)$$

$$\begin{cases} -c_p w_1 + c_p\sum_{i=1}^{t}\left(d_i^0 - q_i + |\delta 2_{t,i}|\right) + c_p\Omega_t\sqrt{\sum_{i=1}^{t}(\eta 2_{t,i})^2}\leqslant y_t, \quad \forall t\in\left[T\right], \\ \delta 2_{t,i} + \eta 2_{t,i} = \widetilde{d}_i, \quad \forall t\in\left[T\right],\ i\in\left[t\right], \end{cases} \quad (22)$$

$$\begin{cases} e_h w_1 - e_h \sum_{i=1}^{t} \left(d_i^0 - q_i - \left|\delta 3_{t,i}\right|\right) + e_h \Omega_t \sqrt{\sum_{i=1}^{t} \left(\eta 3_{t,i}\right)^2} \leqslant z_t, \quad \forall t \in \left[T\right], \\ \delta 3_{t,i} + \eta 3_{t,i} = \widetilde{d}_i, \quad \forall t \in \left[T\right], \ i \in \left[t\right], \end{cases} \tag{23}$$

$$\begin{cases} w_1 - \sum_{i=a}^{t} \left(d_i^0 - q_i - \left|\delta 4_{t,i}\right|\right) + \Omega_t \sqrt{\sum_{i=1}^{t} \left(\eta 4_{t,i}\right)^2} \leqslant M, \quad \forall t \in [T], \\ \delta 4_{t,i} + \eta 4_{t,i} = \widetilde{d}_i, \quad \forall t \in [T], \ i \in [t]. \end{cases} \tag{24}$$

Proof 2. The proof of Theorem 2 is in Appendix B. □

Therefore, based on Theorems 1 and 2, the robust inventory model (C) is equivalent to the following model:

$$\left(D\right): \quad \min_{\mu} \quad \alpha \sum_{t=1}^{T} \left[\phi x_t + \sum_{n=1}^{N} c_n q_t \omega_n\right] + \beta \sum_{t=1}^{T} y_t + \psi[\tau(E^+ - E^-)]$$
$$\text{s.t.} \quad \text{Constraints } (4), (9), \text{ and } (20) - (24),$$

where $\mu = (\boldsymbol{q}, \boldsymbol{y}, \boldsymbol{z}, \boldsymbol{\lambda}, \boldsymbol{w}, \boldsymbol{\delta}, \boldsymbol{\eta})$, $\boldsymbol{q} = (q_t)_{t\in[T]}$, $\boldsymbol{y} = (y_t)_{t\in[T]}$, $\boldsymbol{z} = (z_t)_{t\in[T]}$, $\boldsymbol{\lambda} = (\lambda_k)_{k\in[K]}$, $\boldsymbol{w} = (w_k)_{k\in[K]}$, $\boldsymbol{\delta} = (\delta 1_{ti}, \delta 2_{ti}, \delta 3_{ti}, \delta 4_{ti})_{t\in[T], i\in[t]}$, and $\boldsymbol{\eta} = (\eta 1_{ti}, \eta 2_{ti}, \eta 3_{ti}, \eta 4_{ti})_{t\in[T], i\in[t]}$.

Remark 2. In model (D), the constraints (21)–(24) are second-order cone constraints, and the variables x_t are binary. Hence, model (D) is a mixed-integer second-order cone-programming (MISOCP), which is an SOCP problem with integrality constraints on some variables (Miyashiro & Takano, 2015). The SOCP, linear programming (LP), conic quadratic programming (CQP) and semi-definite optimization programming (SDP) are computationally tractable conic programming (Alizadeh & Goldfarb, 2003; Ben-Tal et al., 2009). LP can be solved by the simplex method, and SOCP, CQP, and SDP can be solved by the interior point method. The standard way to solve the MISOCP problem is by applying a branch-and-bound procedure that solves an SOCP problem at each node. Such branch-and-bound algorithms have been implemented in some commercial solvers such as the state-of-the-art CPLEX solver (Dinler, Tural, & Iyigun, 2015). In the next section for case study, the optimal decisions are provided by CPLEX solver.

In this section, we have determined the order weight of each supplier through AHP technique, and transformed the robust inventory model into the computable model (D). In the next section, we will carry out numerical experiments to verify the feasibility of our robust multi-supplier multi-period inventory model. .

5. A case study

In this section, we conduct numerical experiments in the following four aspects. Firstly, the validity of our proposed inventory model is verified by comparing the proposed robust model with its nominal model. Secondly, the optimal decision under multi-supplier is compared with that under single-supplier in terms of the optimal order quantity and optimal cost. Thirdly, sensitivity analysis about carbon cap, the price of per unit carbon emission, and weight factors are performed. Finally, several managerial insights are given to the manager. All numerical experiments are solved by the CPLEX 12.8.0 optimization software on an Inter(R) Core(TM) i7-6500U 2.50 GHz personal computer with 8 GB RAM operating under Windows 10 (64bit).

5.1. Problem background and data description

As far as we know, there are 18 Apple assembly plants in the world. 14 of these plants are located in China, such as Foxconn, Pegatron, Wistron and BYD. Most of the world's iPhones are assembled in China. In our numerical experiments, for the inventory management problem of iPhone11, we take Foxconn, Pegatron and Wistron as three iPhone suppliers. After all the iPhone 11 phones are assembled at Foxconn, Pegatron and Wistron, they will enter the local tax process. If these iPhone11 are sold abroad, they will be taxed for export. Otherwise, these iPhone11 are sold domestically. They are taxed for domestic sales. Then these iPhone11 will be shipped to the DHL Shanghai warehouse (Apple's logistics supplier). DHL Shanghai warehouse will deliver these iPhone11 to sub-warehouses in China. Among the three suppliers, Foxconn is the largest, Pegatron the second, and Wistron the smallest in terms of enterprise size and production lines. However, Foxconn is the furthest away from the DHL warehouse compared with Pegatron and Wistron. The sales process of iPhone11 in China is shown in Fig. 3.

From https://www.taosj.com and http://hangye.manmanbuy.com, we obtain the sales amount of iPhone11 sold from Suning(an e-commerce

Table 7
The judgment matrix of evaluation criteria.

	Pq	Oc	Sl	Ec
Pq	1	3	4	6
Oc	1/3	1	3	4
Sl	1/4	1/3	1	3
Ec	1/6	1/4	1/3	1

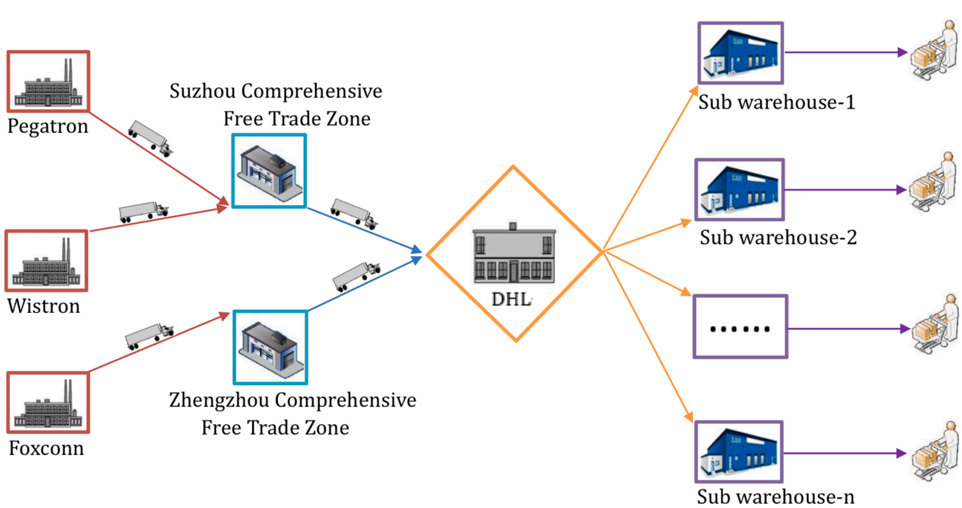

Fig. 3. The sales process of iPhone11 in the mainland of China.

Table 8
The judgment matrix of suppliers about Pq.

Pq	Foxconn	Pegatron	Wistron
Foxconn	1	5	4
Pegatron	1/5	1	2
Wistron	1/4	1/2	1

Table 9
The judgment matrix of suppliers about Oc.

Oc	Foxconn	Pegatron	Wistron
Foxconn	1	2	3
Pegatron	1/2	1	2
Wistron	1/3	1/2	1

platform) in each month from January to June in 2020. We take the six sales amounts as the nominal demand values of the DHL warehouse, that is $D^0 = \{d_t^0\}_{t=1}^{6}$={54729, 68303, 72733, 60533, 77470, 145106}. We assume $\widetilde{d}_t$ is 5% of the nominal value. Referring to Mohammed, Selim, Hassan, and Syed (2017), in this paper, we consider that the transport vehicles are all the same type of mid-size trucks, and $e_0^l = 0.1008 \times 10^{-3}$ g, which is the nominal value of the unit carbon emission in transportation. We assume e_k^l is 5% of the nominal value. The distances of three suppliers to the DHL warehouse are 954 km, 94 km, and 94 km, respectively, which are obtained by Baidu Maps. A fully packaged iPhone11 weighs about 400 g. The unit carbon emission from storing product is $e^h = 0.5$ g. We set carbon cap $E^{cap} = 25000$ g and unit price of carbon emission $\tau = 2.5$. We assume that the unit price of product is inversely proportional to the order quantity. According to the order weight of each supplier ($\omega_1 = 0.6, \omega_2 = 0.25, \omega_3 = 0.15$, the calculation process of the weights is shown in the next subsection), we let $c_1 = 6.54, c_2 = 6.7, c_3 = 6.8, c_h = 4$ and $c_p = 12$. According to the production size of each supplier, we let $\gamma_1 = 130000$, $\gamma_2 = 120000$, and $\gamma_3 = 110000$ in each period. The initial inventory is $w_1 = M = 15000$. Without loss of generality, we first set weight factors α, β, and ψ are equal to 1.

5.2. Determining the weight of supplier

Comprehensively considering the product quality, ordering cost, service level, and emergency capacity of Foxconn, Pegatron and Wistron, we have the judgment matrices in Tables 7–11, which pass the consistency test. Now, we can calculate the weights of criteria and suppliers about each criterion. The final weight matrix is got (See Table 12).

According to the final weight matrix and weight calculation formula (17), we obtain the corresponding order weights of the three suppliers: 0.6, 0.25, and 0.15.

5.3. Computational results

Model (B) based on multi-supplier, deterministic demand, and deterministic unit carbon emission in transportation is called the nominal model. We separately calculate the optimal order quantities provided by the nominal model and robust model of each period. The optimal order quantities are shown in Fig. 4.

From Fig. 4, we can observe that there are some differences between the two models in terms of the optimal order quantity. The order quantity provided by the robust model is larger than that of the nominal model in each period. In addition, by calculation, the total cost of the robust model is 3840510.37, and the total cost of the nominal model is 3109188.72. The former is higher than the latter and the excess is used for robust solution to immunize the influence of the uncertainties.

Table 10
The judgment matrix of suppliers about Sl.

Sl	Foxconn	Pegatron	Wistron
Foxconn	1	3	4
Pegatron	1/3	1	3
Wistron	1/4	1/3	1

Table 11
The judgment matrix of suppliers about Ec.

Ec	Foxconn	Pegatron	Wistron
Foxconn	1	4	3
Pegatron	1/4	1	2
Wistron	1/3	1/2	1

5.4. The price of robustness

To further compare the nominal model and the robust model, we define the price of robustness(PR) as follows:

$$PR = \frac{\text{Robust}^* - \text{Nominal}^*}{\text{Nominal}^*} \times 100\%,$$

where $(\cdot)^*$ represents the optimal cost.

In fact, PR represents the price that the robust solution resists uncertainty. After calculation, the price is PR = 23.5%. It means that the cost of the robust solution increases only 23.5% compared with the nominal solution. That is to say the robust solution can resist uncertainty at a small price.

5.5. Multi-supplier VS. single-supplier

Now, we compare the total costs in the two cases of single-supplier ordering way and multi-supplier ordering way. It is assumed that all orders come from Foxconn, Pegatron and Wistron, respectively. The fluctuation values of demands are 5% of these nominal values. If all orders of iPhone11 are completed by Foxconn in Zhengzhou, then we get $N = 1, c_1 = 6.54$, and $s_1 = 954$. When all orders of iPhone11 are completed by Pegatron in Kunshan, we have $N = 1, c_2 = 6.7$, and $s_2 = 94$. If all the orders of iPhone11 are completed by Wistron in Kunshan, then we get $N = 1, c_3 = 6.8$, and $s_3 = 94$. To ensure fairness, the other parameters are all the same.

The optimal order quantities and total costs for different ordering ways are listed in Table 13. We can find that the optimal order quantities are equal whether the order from multi-supplier or single-supplier in the first five periods. The reasons for this phenomenon are as follows: in the first five periods, compared to the production capacity of each supplier, the demands are smaller, so the order quantities are smaller. In this case, the orders can be fulfilled by multi-supplier or only any single-supplier. Therefore, the optimal order quantities are equal for different ordering ways in the first five periods. However, since the demand in the sixth period suddenly increases, a sudden increase in order quantity occurs correspondingly. The order quantity equals the production capacity. When all orders only come from Foxconn, Pegatron or Wistron, a shortage occurs, which brings shortage cost. It tells us that single-supplier is not enough to cope with emergencies such as sudden increase in demand. In contrast, ordering from multi-supplier effectively prevents the occurrence of shortage.

Table 12
The final weight matrix.

	Weight	Foxconn	Pegatron	Wistron
Product quality	0.545	0.68	0.2	0.13
Ordering cost	0.26	0.54	0.3	0.16
Service level	0.125	0.61	0.27	0.12
Emergency capacity	0.07	0.12	0.56	0.32

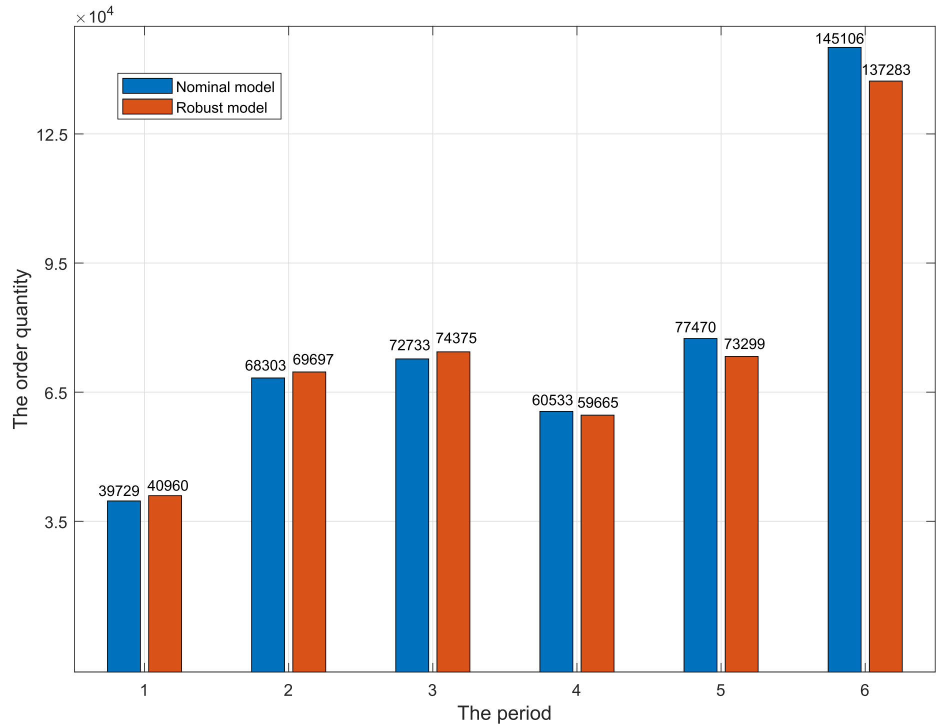

Fig. 4. The optimal solutions of nominal model and robust model.

Table 13
The optimal order quantity and total cost about different ordering ways.

Ordering way	The order quantity in each period						Total cost
	1	2	3	4	5	6	
Multi-supplier	40960	69697	74375	59665	73299	137283	3840510
Foxconn	40960	69697	74375	59665	73299	130000	3886730
Pegatron	40960	69697	74375	59665	73299	120000	3901914
Wistron	40960	69697	74375	59665	73299	110000	3997448

The cost of ordering from multi-supplier is 3840510.37, which is the smallest. In other words, ordering from multi-supplier not only doesn't increase the cost, but reduces the cost. More importantly, ordering from multi-supplier can effectively avoid the occurrence of supply disruption, and induce healthy competition among suppliers at the same time. The comparison of the optimal cost under different ordering ways is shown in Fig. 5.

To further compare the multi-supplier ordering way and single-supplier ordering way, we conduct some experiments based on another set of nominal demands $D^0 = \{d_t^0\}_{t=1}^6$={129815, 79283, 128293, 138621, 108416, 199825}. The other parameters are unchanged. The experimental results are listed in Table 14. We observe that the optimal order quantities and total costs for different ordering ways are different. About single-supplier ordering way, when the demand exceeds production capacity, the optimal order quantity equals production capacity, which is unable to meet the demand. This leads to product shortage, which brings a high penalty cost. Nevertheless, ordering from multi-supplier prevents the occurrence of shortage. Therefore, the total cost of multi-supplier ordering way is the smallest. This also illustrates the superiority of the multi-supplier ordering way.

5.6. The effects of E^{cap} and τ

In this subsection, we explore the influence of the carbon cap and unit price of carbon emission on the total cost and environmental cost. First of all, let the unit price be $\tau = 2.5$, the carbon cap be $E^{cap} = 23000, 25000, 27000, 29000, 31000, 33000, 37000$. Other parameters remain unchanged. The experimental results are shown in Table 15. We observe that the total cost and the environmental cost decreases as E^{cap} increases. This trend is consistent with the actual situation. The reason is that the enterprise buys fewer and fewer credits for carbon emissions in the carbon trading market as E^{cap} grows larger. When E^{cap} reaches 29000, the enterprise can even sell its remaining carbon emissions credits to make a profit.

Next, let $\tau = 2, 2.5, 3, 3.5, 4$. Other parameters remain unchanged. We explore how the total cost and environmental cost vary as E^{cap} and τ vary. The experimental results are shown in Figs. 6 and 7. In order to explain the experimental results conveniently, we call the carbon cap, under which the enterprise neither sells nor buys the credits for carbon emissions in the carbon trading market, the carbon trading threshold. It is denoted as E^{Δ}. By calculation, we get $E^{\Delta} = 31525$ (the red line in Fig. 7).

Figs. 6 and 7 show that the carbon cap and unit price of carbon emission affect indeed the total cost and environmental cost. When $E^{cap} \leqslant E^{\Delta}$, the total cost and environmental cost decrease as τ decreases. But when $E^{cap} \geqslant E^{\Delta}$, the total cost and environmental cost decrease as τ increases. That's mainly because, when $E^{cap} \leqslant E^{\Delta}$, the enterprise needs to buy the credits for carbon emissions, and when $E^{cap} \geqslant E^{\Delta}$, enterprise doesn't need to buy credits, even it can sell its remaining credits.

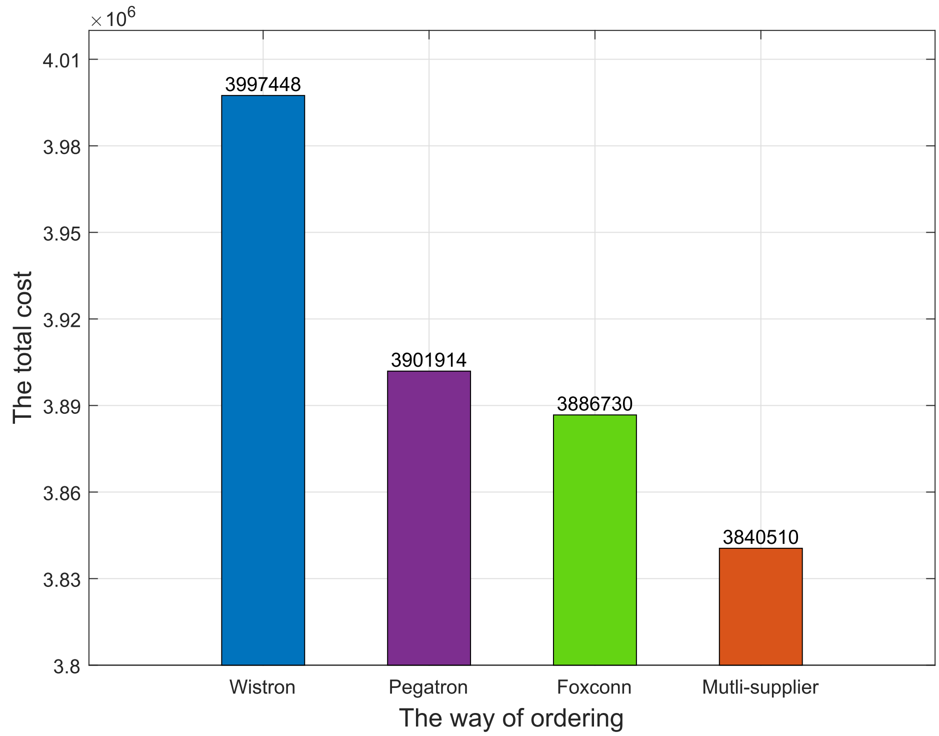

Fig. 5. The comparison among the total costs of different ordering ways.

Table 14
The influence of nominal demands D^0 on optimal order quantity and total cost.

Ordering way	The order quantity in each period						Total cost
	1	2	3	4	5	6	
Multi-supplier	117736	80922	123808	131723	102427	188932	6879413
Foxconn	117736	80922	123808	130000	104150	130000	7219725
Pegatron	117736	82241	120000	120000	116639	120000	7399894
Wistron	110000	89977	110000	110000	110000	110000	8429563

Table 15
The total cost and environmental cost under different E^{cap}.

τ	E^{cap}	E^+	E^-	Total cost	Environmental cost
2.5	29000	2525	0	3830510	6312
2.5	31000	525	0	3825510	1312
2.5	33000	0	1475	3820210	−3688
2.5	35000	0	3475	3815510	−8688

5.7. The effects of weights α, β, and ψ

In this subsection, to further analyze the effects of weights α,β, and ψ on the total cost, order cost, holding cost/shortage cost, environmental cost and order quantity, sensitivity analysis on weights are performed. For convenience, we introduce Part1, Part2, and Part3 to denote the order cost, holding cost/shortage cost, environmental cost, respectively.

$$\text{Part1} = \sum_{t=1}^{T}\left[\phi x_t + \sum_{n=1}^{N} c_n q_t \omega_n\right],$$

$$\text{Part2} = \sum_{t=1}^{T} y_t,$$

$$\text{Part3} = \tau(E^+ - E^-).$$

Now, the total cost is the sum of Part1, Part2, and Part3. That is, Totalcost $=$ Part1 $+$ Part2 $+$ Part3.

Firstly, let $\alpha = 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2$. Other parameters remain unchanged. The optimal order quantities under different α are listed in Table 16. Total cost, Part1, Part2, and Part3 under different α are illustrated in Fig. 8.

From Table 16, we see the optimal order quantity of each period is unchanged when $\alpha \leqslant 1.6$. Decision maker doesn't order products in the sixth period when $\alpha > 1.6$, which directly leads to the reductions in Part1 and Part3. Meanwhile, it leads to the occurrence of product shortage. So, Part2 increases when α changes from 1.6 to 1.8. The sum of the reductions in Part1 and Part3 is less than the increase in Part2. So the total cost increases when α changes from 1.6 to 1.8 (See Fig. 8).

Then, let $\beta = 0.2, 0.4, 0.6, 0.8, 1, 1.2$. Other parameters remain unchanged. The optimal order quantities under different β are listed in Table 17. Total cost, Part1, Part2, and Part3 under different β are drawn on Fig. 9.

Table 17 implies that the decision maker doesn't order products in the fifth period and sixth period when $\beta = 0.2$, and doesn't order products only in the sixth period when $\beta = 0.4$. If $\beta > 0.4$, the orders occur in all periods, and the optimal order quantity of each period remains the same. As β increases, the decision maker pays more and more attention to Part2, and the amount of shortage becomes smaller. Therefore, Total cost and Part2 decrease with β, and Part1 and Part3 increase with β (See Fig. 9).

Finally, let $\psi = 15, 20, 25, 30.8, 32, 35$, other parameters remain unchanged. The optimal order quantities under different ψ are listed in Table 18. Total cost, Part1, Part2, and Part3 under different β are illustrated in Fig. 10.

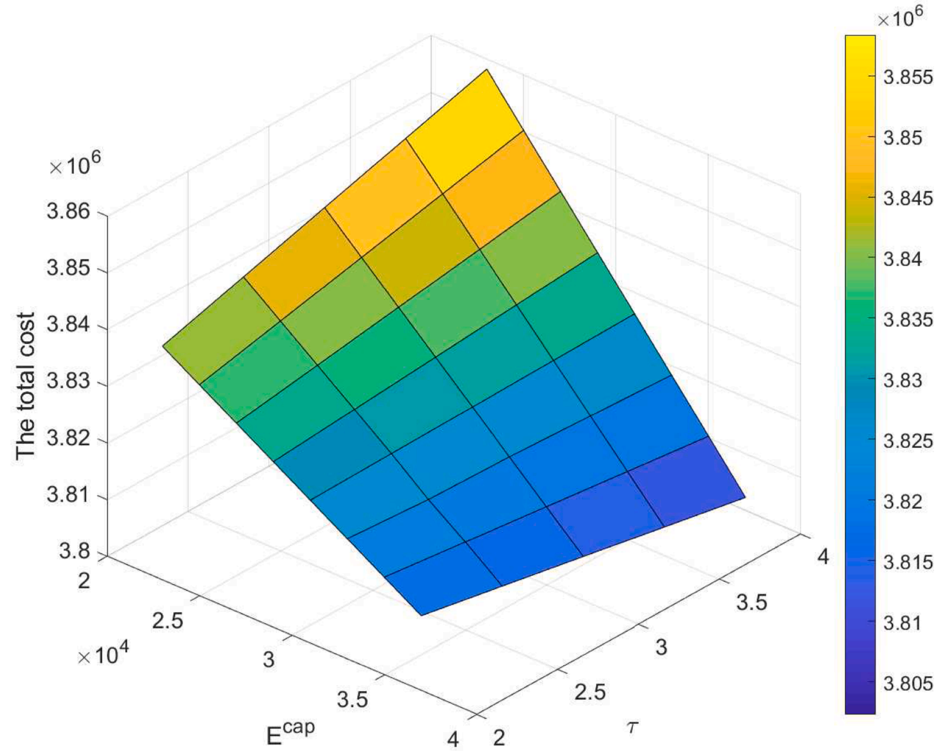

Fig. 6. The total cost under different E^{cap} and τ.

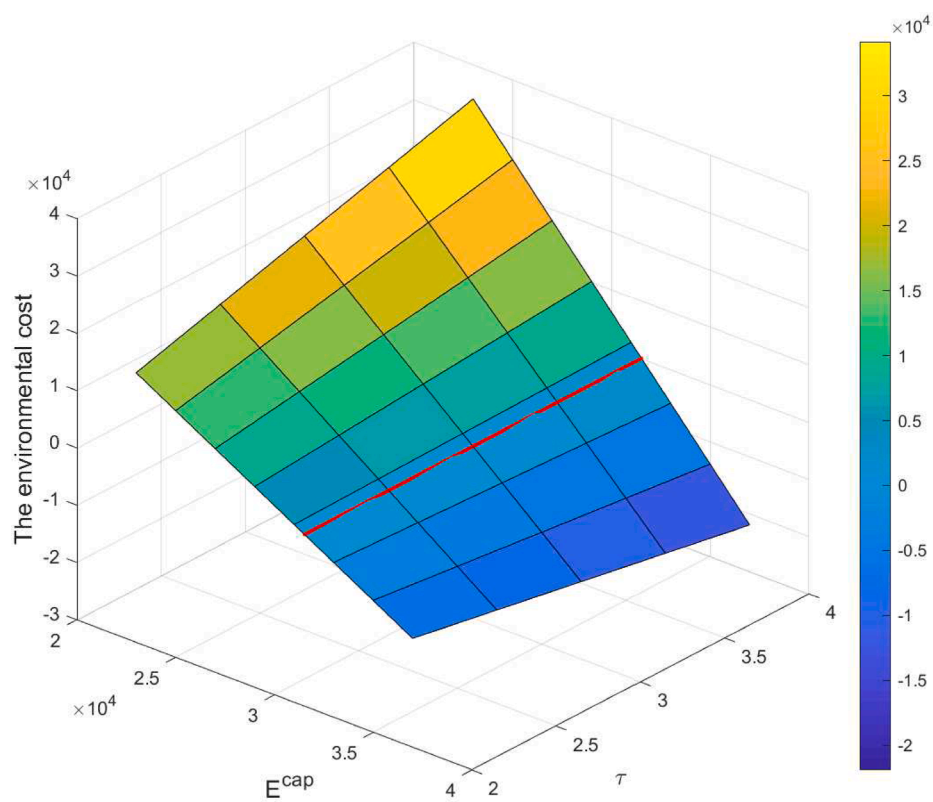

Fig. 7. The environmental cost under different E^{cap} and τ.

Table 16
The experimental results under different α.

α	The order quantity in each period						Total cost	Part1	Part2	Part3
	1	2	3	4	5	6				
0.8, 1, 1.2, 1.4, 1.6	40960	69697	74375	59665	73299	137283	3840510	3043492	780706	16312
1.8, 2	40960	69697	74375	59665	73299	0	4550478	2129816	2428103	-7440

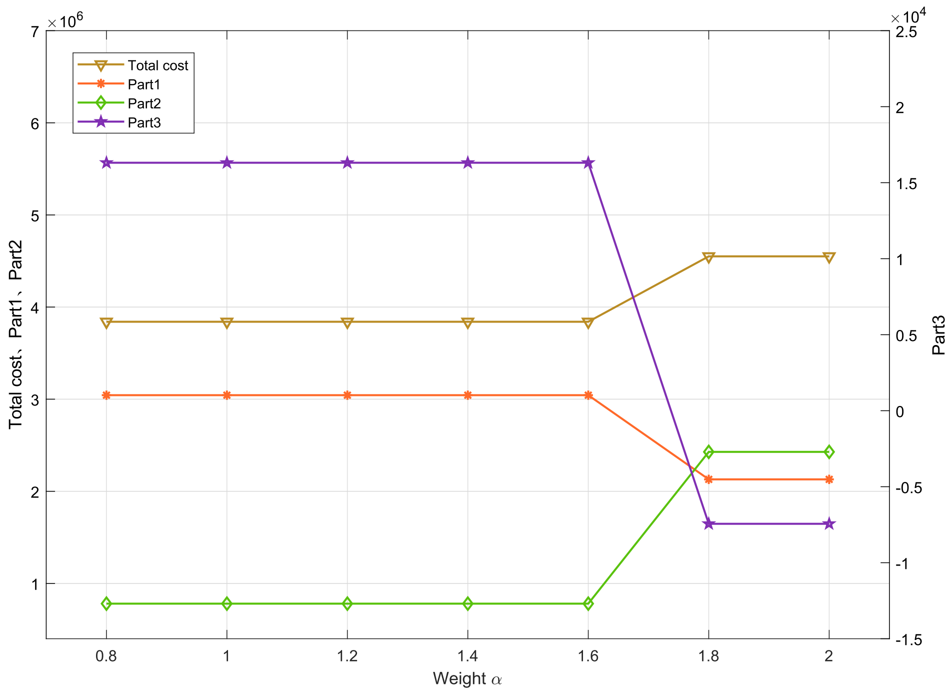

Fig. 8. The total cost, Part1, Part2, and Part3 under different α.

Table 17
The experimental results under different β.

β	The order quantity in each period						Total cost	Part1	Part2	Part3
	1	2	3	4	5	6				
0.2	40960	69697	74375	59665	0	0	5806793	1639649	4187279	−20134
0.4	40960	69697	74375	59665	73299	0	4550478	2129816	2428103	−7440
0.6, 0.8, 1, 1.2	40960	69697	74375	59665	73299	137283	3840510	3043492	780706	16312

In Table 18, when $\psi < 30.8$, the order quantity in each period remains the same. When $\psi = 30.8$, decision maker only reduces the order quantity in the sixth period, and keeps the order quantities in the first five periods unchanged. When $\psi > 30.8$, the decision maker doesn't order products in the sixth period. In contrast to $\psi < 30.8$, when $\psi = 30.8$, the sum of reductions in Part1 and Part3 is less than the increase in Part2, then the total cost increases slightly. Compared with $\psi = 30.8$, when $\psi > 30.8$, the total cost and Part2 increase, and Part1 and Part3 decrease. Therefore, the total cost and Part2 increase with ψ, while Part1 and Part3 decrease with ψ (See Fig. 10). The larger ψ indicates the decision maker gives high priority to the environmental impact of carbon emissions.

5.8. Managerial insights

Based on the above numerical experiments and result analysis, we propose the following three managerial insights:

- The market demand and unit carbon emission in transportation are uncertain in reality. The decision maker shouldn't use a model based on deterministic market demand and carbon emission to manage inventory. In addition, the optimal cost given by the robust model is slightly higher than that of the nominal model. That is to say, the robust optimal decision is resistant to the uncertainty about demand and carbon emission in transportation at a small price. Thus, when the market demand and unit carbon emission in transportation are uncertain and their distributions are unknown, to immunize the influence of the uncertainties, the decision maker can use our robust optimization model for inventory management.
- According to the case study, the single-supplier ordering way is not enough to deal with some emergency circumstances, for example, the demand's suddenly increase, supply disruption of single-supplier due to natural disasters. So, the orders only from single-supplier can lead to supply shortage and further bring shortage cost. By comparison, due to the diversity of procurement channels, the multi-supplier ordering way can effectively alleviate the supply shortage. Therefore, the decision maker should order from multi-supplier when there are multiple potential suppliers, and determine the order weight of each supplier before placing an order.

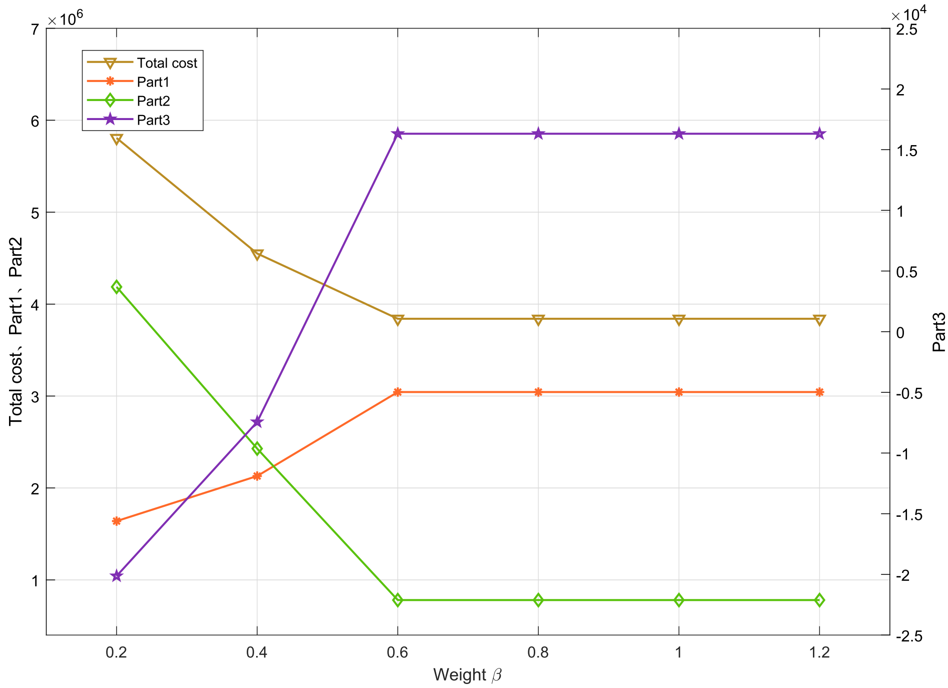

Fig. 9. The total cost, Part1, Part2, and Part3 under different β.

Table 18
The experimental results under different ψ.

ψ	The order quantity in each period						Total cost	Part1	Part2	Part3
	1	2	3	4	5	6				
15, 20, 25	40960	69697	74375	59665	73299	137283	3840510	3043492	780706	16312
30.8	40960	69697	74375	59665	73299	128244	3887381	2983914	888718	14744
32, 35	40960	69697	74375	59665	73299	0	4550478	2129816	2428103	-7440

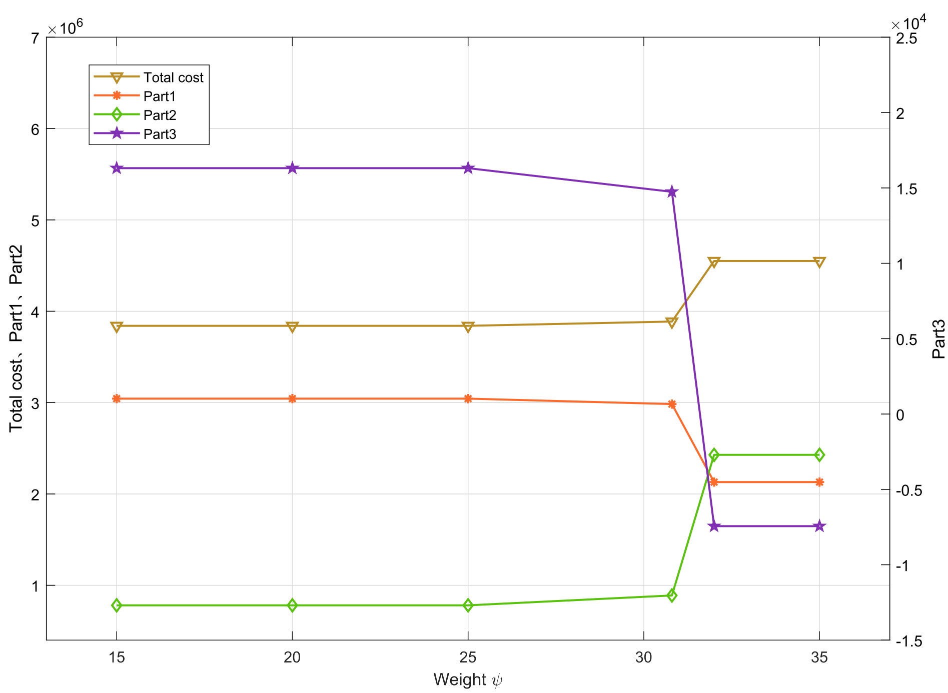

Fig. 10. The total cost, Part1, Part2, and Part3 under different ψ.

- We use the case study of iPhone11 to verify the effectiveness of the robust inventory model. Our robust multi-supplier multi-period inventory model can also be flexibly applied to other products. We take product quality, ordering cost, service level, and emergency capacity as evaluation criteria for each supplier, and calculate the order weight. When decision maker applies our robust inventory model to other products, he only needs to adjust the evaluation criteria to evaluate suppliers according to the practical need of the problem, and obtain the order weight of each supplier.

6. Conclusion and future research

In this paper, we proposed a robust multi-supplier multi-period inventory model with uncertain market demand and uncertain unit carbon emission in transportation. Our robust inventory model considers the orders from multi-supplier. About multi-supplier, we took product quality, ordering cost, service level, and emergency capacity as evaluation criteria, used Analytic Hierarchy Process (AHP) technique to comprehensively evaluate each supplier, and further gave the corresponding score of each supplier. According to the score, we got the order weight of each supplier.

We simultaneously considered two uncertain parameters: uncertain market demand and unit carbon emission in transportation. About these two uncertain parameters, we constructed Box+Ball and Budget uncertainty sets to characterize them. The proposed robust inventory model based on these two uncertainty sets is a semi-infinite programming model and not easy to solve directly. We used cone duality theory to transform the semi-infinite programming model into a mixed-integer second-order cone programming model, which can be solved by general commercial software. Hence, the proposed robust multi-supplier multi-period inventory model with uncertain demand and uncertain unit carbon emission is practical.

We took the DHL Shanghai warehouse as an example to conduct numerical experiments. The experimental results illustrated that our robust inventory model is feasible and effective, and the optimal decision given by our robust model is resistant to the uncertainty about demand and unit carbon emission in transportation at a small price. The more important is that multi-supplier can alleviate the occurrence of supply disruption and supply shortage. So, when the enterprise has multiple potential suppliers, the decision maker should order from multi-supplier.

The main limitation of this research is that the proposed inventory model assumes the distribution of perturbation parameter is free. The partial distribution information of uncertain market demand and unit carbon emission in transportation is not fully utilized. At present, we consider researchers to have the following limitation: the types and numbers of existing perturbation sets are limited. If the researchers want to apply the RO method to solve the inventory problem with uncertain parameters, they can only choose the appropriate perturbation sets from Budget, Box, Ball, and their different intersections. Therefore, constructing a new perturbation set which can guarantee the model's computational tractability is a challenge for the researchers.

There are some suggestions about future research. The first is to explore a multi-objective inventory optimization model with primary objective, such as minimizing the total cost (e.g., the sum of order cost and holding cost/shortage cost) and secondary objective, such as minimizing the total carbon emissions (e.g., the sum of carbon emissions in transportation and storage). The second is to study the multi-supplier multi-period inventory problem under uncertain demand and carbon emission with known partial distribution information. If the information is about the probability distribution, the stochastic distributionally robust optimization method (Ben-Tal et al., 2009) can be used to establish an inventory model to decide the optimal order quantity. When the information is about the possibility distribution, the researchers can apply the fuzzy distributionally robust optimization method (Liu, Chen, & Liu, 2021) to build an inventory model to seek the optimal order quantity.

CRediT authorship contribution statement

Yuqiang Feng: Methodology, Data curation, Writing - original draft, Software. **Yankui Liu:** Conceptualization, Methodology, Validation. **Yanju Chen:** Visualization, Supervision, Validation, Writing - review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Proof of Theorem 1

Here, we prove that the robust counterpart of uncertain constraint (12) can be represented by the system (20) based on $\mathcal{Z}_{\text{Budget}}$.

Proof 3. The perturbation set $\mathcal{Z}_{\text{Budget}}$ has the following conic representation:

$$\begin{aligned} \mathcal{Z}_{\text{Budget}} &= \left\{\boldsymbol{\xi} \in \mathbb{R}^K : \|\boldsymbol{\xi}\|_\infty \leqslant 1, \|\boldsymbol{\xi}\|_1 \leqslant \Gamma\right\} \\ &= \left\{\boldsymbol{\xi} \in \mathbb{R}^K : P_1\boldsymbol{\xi} + p_1 \in \mathcal{G}^1, P_2\boldsymbol{\xi} + p_2 \in \mathcal{G}^2\right\}, \end{aligned}$$

where

$$P_1\boldsymbol{\xi} = [\boldsymbol{\xi}; 0], p_1 = [0_{K\times 1}; 1], \mathcal{G}^1 = \left\{(\chi, \sigma) \in \mathbb{R}^K \times \mathbb{R} : \sigma \geqslant \|\chi\|_\infty\right\},$$
$$P_2\boldsymbol{\xi} = [\boldsymbol{\xi}; 0], p_2 = [0_{K\times 1}; \Gamma], \mathcal{G}^2 = \left\{(\chi, \sigma) \in \mathbb{R}^K \times \mathbb{R} : \sigma \geqslant \|\chi\|_1\right\}.$$

We use $\mathcal{G}_*$ for the dual cone of $\mathcal{G}$, where $\mathcal{G}_*^1 = \{(\chi, \sigma) \in \mathbb{R}^K \times \mathbb{R} : \sigma \geqslant \|\chi\|_1\}, \mathcal{G}_*^2 = \mathcal{G}^1$.

Setting $g^1 = [\nu_1; \tau_1], g^2 = [\nu_2; \tau_2]$ with one-dimensional τ_1, τ_2 and K-dimensional ν_1, ν_2, then the semi-infinite constraint (12) can be represented by the following system of conic inequalities in variables ν, τ, q:

$$\begin{cases} \tau_1 + \Gamma\tau_2 \leqslant + e_0^l \sum_{t=1}^{T}\left(\sum_{n=1}^{N} s_n\omega_n q_t + z_t\right) + E^- \leqslant E^{cap} + E^+, \\ (\nu_1)_k + (\nu_2)_k = e_k^l \sum_{n=1}^{N} s_n\omega_n q_t, \quad \forall k \in \left[K\right], \\ \|\nu_1\|_1 \leqslant \tau_1, \quad \left[\Leftrightarrow \left[\nu_1;\tau_1\right] \in \mathcal{G}_*^1\right], \\ \|\nu_2\|_\infty \leqslant \tau_2, \quad \left[\Leftrightarrow \left[\nu_2;\tau_2\right] \in \mathcal{G}_*^2\right]. \end{cases}$$

Eliminating τ_1, τ_2 from the above constraints, we get

$$\begin{cases} \sum_{k=1}^{K} |(\nu_1)_k| + \Gamma \max_k |(\nu_2)_k| + e_0^l \sum_{t=1}^{T}\left(\sum_{n=1}^{N} s_n\omega_n q_t + z_t\right) + E^- \leqslant E^{cap} + E^+, \\ (\nu_1)_k + (\nu_2)_k = e_k^l \sum_{n=1}^{N} s_n\omega_n q_t, \quad \forall k \in [K]. \end{cases}$$

Letting $\lambda = \nu_1, w = \nu_2$, we obtain system (20). □

Appendix B. Proof of Theorem 2

We prove that the robust counterpart of uncertain constraint (13) can be represented by the system (21) under $\mathcal{Z}_{\text{Box+Ball}}$.

Proof 4. The perturbation set $\mathcal{Z}_{\text{Box+Ball}}$ is the intersection of two uncertainty sets (Ball and Box). For a given $t \in [T]$, this perturbation set has the following conic representation:

$$\mathcal{Z}_{\text{Box+Ball}} = \left\{ \zeta \in \mathbb{R}^T : -1 \leqslant \zeta_i \leq 1, \ \sqrt{\sum_{i=1}^{t} \zeta_i^2} \leqslant \Omega_t, \ i \leqslant t, t \leqslant T \right\}$$
$$= \left\{ \zeta \in \mathbb{R}^T : P_1\zeta + p_1 \in \mathcal{G}^3, P_2\zeta + p_2 \in \mathcal{G}^4 \right\},$$

where

$$P_1\zeta = [\zeta;0], p_1 = [0;1], \mathcal{G}^3 = \left\{(\chi,\sigma) \in \mathbb{R}^T \times \mathbb{R} : \sigma \geqslant \|\chi\|_\infty \right\}, \tag{25}$$

$$P_2\zeta = \left[\zeta;0\right], p_2 = \left[0;\Omega\right], \mathcal{G}^4 = \left\{\left(\chi,\sigma\right) \in \mathbb{R}^T \times \mathbb{R} : \sigma \geqslant \|\chi\|_2 \right\}. \tag{26}$$

We use $\mathcal{G}_*$ for the dual cone of $\mathcal{G}$, where $\mathcal{G}_*^3 = \{(\chi,\sigma) \in \mathbb{R}^T \times \mathbb{R} : \sigma \geqslant \|\chi\|_1 \}$, and $\mathcal{G}_*^4 = \mathcal{G}^4$.

Setting $\rho^1 = [\nu_3;\tau_3], \rho^2 = [\nu_4;\tau_4]$ with one-dimensional τ_3, τ_4 and T-dimensional ν_3,ν_4, then the semi-infinite constraint (13) can be represented by the following system of conic inequalities in variables ν, τ, q:

$$\begin{cases} \tau_3 + \Omega_t\tau_4 \leqslant \frac{y_t}{c_h} - w_1 + \sum_{i=1}^{t} d_i^0 - \sum_{i=1}^{t} q_i, \\ (\nu_3 + \nu_4)_i = \widetilde{d}_i, \quad \forall i \in \left[t\right], \\ \|\nu_3\|_1 \leqslant \tau_3, \quad \left[\Leftrightarrow \left[\nu_3;\tau_3\right] \in \mathcal{G}_*^3\right], \\ \|\nu_4\|_2 \leqslant \tau_4, \quad \left[\Leftrightarrow \left[\nu_4;\tau_4\right] \in \mathcal{G}_*^4\right]. \end{cases}$$

Eliminating τ_3, τ_4 from the above constraints, one get

$$\begin{cases} c_h w_1 - c_h \sum_{i=1}^{t} \left(d_i^0 - q_i - |(\nu_3)_i|\right) + c_h\Omega_t \sqrt{\sum_{i=1}^{t} (\nu_4)_i^2} \leqslant y_t, \\ (\nu_3)_i + (\nu_4)_i = \widetilde{d}_i, \quad \forall i \in \left[t\right]. \end{cases}$$

Letting $\delta 1 = \nu_3, \eta 1 = \nu_4$, we gain system (21).

Note: the proof processes of the robust counterparts of uncertain constraints (14)–(16) can be represented by systems (22)–(24) are similar to the above proof process. □

References

Alizadeh, F., & Goldfarb, D. (2003). Second-order cone programming. *Mathematical Programming, 95*(1), 3–51.

Alinovi, A., Bottani, E., & Montanari, R. (2012). Reverse logistics: a stochastic EOQ-based inventory control model for mixed manufacturing/remanufacturing systems with return policies. *International Journal of Production Research, 50*(5), 1243–1264.

Bertsimas, D., & Sim, M. (2004). The price of robustness. *Operations Research, 52*(1), 35–53.

Bertsimas, D., & Thiele, A. (2006). A robust optimization approach to inventory theory. *Operations Research, 54*(1), 150–168.

Ben-Tal, A., El Ghaoui, L., & Nemirovski, A. (2009). *Robust Optimization*. Princeton University Press.

Bozorgi, A. (2016). Multi-product inventory model for cold items with cost and emission consideration. *International Journal of Production Economics, 176*, 123–142.

Chakraborty, T., Chauhan, S., & Ouhimmou, M. (2020). Mitigating supply disruption with a backup supplier under uncertain demand: competition vs. cooperation. *International Journal of Production Research, 58*(12), 3618–3649.

Craven, M., Liu, L., Mysore, M., & Wilson, M. (2020). COVID-19: Implications for business. *McKinsey & Company*, 1–8.

Dada, M., Petruzzi, N. C., & Schwarz, L. B. (2007). A newsvendors procurement problem when suppliers are unreliable. *Manufacturing & Service Operations Management, 9*(1), 9–32.

Deshmukh, A. J., & Chaudhari, A. A. (2011). A review for supplier selection criteria and methods. In *Technology Systems And Management* (pp. 283–291). Berlin, Heidelberg: Springer.

Dinler, D., Tural, M., & Iyigun, C. (2015). Heuristics for a continuous multi-facility location problem with demand regions. *Computers & Operations Research, 62*, 237–256.

Guo, Z., Tian, S., & Liu, Y. (2017). A multiproduct single-period inventory, management problem under variable possibility distributions. *Mathematical Problems in Engineering, 2017*, 2159281.

Guo, Z., & Liu, Y. (2018). Modelling single-period inventory problem by distributionally robust fuzzy optimization method. *Journal of Intelligent & Fuzzy Systems, 35*(1), 1007–1019.

Gao, X., Chen, S., Tang, H., & Zhang, H. (2020). Study of optimal order policy for a multi-period multi-raw material inventory management problem under carbon emission constraint. *Computers & Industrial Engineering, 148*, 106693.

Hua, G., Cheng, T. C. E., & Wang, S. (2009). Managing carbon footprints in inventory control. *SSRN Electronic Journal, 132*(2), 178–185.

Hovelaque, V., & Bironneau, L. (2015). The carbon-constrained EOQ model with carbon emission dependent demand. *International Journal of Production Economics, 164*, 285–291.

Hosseini, S., Morshedlou, N., Ivanov, D., Sarder, M. D., Barker, K., & Al Khaled, A. (2019). Resilient supplier selection and optimal order allocation under disruption risks. *International Journal of Production Economics, 213*, 124–137.

Hosseini, S., Ivanov, D., & Dolgui, A. (2019). Review of quantitative methods for supply chain resilience analysis. *Transportation Research Part E: Logistics and Transportation Review, 125*, 285–307.

Huang, Y., Fang, C., & Lin, Y. (2020). Inventory management in supply chains with consideration of logistics, green investment and different carbon emissions policies. *Computers & Industrial Engineering, 139*, 106207.

Jadidi, O., Jaber, M., Zolfaghri, S., Pinto, R., & Firouzi, F. (2021). Dynamic pricing and lot sizing for a newsvendor problem with supplier selection, quantity discounts, and limited supply capacity. *Computers & Industrial Engineering, 154*, 107113.

Karsak, E. E., & Dursun, M. (2015). An integrated fuzzy MCDM approach for supplier evaluation and selection. *Computers & Industrial Engineering, 82*, 82–93.

Liao, H., & Deng, Q. (2018). A carbon-constrained EOQ model with uncertain demand for remanufactured products. *Journal of Cleaner Production, 199*, 334–347.

Li, Y., & Liu, Y. (2019). A risk-averse multi-item inventory problem with uncertain demand. *Journal of Data, Information and Management, 1*(3), 77–90.

Li, Z., & Hai, J. (2019). Inventory management for one warehouse multi-retailer systems with carbon emission costs. *Computers & Industrial Engineering, 130*, 565–574.

Li, T., Fang, W., & Baykal-Gürsoy, M. (2021). Two-stage inventory management with financing under demand updates. *International Journal of Production Economics, 232*, 107915.

Liu, N., Chen, Y., & Liu, Y. (2021). Approximating credibilistic constraints by robust counterparts of uncertain linear inequality. *Iranian Journal of Fuzzy Systems, 18*(5), 37–51.

Mohammadi, M., Torabi, S., & Tavakkoli-Moghaddam, R. (2014). Sustainable hub location under mixed uncertainty. *Transportation Research Part E: Logistics and Transportation Review, 62*, 89–115.

Miyashiro, R., & Takano, Y. (2015). Mixed integer second-order cone programming formulations for variable selection in linear regression. *European Journal of Operational Research, 247*(3), 721–731.

Mutha, A., Bansal, S., & Guide, V. (2016). Managing demand uncertainty through core acquisition in remanufacturing. *Production and Operations Management, 25*(8), 1449–1464.

Mohammed, F., Selim, S. Z., Hassan, A., & Syed, M. (2017). Multi-period planning of closed-loop supply chain with carbon policies under uncertainty. *Transportation Research Part D: Transport and Environment, 51*, 146–172.

Mishra, U., Wu, J., & Sarkar, B. (2021). Optimum sustainable inventory management with backorder and deterioration under controllable carbon emissions. *Journal of Cleaner Production, 279*, 123699.

Prékopa, A. (2013). *Stochastic Programming*. Springer Science & Business Media.

Soyster, A. L. (1973). Convex programming with set-inclusive constraints and applications to inexact linear programming. *Operations Research, 21*(5), 1154–1157.

Song, Z., Tang, W., Zhao, R., & Zhang, G. (2021). Inventory strategy of the risk averse supplier and overconfident manufacturer with uncertain demand. *International Journal of Production Economics, 234*, 108066.

Silbermayr, L., & Minner, S. (2014). A multiple sourcing inventory model under disruption risk. *International Journal of Production Economics, 149*, 37–46.

Song, H., Ran, L., & Shang, J. (2017). Multi-period optimization with loss-averse customer behavior: Joint pricing and inventory decisions with stochastic demand. *Expert Systems with Applications, 72*, 421–429.

Svoboda, J., Minner, S., & Yao, M. (2021). Typology and literature review on multiple supplier inventory control models. *European Journal of Operational Research, 293*, 1–23.

Taleizadeh, A. A., Shavandi, H., & Haji, R. (2011). Constrained single period problem under demand uncertainty. *Scientia Iranica, 18*(6), 1553–1563.

Tajik, N., Tavakkoli-Moghaddam, R., Vahdani, B., & Mousavi, S. M. (2014). A robust optimization approach for pollution routing problem with pickup and delivery under uncertainty. *Journal of Manufacturing Systems, 33*(2), 277–286.

Tao, Z., & Tang, Z. (2015). *An Integrated Inventory Control and Transportation Model Under Carbon Emission Trade*. Berlin Heidelberg: Springer.

Thorsen, A., & Yao, T. (2017). Robust inventory control under demand and lead time uncertainty. *Annals of Operations Research, 257*(1), 207–236.

Thomas, P. G., & Doherty, P. C. (1980). The Analytic Hierarchy. In *Process: Planning, Priority Setting, Resource Allocation*. New York.: McGraw-Hill.

Tiwari, S., Daryanto, Y., & Wee, H. M. (2018). Sustainable inventory management with deteriorating and imperfect quality items considering carbon emission. *Journal of Cleaner Production, 192*, 281–292.

Torabi, S. A., Baghersad, M., & Mansouri, S. A. (2015). Resilient supplier selection and order allocation under operational and disruption risks. *Transportation Research Part E: Logistics and Transportation Review, 79*, 22–48.

Wang, T. K., Zhang, Q., Chong, H. Y., & Wang, X. (2017). Integrated supplier selection framework in a resilient construction supply chain: An approach via analytic hierarchy process (AHP) and grey relational analysis (GRA). *Sustainability, 9*(2), 289.

Wei, C., Li, Y., & Cai, X. (2011). Robust optimal policies of production and inventory with uncertain returns and demand. *International Journal of Production Economics, 134*(2), 357–367.

Yin, F., Chen, Y., Song, F., & Liu, Y. (2019). A new distributionally robust p-hub median problem with uncertain carbon emissions and its tractable approximation method. *Applied Mathematical Modelling, 74*, 668–693.