Robust Pricing for a Dual-channel Green Supply Chain under Fuzzy Demand Ambiguity

Huili Pei, Yankui Liu, Hongliang Li

***Abstract*—The pricing decision is a fundamental problem in a two-echelon dual-channel supply chain and it is a challenging issue to get the optimal pricing strategy due to high demand ambiguity. Although this problem has been studied by many researchers, the market demand is usually assumed to be deterministic or described as a fuzzy variable with exact distribution information in most of the existing literature. In this paper, the uncertain demand is assumed to possess fuzzy uncertainty instead of stochastic nature. In the case that the demand distribution information is partially available, this paper proposes a new uncertainty distribution set to depict the demand distribution uncertainty based on type-2 fuzzy theory. Then, a novel robust pricing game modeling framework is developed for the dual-channel green supply chain with the demand distribution varying in the proposed uncertainty distribution set. Finally, a new method is proposed for obtaining the robust equilibrium decisions of supply chain members. Numerical analysis and comparisons are conducted to demonstrate the impacts of the demand ambiguity on the manufacturer's equilibrium channel choice strategy. We also demonstrate our solution results are robust in the setting that the prices of the green products are not equal in the retail channel and the direct channel.**

***Index Terms*—Dual-channel green supply chain; Type-2 fuzzy variable; Robust pricing; Demand ambiguity; Game theory**

I. Introduction

With the rapid development of Internet and related information technology, more and more consumers choose to purchase through online channel. In 2021, the 48th statistical report on the development of Internet released by China Internet Network Information Center (CNNIC) showed that the number of Internet users in China has reached 1.011 billion. In the first half of 2021, the online retail sales reached 6113.3 billion yuan. In order to meet consumers' purchase choices, more and more manufacturers have opened online direct selling channel. For example, computer manufacturers (Apple, Dell and Lenovo), cosmetics industry (Estee Lauder), beverage and food industry (Coca Cola), sports industry (Nike), supermarkets (Carrefour, WalMart) and electronics manufacturers (Samsung and Sony) are selling their products through both online channel and the retail channel [1]. That is, they own dual-channel: retail channel and direct channel.

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Implementation of dual-channel helps the manufacturers to create their own market and obtain more benefits from the growing demand. Meanwhile, adding a direct channel may also place the manufacturer in competition with the existing retailers ([2], [3]). In order to compete for consumers, it is inevitable to blindly reduce the sales price which will affect the profits of the supply chain members. Dual-channel also makes consumers compare price information between the retail and online stores to make sure that they can buy the most suitable product at the lowest price. Therefore, pricing decisions are very critical for the manufacturers in dual-channel supply chain.

Many uncertain factors (for instance, material costs, customer incomes, workers' expenses and technology improvements) exist in the real world and the uncertain factors usually make the demand ambiguous and unpredictable which will affect pricing decisions in dual-channel supply chain. Therefore, how to characterize the uncertain demand becomes very important for the decision-makers. In the existing literature ([4]-[6]), the market demand is assumed to be deterministic or described as a random variable with known probability distribution. In fact, the exact probability distribution of demand may be difficult to obtain from limited historical data. Moreover, in some cases, it may be impossible to collect data ([7]). To address this situation, in our model, the demand is assumed to possess fuzzy uncertainty instead of stochastic nature. In this case, the demand information can be approximately estimated based on the experiences or subjective judgments of experts. For example, Jamali *et al.* [8] studied pricing problem in a dual-channel green supply chain by characterizing the uncertain demand as a fuzzy variable with fixed possibility distribution. However, the exact possibility distribution or membership function is difficult to obtain due to insufficient cognition.

Motivated by the above discussions, in this paper, we assume that the manufacturer only knows that the demand distribution varies in some uncertainty distribution set instead of the exact demand distribution. In the presence of the demand ambiguity, this paper aims to address the following four questions: (Q1) How to depict the imprecise distribution of uncertain demand with limited historical data? (Q2) What is the structure of the optimal solution like? (Q3) How do the uncertain parameters in the uncertainty set affect the manufacturer's channel choice strategy? (Q4) Are the results robust for other settings?

To answer the above questions, this paper studies the pricing decision problem of a two-echelon dual-channel green supply chain assuming that the demand distribution is unknown and only partial demand distribution information is available. In order to depict the imprecise distribution of uncertain demand, we introduce a new uncertainty distribution set based on type-2 fuzzy theory and the uncertainty distribution set is constructed based on PLI type-2 triangular fuzzy variable. Based on the proposed uncertainty distribution set, a novel robust pricing game modeling framework is developed for the dual-channel green supply chain. We also propose a new method to obtain the corresponding robust equilibrium decisions. Also, the effects of the uncertainty perturbation parameters on the manufacturer's channel choice strategy are investigated.

In comparison to the existing literature, the main contributions of the present paper include the following aspects.

• Based on type-2 fuzzy theory, a new uncertainty distribution set is proposed to depict the ambiguous distribution of the uncertain demand in the case that the demand distribution is partially available. The parameters embedded in the variable distributions facilitates the managers to make wise decisions according to their attitudes toward uncertainty.

• A novel robust pricing game modeling framework is provided via the proposed new uncertainty distribution set for a two-echelon dual-channel green supply chain. A new method is presented to derive the robust equilibrium decisions of the robust pricing game model.

• Numerical analysis are conducted to investigate how the uncertainty perturbation parameters affect the manufacturer's channel choice strategy. The numerical results show that the manufacturer's channel selection strategy changes when the uncertain perturbation parameters change. Specifically, when the values of uncertainty perturbation parameters are relatively small, opening a direct selling channel is not beneficial to the manufacturer. When the values of uncertainty perturbation parameters are relatively large, the manufacturer is willing to own a direct selling channel.

• We demonstrate our results are robust in the setting that the prices of the green products are not equal in the retail channel and the direct channel.

The remainder of this paper is organized as follows. In Section II, we review the relevant literature. Section III introduces the robust pricing game model. The robust equilibrium decisions are derived in Section IV and numerical results are presented in Section V. We present an extension of our model in Section VI and conclude the paper in Section VII.

II. Literature review

Literature mostly related to our work can be categorized into the following two streams: fuzzy pricing models in dual-channel supply chain and fuzzy optimization theory.

A. Fuzzy pricing models in dual-channel supply chain

Determining the product prices of both channels in dual-channel supply chain is an important issue. Pricing decisions of dual-channel supply chain have been considered by many researchers. There exist many pricing models about dual-channel supply chain. Here we only focus on fuzzy pricing models in dual-channel supply chain. We apply the methodology used in [9]-[11] to gather the fuzzy pricing models in dual-channel supply chain using Web of Science search engine, and 14 papers ([8], [12]-[24]) are obtained. For example, Soleimani [13] analyzed the pricing decisions of a dual-channel supply chain characterizing the manufacturing cost and the customer demand as fuzzy variables. Jamali *et al*. [8] addressed green product pricing decision considering dual distribution channels under fuzzy conditions. They characterized production cost of the green product, cost coefficient of the product greenness level, and demand as fuzzy variables with known distributions. Yang *et al.* [23] modeled the environmental responsibility behaviors of both manufacturer and consumers to study the dual-channel structure strategy of a green manufacturer and further examined its environmental performance under fuzzy demand. They showed that consumer demand uncertainty encourages the manufacturer to open his online channel. Karthick and Uthayakumar [24] considered the pricing problems in a dual-channel supply chain where the demands are uncertain or ambiguous and are treated as trapezoidal fuzzy numbers.

In dealing with fuzzy demand, the above mentioned literature usually assumed that the exact possibility distribution or membership function of the uncertain demand is accurately known. In fact, it is impossible to obtain the exact possibility distribution or membership function of uncertain model parameters due to insufficient cognition and limited cognitive means. Different from the existing literature, we consider the demand distribution uncertainty and characterize the uncertain demand using an uncertainty distribution set.

B. Fuzzy optimization theory

It is well known that pricing decisions in dual-channel supply chain are often affected by the uncertain market demand. And the exact distribution of the uncertain demand is difficult to obtain because of the rapid changes in real-life situations. In this case, the distribution information of the uncertain demand can be approximately estimated based on the experts' judgments, intuitions and experiences. To address this issue, type-2 fuzzy theory has been employed which can date back to Zadeh [25] who first proposed the concept of type-2 fuzzy set. In order to make type-2 fuzzy sets easy to use and understand, Mendel and John [26] established some basic terms for type-2 fuzzy set. Subsequently, type-2 fuzzy theory has been increasingly used and has become one of the most important approaches to characterize subjective uncertainty ([27], [28]). For instances, Wu and Mendel [29] introduced the centroid, cardinality, fuzziness, variance and skewness of an interval type-2 fuzzy set as measures of uncertainty. Liu and Liu [30] developed an axiomatic method called fuzzy possibility theory, in which the variable based method is used instead of the set based method to deal with type-2 fuzziness. Ngan [31] provided a framework called probabilistic linguistic computing in the context of general type-2 fuzzy setting. Tahayori *et al.* [32] presented a universal methodology for generating an interval type-2 fuzzy set membership function from a collection of type-1 fuzzy sets. At the same time, type-2 fuzzy set has been successfully applied to many practical applications. For example, Pagola *et al.* [33] proposed a new fuzzy thresholding algorithm, in which an expert can select multiple membership functions to construct an interval type-2 fuzzy set such that the length of the interval represents the uncertainty of the expert. Based on type-2 fuzzy theory, Bai *et al.* [34] developed a distributionally robust sustainable development model where the uncertain per capita gross domestic product, per capita electricity consumption and per capita greenhouse gas emissions are characterized by parametric interval-valued possibility distributions and their associated uncertainty distribution sets. Muhuri *et al.* [35] addressed the multi-objective reliability-redundancy allocation problem where the component parameters are modeled as interval type-2 fuzzy numbers. They showed that their approach outperforms classical as well as other type-1 fuzzy number based approaches. Kundu *et al.* [36] proposed a fuzzy multi-criteria group decision-making method based on ranking interval type-2 fuzzy variables and applied it to a transportation mode selection problem. Guo and Liu [37] developed a distributionally robust fuzzy optimization model for single-period inventory problem, in which the uncertain market demand is characterized by generalized parametric interval-valued possibility distribution. In this paper, we propose a new method to depict the ambiguous demand distribution and develop a novel robust game modeling framework for the dual-channel supply chain pricing problem which is completely different from [34], [37].

III. The Model

We consider a two-echelon dual-channel green supply chain which consists of a manufacturer and a retailer, where the manufacturer makes and sells the green products through both the direct channel and the retail channel. Customers can purchase products through either of the two channels according to their preference. The manufacturer is assumed to produce the green products with greening degree β at a cost of c. The manufacturer sells the product to the retailer at the wholesale price ω and then the retailer sells it to the customers through the retail channel at a retail price p_r. The manufacturer also has an option to sell the green products directly to the end consumers at price p_d.

This paper assumes that the price in the direct channel is equal to that in the retail channel in order to reduce channel conflict, that is, $p_r = p_d = p$ (the analysis of the nonconsistent pricing strategy will be discussed in Section VI). To avoid the trivial case, we assume $p > \omega > c$.

We model the demands faced by the manufacturer and the retailer as linear functions of both the greening degree β and the sale price p. The demands of both channels are assumed decreasing in the sale price p and increasing in the greening level β, which is similar to [38]. Consequently, the demand functions can be represented as $D_r = \gamma a - \eta p + \delta_r\beta, D_d = (1-\gamma)a - \eta p + \delta_d\beta$, where a is the market size, η is the sensitivity of demand to price changes. $\gamma (0 < \gamma < 1)$ is the degree of customer loyalty to the retail channel, and correspondingly, $1-\gamma$ represents the degree of customer loyalty to the direct channel. δ_r and δ_d represent the demand expansion effectiveness coefficients of the greening innovation in the retail channel and the direct channel, respectively. The assumption of linearity with respect to price and greening level has been made in many of operations management and marketing literature ([38]-[40]).

Both the direct and the retail channels have their own customers. For the same green level, the impact of retail channel on customers is greater than that of direct channel because customers can thoroughly check the green products when they purchase the products, which means $\delta_r > \delta_d$.

The manufacturer must invest additional cost to realize the green innovation based on the original production process. The cost of green innovation is assumed to be quadratic([23], [41]). Therefore, the extra cost to produce the green products is $C(\beta) = \nu\beta^2$, where ν is an investment coefficient of the green degree per unit. Table I summarizes the notations used in this paper.

There are industries, such as GM, Toyota, Canon, Xerox, and HP, where the manufacturers have more power than the retailers. So, we discuss the pricing policies under manufacturer-led Stackelberg game framework where the manufacturer as the leader and the retailer as the follower.

The sequence of events is as follows:

(1) The manufacturer first announces the wholesale price and the green level of the green products;

(2) After observing the wholesale price and the green level of the products, the retailer sets the retail price.

A. The benchmark

Based on the above descriptions and assumptions, the benchmark model can be represented as the following optimization model:

$$\begin{array}{ll} \max\limits_{\omega,\beta} & \pi_m(p,\omega,\beta) \\ \text{s.t.} & p \in \arg\max\limits_{p} \pi_r(p). \end{array} \tag{1}$$

Let us denote by ω^b, β^b and p^b the equilibrium solutions to the benchmark model defined in Equation (1). By using backward induction, Lemma 1 gives the equilibrium decisions.

Lemma 1: If $\nu > \frac{\delta_d^2+6\delta_r\delta_d-3\delta_r^2}{12\eta}$, we have the following equilibrium decisions:

(a) The manufacturer's optimal decisions are

$$\begin{aligned} \omega^b &= \tfrac{1}{\eta\varphi}[a\gamma\psi + a(\delta_r\delta_d - \delta_d^2 - \psi) + 2\eta c(2\delta_r\delta_d - \psi)], \\ \beta^b &= \tfrac{1}{\varphi}[2a\gamma(\delta_d - 3\delta_r) + a(\delta_d + 3\delta_r) - 4\eta c\delta_d]. \end{aligned}$$

(b) The retailer's optimal retail price is

$$\begin{aligned} p^b = \ & \tfrac{1}{\varphi\eta}[2a(\gamma(2\eta\nu - \delta_r\delta_d + \delta_r^2) + \eta\nu + \delta_r^2) \\ & + \eta c(\psi - 4\delta_r\delta_d)], \end{aligned}$$

where $\varphi = 12\eta\nu - \delta_d^2 - 6\delta_r\delta_d + 3\delta_r^2, \psi = 4\eta\nu - \delta_d^2 + \delta_r^2$. All proofs in this paper are included in Appendix A.

B. Distributionally robust pricing model

In the benchmark model, the market size a is assumed to be deterministic. Generally, the market size is uncertain due to the impacts of the economic environment and business conditions. In this section, we consider the distribution μ_ξ of the uncertain market size ξ is partially available and varies in the uncertainty distribution set $\mathcal{U}$ which will be defined in Section IV. We assume that both parties in the Stackelberg game are risk-neutral and plan for the worst case.

TABLE I: Notations

Symbols	Description
p_r	Retail price of the retail channel
p_d	Selling price of the direct channel
ω	Wholesale price
a	The deterministic market size
ξ	The uncertain market size
β	Green degree of the green products
γ	The degree of customer loyalty to the retail channel
η	The sensitivity of demand to price changes
δ_r	Demand expansion effectiveness coefficient of the greening innovation in the retail channel
δ_d	Demand expansion effectiveness coefficient of the greening innovation in the direct channel
ν	Investment coefficient of the green degree per unit
D_r	The demand of retail channel in consistent price strategy
D_d	The demand of direct channel in consistent price strategy
$\overline{D}_r$	The demand of retail channel in nonconsistent price strategy
$\overline{D}_d$	The demand of direct channel in nonconsistent price strategy
$\mathcal{U}$	The uncertainty distribution set
Δ_l	Left span
Δ_r	Right span

On the basis of the uncertainty distribution set, the uncertain pricing model is represented as follows

$$\left\{\begin{array}{ll} \max\limits_{\omega,\beta} & \mathrm{E}[\pi_m(p,\omega,\beta;\xi)], \\ \text{s.t.} & p \in \arg\max\limits_{p} \mathrm{E}[\pi_r(p;\xi)]. \end{array}\right\}_{\mu_\xi \in \mathcal{U}.} \tag{2}$$

where $\pi_m(p,\omega,\beta;\xi) = (p-c)D_d + (\omega-c)D_r - \nu\beta^2$ represents the manufacturer's profit with the first two terms indicating the profits of products sold through the direct channel and the retail channel, respectively, and the last term denoting the extra cost for producing the green products, $\pi_r(p;\xi) = (p-\omega)D_r$ represents the profit of the retailer and E is the expected value operator of fuzzy variable ([42]-[44]).

It is evident that model (2) is a family of fuzzy expected value models with μ_ξ varying in the uncertainty distribution set $\mathcal{U}$.

Clearly, the inherent difficulty is that a collection of expected value models is not associated with the concepts of optimal solution and optimal value. Therefore, how to define these concepts to model (2) depends on the underlying decision environment. Here we concentrate on the following decision-making environment:

(A1) The manufacturer's decisions and the retailer's decision in model (2) represent "here and now" decisions;

(A2) The manufacturer and the retailer must be fully responsible for the consequences of the decisions to be made when and only when μ_ξ belongs to the corresponding uncertainty distribution set $\mathcal{U}$.

The above assumptions can determine a meaningful feasible solution to model (2) based on the worst-case criterion, which is called a distributionally robust feasible solution. Thus, the robust counterpart of model (2) is formally written as

$$\begin{array}{ll} \max\limits_{\omega,\beta} & \inf\limits_{\mu_\xi \in \mathcal{U}} \mathrm{E}[\pi_m(p,\omega,\beta;\xi)] \\ \text{s.t.} & p \in \arg\max\limits_{p} \inf\limits_{\mu_\xi \in \mathcal{U}} \mathrm{E}[\pi_r(p;\xi)]. \end{array} \tag{3}$$

So far, we have obtained a robust counterpart model (3). The optimal solution and optimal value of the robust counterpart model (3) are called the distributionally robust optimal solution and optimal value, respectively.

It is evident that model (3) includes infinitely many integrals, which are computationally tractable under our uncertainty distribution set $\mathcal{U}$. In order to derive the equivalent expression of robust counterpart model (3), we will define the uncertainty distribution set in the subsequent section.

IV. Model analysis

Usually, the market size is assumed to be deterministic in the existing literature ([4], [5]). Actually, the market size a is uncertain due to the innovation and market turbulence and varies on a bounded interval, assuming $a \in [a(1-\Delta_l), a(1+\Delta_r)]$. So we can use triangular fuzzy variable to characterize the distribution uncertainty of the uncertain market size ξ. Furthermore, in order to describe the distribution perturbation of the uncertain market size ξ, we assume the uncertain market size ξ is represented by a parametric level interval (PLI) type-2 triangular fuzzy variable $\mathrm{Tri}[r_1,r_2,r_3;\theta], \theta = (\theta_l^1,\theta_r^1,\theta_l^2,\theta_r^2)$, where $r_1 = a(1-\Delta_l), r_2 = a, r_3 = a(1+\Delta_r)$. In this section, a new uncertainty distribution set $\mathcal{U}$ is introduced. For some concepts and notations used but not defined in this section, the interested reader may refer to the related literature [45].

Uncertainty distribution set Let the uncertain market size $\xi \sim \mathrm{Tri}[r_1,r_2,r_3;\theta], \theta = (\theta_l^1,\theta_r^1,\theta_l^2,\theta_r^2)$ be a PLI type-2 triangular fuzzy variable and ξ^λ be its λ selection variable. For any given $\lambda \in [0,1]$, the distribution of ξ^λ is denoted by $\mu_{\xi^\lambda}(x;\theta)$ which is determined by the nested sets $\{[x_u^L, x_u^R] : x_u^L \in J_u^L, x_u^R \in J_u^R, u \in [0,1]\}$, where x_u^L, x_u^R are represented as

$$\begin{aligned} x_u^L = \; & \lambda(r_1 + u(r_2-r_1) - \theta_l^1 \min\{u,1-u\}) \\ & +(1-\lambda)(r_1 + u(r_2-r_1) + \theta_r^1 \min\{u,1-u\}), \\ x_u^R = \; & \lambda(r_3 - u(r_3-r_2) - \theta_l^2 \min\{u,1-u\}) \\ & +(1-\lambda)(r_3 - u(r_3-r_2) + \theta_r^2 \min\{u,1-u\}). \end{aligned}$$

Then an uncertainty distribution set $\mathcal{U}$ associated with ξ is defined as follows

$$\begin{aligned} \mathcal{U} = \; & \{\mu_{\xi^\lambda}(x;\theta) | \mu_{\xi^\lambda}(x;\theta) \text{is determined by the nested sets} \\ & \{[x_u^L, x_u^R] : x_u^L \in J_u^L, x_u^R \in J_u^R, u \in [0,1]\}, \text{where} \\ & \lambda \in [0,1]\}. \end{aligned} \tag{4}$$

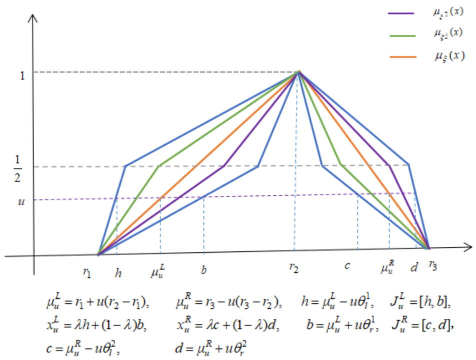

Fig. 1: The variable possibility distribution of uncertain demand ξ.

Evidently, there are infinitely many possibility distributions $\mu_{\xi^\lambda}(x;\theta)$ in the uncertainty distribution set $\mathcal{U}$. For the uncertain market size ξ, the possibility distribution of the selection variable ξ^λ is plotted in Figure 1.

According to the reviewers' valuable suggestions on why this paper uses PLI type-2 triangular fuzzy variable to describe the uncertain demand, we give some explanations and summarize our concern in following Remark.

Remark: The reason that we use PLI type-2 triangular fuzzy variable to characterize the uncertain demand includes the following two aspects. First, in our model, based on the limited sales data and subjective judgments of expert, we can know the most possible demand quantity, and also learn about the uncertain demand varies on a bounded interval via some limited sales data. In this case, the uncertain demand can be represented by a PLI type-2 triangular fuzzy variable. In addition, it is known that type-2 triangular fuzzy variable is one of the most commonly used fuzzy variables in the literature ([34], [46], [47], [48]). In fact, other PLI type-2 fuzzy variables can also be used to construct uncertainty distribution set. For example, if the possible demand quantity can be estimated to vary in an interval from historical sales data, the uncertain demand can be modeled as a PLI type-2 trapezoidal fuzzy variable.

We next give a real case example to illustrate how to construct the uncertainty distribution set $\mathcal{U}$ to depict the imprecise distribution of uncertain demand with limited historical data.

Example: Consider a case about a manufacturer that sells its air conditioners through online and offline channels. Before the summer, the manufacturer plans to produce a fixed number of air conditioners. However, the exact demand distribution information is not available due to market turbulence and product innovation. Based on the limited sales data and subjective judgments of experts, the number of market demand ξ is between 800 and 1300 during a sales cycle and the most possible sales quantity is 1000. According to the sales experience, we can take the triangular possibility distribution $\mathrm{Tri}[800, 1000, 1300]$ as the nominal distribution of the market demand ξ. The true demand distribution fluctuates around the nominal distribution. In order to characterize the perturbation of the nominal distribution, we begin with the cut set of the nominal distribution. The $u-$cut of the nominal distribution is assumed to be $[T_u^L, T_u^R]$, where $T_u^L = 800 + u(1000 - 800), T_u^R = 1300 - u(1300 - 1000)$ are fixed. In fact, the endpoints T_u^L and T_u^R are difficult to determine. Thus, we construct two intervals J_u^L, J_u^R through introducing parameters and T_u^L, T_u^R vary in J_u^L, J_u^R, respectively.

$$
\begin{aligned}
J_u^L = \;& [800 + u(1000-800) - \theta_l^1 \min\{u, 1-u\},\\
& 800 + u(1000-800) + \theta_r^1 \min\{u, 1-u\}],
\end{aligned}
$$

$$
\begin{aligned}
J_u^R = \;& [1300 - u(1300-1000) - \theta_l^2 \min\{u, 1-u\},\\
& 1300 - u(1300-1000) + \theta_r^2 \min\{u, 1-u\}]
\end{aligned}
$$

According to Zadeh extension principle, for any given $\lambda \in [0,1]$, the possibility distribution fluctuates around the nominal distribution and is determined by the nested sets $\{[x_u^L, x_u^R] : x_u^L \in J_u^L, x_u^R \in J_u^R, u \in [0,1]\}$, where x_u^L, x_u^R are represented as

$$
\begin{aligned}
x_u^L = \;& \lambda(800 + u(1000-800) - \theta_l^1 \min\{u, 1-u\})\\
& +(1-\lambda)(800 + u(1000-800) + \theta_r^1 \min\{u, 1-u\}),
\end{aligned}
$$

$$
\begin{aligned}
x_u^R = \;& \lambda(1300 - u(1300-1000) - \theta_l^2 \min\{u, 1-u\})\\
& +(1-\lambda)(1300 - u(1300-1000) + \theta_r^2 \min\{u, 1-u\}).
\end{aligned}
$$

Using the above construction method, we can obtain the uncertainty distribution set.

Based on the definition of the uncertainty distribution set, the expected profits of the manufacturer and the retailer can be calculated as follows

$$
\begin{aligned}
\mathrm{E}[\pi_m(p,\omega,\beta;\xi^\lambda)] = \;& -\tfrac{\lambda}{8}[\gamma\omega + (1-\gamma)p - c](\theta_r^1 + \theta_l^1 + \theta_r^2\\
& +\theta_l^2) + [\gamma\omega + (1-\gamma)p - c](\tfrac{\theta_r^1+\theta_r^2}{8}\\
& +\tfrac{r_1+2r_2+r_3}{4}) - \eta p(\omega + p) + (\delta_r\omega +\\
& \delta_d p)\beta - c((\delta_r + \delta_d)\beta - 2\eta p) - \nu\beta^2.
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\mathrm{E}[\pi_r(p;\xi^\lambda)] = \;& -\tfrac{\lambda}{8}\gamma(p-\omega)(\theta_r^1 + \theta_l^1 + \theta_r^2 + \theta_l^2)\\
& +\gamma(p-\omega)(\tfrac{\theta_r^1+\theta_r^2}{8} + \tfrac{r_1+2r_2+r_3}{4})\\
& +(p-\omega)(\delta_r\beta - \eta p).
\end{aligned} \tag{6}
$$

For simplicity, we introduce the following notations

$$
\begin{aligned}
g_1(p,\omega,\beta) \;&=\; (-\tfrac{\theta_l^1+\theta_l^2}{8} + \tfrac{r_1+2r_2+r_3}{4})[p(1-\gamma) + \omega\gamma - c]\\
& \quad -\eta p(\omega + p) + (\delta_r\omega + \delta_d p)\beta\\
& \quad -c((\delta_r + \delta_d)\beta - 2\eta p) - \nu\beta^2,\\
g_2(p) \;&=\; \gamma(p-\omega)(-\tfrac{\theta_l^1+\theta_l^2}{8} + \tfrac{r_1+2r_2+r_3}{4}) + (p-\omega)\\
& \quad (\delta_r\beta - \eta p).
\end{aligned}
$$

Using the notations above, the robust values of (5) and (6) can be rewritten as

$$
\inf_{\mu_{\xi^\lambda} \in \mathcal{U}} \mathrm{E}[\pi_m(p,\omega,\beta;\xi^\lambda)] \;= g_1(p,\omega,\beta), \tag{7}
$$

$$
\inf_{\mu_{\xi^\lambda} \in \mathcal{U}} \mathrm{E}[\pi_r(p;\xi^\lambda)] \;= g_2(p). \tag{8}
$$

The robust counterpart model (3) can be equivalently represented as follows

$$
\begin{aligned}
\max_{\omega,\beta} \quad & g_1(p,\omega,\beta)\\
\text{s.t.} \quad & p \in \arg\max_{p} g_2(p).
\end{aligned} \tag{9}
$$

Let us denote by ω^R, β^R and p^R the equilibrium solutions to model (9). By using backward induction, Proposition 1 gives the equilibrium decisions.

Proposition 1: If $\nu > \frac{\delta_d^2+6\delta_r\delta_d-3\delta_r^2}{12\eta}$, we have the following robust equilibrium decisions:

(a) The manufacturer's robust equilibrium decisions are

$$\begin{aligned}\omega^R &= \omega^b + \frac{1}{\eta\varphi}(m-a)(\gamma\psi+\delta_r\delta_d-\delta_d^2-\psi),\\ \beta^R &= \beta^b + \frac{1}{\varphi}(m-a)[2\gamma(\delta_d-3\delta_r)+(\delta_d+3\delta_r)].\end{aligned}$$

(b) The retailer's robust equilibrium retail price is

$$p^R = p^b + \frac{2}{\varphi\eta}(m-a)[\gamma(2\eta\nu-\delta_r\delta_d+\delta_r^2)+\eta\nu+\delta_r^2],$$

where $\varphi = 12\eta\nu-\delta_d^2-6\delta_r\delta_d+3\delta_r^2, \psi = 4\eta\nu-\delta_d^2+\delta_r^2, m = -\frac{\theta_l^1+\theta_l^2}{8}+\frac{r_1+2r_2+r_3}{4}$.

Based on Proposition 1, we study sensitivity analysis about the degree of customer loyalty γ and the green degree β in the distributionally robust manufacturer-led dual-channel green supply chain. Two Propositions are provided below.

Proposition 2: The following results show how the robust equilibrium decisions vary with respect to γ: $\frac{\partial p^R}{\partial\gamma}>0, \frac{\partial\omega^R}{\partial\gamma}>0$ and $\frac{\partial\beta^R}{\partial\gamma}<0$.

Proposition 2 shows that, with the increase of γ, the greening degree β^R decreases, the wholesale price ω^R and the sales price p^R increase. This is because, as the value of γ decreases, the basic market demand in the direct channel increases, which leads to the increase of the market demand in the direct channel. The manufacturer's profit gained from dual-channel green supply chain increases. This will make the manufacturer invest more in green innovation. On the other hand, for higher value of γ, the market demand in the retail channel increases. Therefore, the manufacturer can set higher wholesale price without affecting the greening degree of the product, which forces the retailer to raise the retail price.

Proposition 3: From Proposition 1, we can obtain $\frac{\partial p^R}{\partial\beta}>0$ and $\frac{\partial\omega^R}{\partial\beta}>0$.

Proposition 3 implies that ω^R increases with the increase of β, as a result, p^R also increases. This is because when the greening level β increases, the production cost also increases. So, the manufacturer sets higher wholesale price and the retailer charges higher retail price in the retail channel and higher selling price in the direct channel. Although the retail price p increases, the customers want to buy more products because the products are more environmental friendly.

V. NUMERICAL ANALYSIS

In this section, we perform a numerical study to validate the behavior of our distributionally robust manufacturer-led dual-channel green supply chain pricing model. It is noted that all experiments are conducted with the values of parameters: $\eta = 0.85$, $\delta_d = 0.75$, $\delta_r = 0.87$, $\nu = 50$ and $c = 100$, which is similar to the existing literature ([4], [49]). We employ Maple to carry out the computations. The numerical experiments are executed on a personal computer (Lenovo with Intel(R) Core(TM) 3.00 GHz CPU and RAM 8.00 GB) by using the Microsoft Windows 10 operating system.

A. Comparison results with fuzzy optimization method

In this subsection, the proposed robust optimization method is compared with fuzzy optimization method. In fuzzy optimization method, the distribution of the uncertain market size is assumed to be a fixed possibility distribution ([8]). For convenience, the fixed possibility distribution is taken as the nominal possibility distribution of the uncertain market size corresponding to $\theta_l^1=\theta_r^1=\theta_l^2=\theta_r^2=0$. The equilibrium decisions for the robust method and fuzzy method are given in Table II. From Table II, we find that the optimal decisions in fuzzy method are no longer optimal in our robust method, that is, a small perturbation of the nominal possibility distribution can affect the optimality of the equilibrium solutions.

B. The impacts of the cost coefficient of green level per unit

In this subsection, we will explore how the cost coefficient ν of green degree per unit influences the dual-channel green supply chain under uncertain market demand. We set $\gamma = 0.4, a = 400, \theta_r^1 = \theta_r^2 = 10$, and the values of ν vary from 10 to 200. Figure 2 shows the effect of ν on the manufacturer's profit in the dual-channel green supply chain under different values of the perturbation parameters. Figure 3 shows the effects of ν on the sales price and the wholesale price in the dual-channel green supply chain. Actually when $\Delta_l = \Delta_r = 0$, model (2) becomes the benchmark model (1). The manufacturer's profit in this case is denoted as π_m^b. When $\Delta_l \neq 0, \Delta_r \neq 0$, model (2) becomes model $\mathbf{M}$ in the case that the perturbation parameters $\theta_l^1=\theta_l^2=\theta_r^1=\theta_r^2=0$, that is the uncertain market size ξ is characterized by a triangular fuzzy variable $\text{Tri}[r_1, r_2, r_3]$. The manufacturer's profit in this case is denoted as π_m^F. From Figures 2 and 3, we have the following observations.

- π_m, p and ω decrease with the increase of ν. This finding implies that higher ν not only discourages the manufacturer from producing more environmental friendly products but also increases the price of the products with the same greening level. This finding shows that the government should provide green subsidies to the companies which are devoted to the green products in order to protect the environment.
- For given θ_r, π_m decreases with the increase of θ_l. This is because the larger the perturbation parameter θ_l, the larger the uncertain degree of the basic demand and the larger the price of robustness, which results in lower π_m.
- When $\Delta_r > \Delta_l$, $\pi_m^b < \pi_m^R < \pi_m^F$. In this case, the expected demand is larger than the deterministic demand. So π_m^b is the smallest. π_m^R is less than π_m^F due to the price of robustness.

When $\Delta_r < \Delta_l$, $\pi_m^b > \pi_m^F > \pi_m^R$. In this case, the expected demand is less than the deterministic demand. So π_m^b is the biggest. π_m^R is also less than π_m^F due to the price of robustness.

When $\Delta_r = \Delta_l$, $\pi_m^b = \pi_m^F > \pi_m^R$. In this case, the expected demand is equal to the deterministic demand. So π_m^b is equal to π_m^F. π_m^R is also less than π_m^F due to the price of robustness.

$$\begin{aligned}\mathbf{M}: \quad &\max_{\omega,\beta} \quad \mathrm{E}[\pi_m(p,\omega,\beta;\xi)],\\ &\text{s.t.} \quad p \in \arg\max_p \mathrm{E}[\pi_r(p;\xi)].\end{aligned}$$

TABLE II: Comparison results with fuzzy method

Method	Equilibrium decisions
Fuzzy	$p^F = \frac{1}{2\varphi\eta}[(r_1 + 2r_2 + r_3)(\gamma(2\eta\nu - \delta_r\delta_d + \delta_r^2) + \eta\nu + \delta_r^2) + 2\eta c(\psi - 4\delta_r\delta_d)]$
	$\omega^F = \frac{1}{4\eta\varphi}[(r_1 + 2r_2 + r_3)(\delta_r\delta_d - \delta_d^2 - (1-\gamma)\psi) + 8\eta c(2\delta_r\delta_d - \psi)]$
	$\beta^F = \frac{1}{4\varphi}[(r_1 + 2r_2 + r_3)(2\gamma\delta_d - 6\gamma\delta_r + \delta_d + 3\delta_r) - 16\eta c\delta_d]$
Robust	$p^R = \frac{1}{4\varphi\eta}[(2(r_1 + 2r_2 + r_3) - (\theta_l^1 + \theta_l^2))(\gamma(2\eta\nu - \delta_r\delta_d + \delta_r^2) + \eta\nu + \delta_r^2) + 8\eta c(\psi - 4\delta_r\delta_d)]$
	$\omega^R = \frac{1}{8\eta\varphi}[(2(r_1 + 2r_2 + r_3) - (\theta_l^1 + \theta_l^2))(\delta_r\delta_d - \delta_d^2 - (1-\gamma)\psi) + 16\eta c(2\delta_r\delta_d - \psi)]$
	$\beta^R = \frac{1}{8\varphi}[(2(r_1 + 2r_2 + r_3) - (\theta_l^1 + \theta_l^2))(2\gamma\delta_d - 6\gamma\delta_r + \delta_d + 3\delta_r) - 32\eta c\delta_d]$

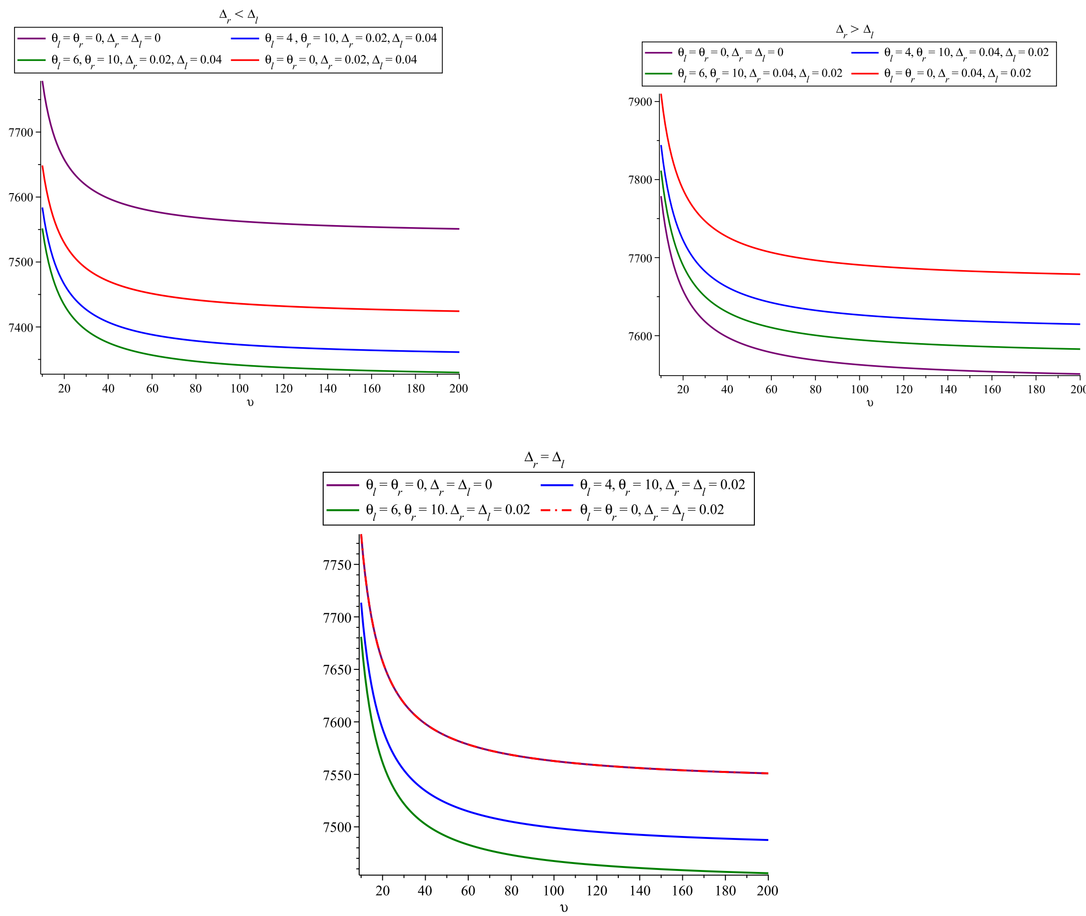

Fig. 2: Comparisons of manufacturer's profits under different ν and θ_l.

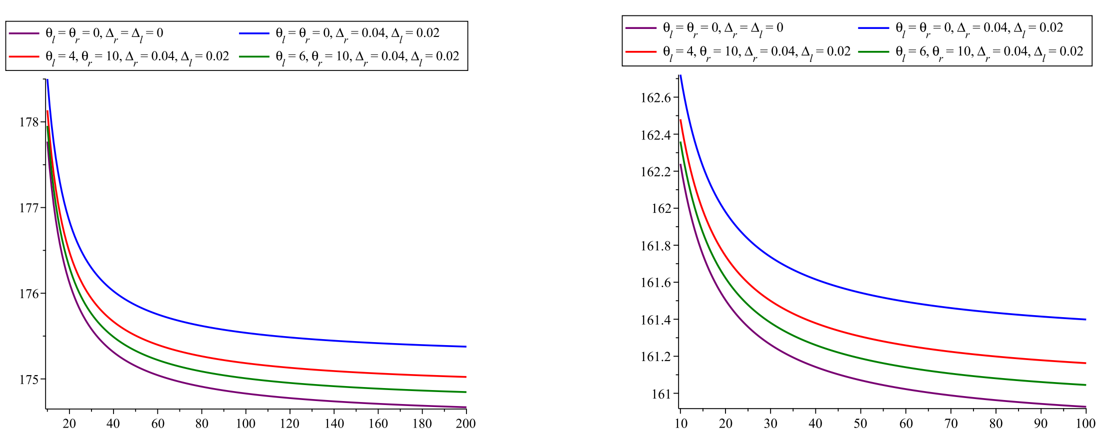

Fig. 3: Effects of ν and θ_l on sales price p and wholesale price ω.

TABLE III: Comparison between robust single-channel and dual-channel supply chain

Equilibrium outcomes	Single-channel	Dual-channel
Green level	$\frac{\delta_r(m-c\eta)}{8\eta\nu-\delta_r^2}$	β^R
Wholesale price	$\frac{4m\nu-c\delta_r^2+4c\eta\nu}{8\eta\nu-\delta_r^2}$	ω^R
Retail price	$\frac{6m\nu-c\delta_r^2+2c\eta\nu}{8\eta\nu-\delta_r^2}$	p^R
Manufacturer's profit	$\frac{\nu(m-c\eta)^2}{8\eta\nu-\delta_r^2}$	π_m^R
Retailer's profit	$\frac{4\eta\nu^2(m-c\eta)^2}{(8\eta\nu-\delta_r^2)^2}$	π_r^R

C. The impacts of the perturbation parameters on manufacturer's channel choice

In this subsection, we will discuss the effects of the perturbation parameters in our distributionally robust pricing model. We want to identify when the manufacturer should open his own direct channel through comparing the manufacturer's and retailer's profits between the single-channel and the dual-channel green supply chains under different values of the perturbation parameters. The robust equilibrium outcomes of the single-channel and the dual-channel green supply chains are summarized in Table III. The robust equilibrium results of the single-channel green supply chain are provided in Appendix B. We assume that π_m^s (π_r^s) and π_m^R (π_r^R) represent the manufacturer's (retailer's) profits in single-channel and dual-channel green supply chain, respectively.

Figure 4 shows the comparisons of the manufacturer's profits between single and dual channel green supply chains under different θ_l^1. From Figure 4, we find that when θ_l^1 is relatively small, the manufacturer's profit in single-channel green supply chain is greater than that of dual-channel green supply chain, π_m^s decreases with θ_l^1 increasing and is less than π_m^R when $\theta_l^1 > \theta_0$. However, the retailer's profit π_r^R in dual-channel green supply chain is always greater than π_r^s in single retail channel green supply chain as θ_l^1 increases. From Figures 4 and 5, we can find that when the market size is uncertain, opening a direct channel is not always profitable for the manufacturer, but it is always beneficial to the retailer. Our result is different from that of [5], who demonstrated that opening a direct channel is always harmful to the retailer.

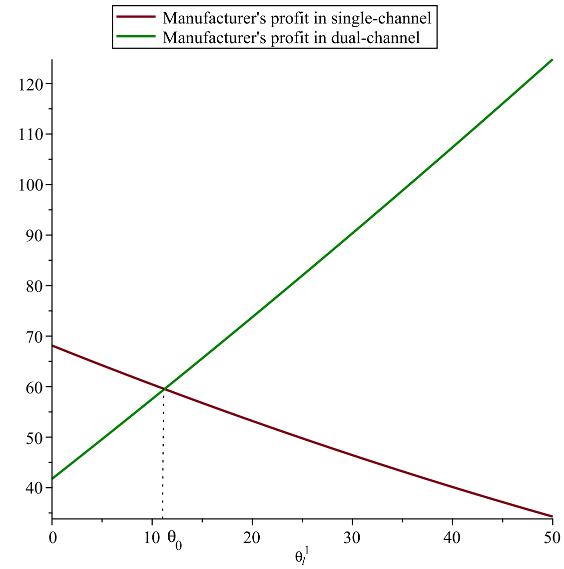

Fig. 4: Comparisons of manufacturer's profits between single and dual channel green supply chains in consistent pricing strategy.

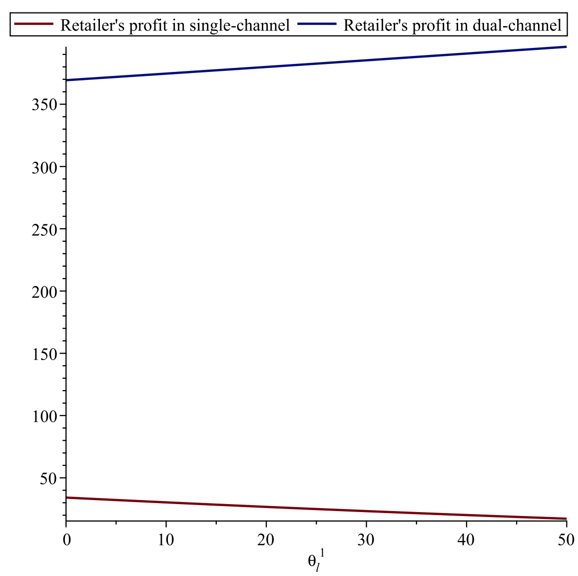

Fig. 5: Comparisons of retailer's profits between single and dual channel green supply chains in consistent pricing strategy.

D. Managerial insights

Based on above numerical analysis, we can get the following managerial insights.

(1) If a decision maker takes the parameters $\Delta_l = \Delta_r = 0$, that is, the market size is deterministic, our distributionally robust pricing model becomes the benchmark model, which is consistent with the model in [5]. As shown in our application example, the robust equilibrium solutions are sensitive to the distribution of the uncertain market size ξ. Therefore, the decision maker cannot ignore the uncertain factors in modeling the pricing problem in dual-channel green supply chain.

(2) By comparing the manufacturer's and the retailer's profits between single-channel and dual-channel green supply chains under different θ_l^1, we find that there exists a threshold θ_0 for the parameter θ_l^1 in the uncertainty set, when the value of the parameter θ_l^1 is less than θ_0, opening a direct channel is not beneficial to the manufacturer and the manufacturer is willing to own a direct channel when the value of the parameter θ_l^1 is greater than θ_0.

(3) In our distributionally robust pricing model, the market size is considered uncertain and characterized by a PLI type-2 fuzzy variable and its associated uncertainty distribution set. The decision maker need to determine two kinds of parameters: the parameters Δ_l, Δ_r and the perturbation parameters θ_l, θ_r. They describe the degree of uncertainty that a type-2 fuzzy variable takes its value. Decision makers can know how the parameters affect their decisions from the sensitivity analysis. Figures 2 and 3 report the impacts of the perturbation parameter θ_l on the distributionally robust optimal solutions

and optimal value when parameters Δ_l, Δ_r take different values.

VI. Extension

So far, all the results and discussions in the above section come from the condition of a consistent pricing strategy. To ensure the robustness of our results, we have conducted additional analysis by making extension to discuss the case in which the prices of the green products are not equal in the retail channel and the direct channel and showing that the key analytical results continue to hold.

The demand functions in the retail channel and the direct channel are assumed as $\overline{D}_r = \gamma a - \beta_1 p_r + \beta_2 p_d + \delta_r \beta, \overline{D}_d = (1-\gamma)a - \beta_1 p_d + \beta_2 p_r + \delta_d \beta$, where β_1 is the price elasticity which represents the manufacturer's (retailer's) demand sensitivity to his(her) own direct selling (retail) price. β_2 is the coefficient of cross-price sensitivity which represents the substitution effect between the two channels whereby an increase in one channel's price lowers its own demand while increasing its competitor's demand. $\beta_1 > \beta_2$ indicates that the impact of ownership price is greater than the impact of cross-price.

The manufacturer's profit function and the retailer's profit function are:

$$\pi_m(p_d, p_r, \omega, \beta) = p_d \overline{D}_d + \omega \overline{D}_r - c(\overline{D}_r + \overline{D}_d) - \nu\beta^2 \tag{10}$$

$$\pi_r(p_r) = (p_r - \omega)\overline{D}_r \tag{11}$$

In this case, we also use the Stackelberg game led by the manufacturer to process this model. First, the manufacturer determines the wholesale price, the direct-selling price and the green level. Then, as follower, the retailer makes decision about the retail price after observing the manufacturer's decisions.

We consider the distribution μ_ξ of the uncertain market size ξ is partially available and varies in the uncertainty distribution set $\mathcal{U}$ defined in (4). We also assume that both parties in the Stackelberg game are risk-neutral and plan for the worst case.

On the basis of uncertainty distribution set, the uncertain pricing model is represented as follows

$$\left\{ \begin{array}{ll} \max\limits_{\omega, p_d, \beta} & \mathrm{E}[\pi_m(p_d, p_r, \omega, \beta; \xi)], \\ \text{s.t.} & p_r \in \arg\max\limits_{p_r} \mathrm{E}[\pi_r(p_r; \xi)], \end{array} \right\}_{\mu_\xi \in \mathcal{U}.} \tag{12}$$

Based on the assumptions (A1), (A2) and worst-case criterion, the robust counterpart of model (12) is formally written as

$$\begin{array}{ll} \max\limits_{\omega, p_d, \beta} & \inf\limits_{\mu_\xi \in \mathcal{U}} \mathrm{E}[\pi_m(p_d, p_r, \omega, \beta; \xi)] \\ \text{s.t.} & p_r \in \arg\max\limits_{p_r} \inf\limits_{\mu_\xi \in \mathcal{U}} \mathrm{E}[\pi_r(p_r; \xi)]. \end{array} \tag{13}$$

According to the definition of the uncertainty distribution set defined in (4), the expected profits of the manufacturer and the retailer can be calculated as follows

$$\begin{aligned} &\mathrm{E}[\pi_m(p_d, p_r, \omega, \beta; \xi^\lambda)] \\ &= -\tfrac{\lambda(\theta_r^1 + \theta_l^1 + \theta_r^2 + \theta_l^2)}{8}[p_d(1-\gamma) + \omega\gamma - c] \\ &+ [p_d(1-\gamma) + \omega\gamma - c](\tfrac{\theta_r^1 + \theta_r^2}{8} + \tfrac{r_1 + 2r_2 + r_3}{4}) \\ &+ p_d[-\beta_1 p_d + \beta_2 p_r + \delta_d \beta] + \omega[-\beta_1 p_r + \beta_2 p_d + \delta_r \beta] \\ &- c[p_d(\beta_2 - \beta_1) + p_r(\beta_2 - \beta_1) + \beta(\delta_r + \delta_d)] - \nu\beta^2 \end{aligned} \tag{14}$$

$$\begin{aligned} \mathrm{E}[\pi_r(p_r; \xi^\lambda)] = &-\tfrac{\lambda}{8}\gamma(p_r - \omega)(\theta_r^1 + \theta_l^1 + \theta_r^2 + \theta_l^2) \\ &+ \gamma(p_r - \omega)(\tfrac{\theta_r^1 + \theta_r^2}{8} + \tfrac{r_1 + 2r_2 + r_3}{4}) \\ &+ (p_r - \omega)[-\beta_1 p_r + \beta_2 p_d + \delta_r \beta] \end{aligned} \tag{15}$$

and

$$\begin{aligned} &\inf_{\mu_{\xi^\lambda} \in \mathcal{U}} \mathrm{E}[\pi_m(p_d, p_r, \omega, \beta; \xi^\lambda)] \\ &= [p_d(1-\gamma) + \omega\gamma - c](-\tfrac{\theta_l^1 + \theta_l^2}{8} + \tfrac{r_1 + 2r_2 + r_3}{4}) \\ &+ p_d[-\beta_1 p_d + \beta_2 p_r + \delta_d \beta] + \omega[-\beta_1 p_r + \beta_2 p_d + \delta_r \beta] \\ &- c[p_d(\beta_2 - \beta_1) + p_r(\beta_2 - \beta_1) + \beta(\delta_r + \delta_d)] - \nu\beta^2 \end{aligned} \tag{16}$$

$$\begin{aligned} &\inf_{\mu_{\xi^\lambda} \in \mathcal{U}} \mathrm{E}[\pi_r(p_r; \xi^\lambda)] \\ &= \gamma(p_r - \omega)(-\tfrac{\theta_l^1 + \theta_l^2}{8} + \tfrac{r_1 + 2r_2 + r_3}{4}) + (p_r - \omega)[-\beta_1 p_r \\ &+ \beta_2 p_d + \delta_r \beta]. \end{aligned} \tag{17}$$

For simplicity, we introduce the following notations

$$\begin{aligned} &f_1(p_d, p_r, \omega, \beta) \\ &= [p_d(1-\gamma) + \omega\gamma - c](-\tfrac{\theta_l^1 + \theta_l^2}{8} + \tfrac{r_1 + 2r_2 + r_3}{4}) \\ &+ p_d[-\beta_1 p_d + \beta_2 p_r + \delta_d \beta] + \omega[-\beta_1 p_r + \beta_2 p_d + \delta_r \beta] \\ &- c[p_d(\beta_2 - \beta_1) + p_r(\beta_2 - \beta_1) + \beta(\delta_r + \delta_d)] - \nu\beta^2 \end{aligned}$$

$$\begin{aligned} f_2(p_r) = &\ \gamma(p_r - \omega)(-\tfrac{\theta_l^1 + \theta_l^2}{8} + \tfrac{r_1 + 2r_2 + r_3}{4}) \\ &+ (p_r - \omega)[-\beta_1 p_r + \beta_2 p_d + \delta_r \beta]. \end{aligned}$$

The robust counterpart model (13) can be equivalently represented as follows

$$\begin{array}{ll} \max\limits_{p_d, \omega, \beta} & f_1(p_d, p_r, \omega, \beta) \\ \text{s.t.} & p_r \in \arg\max\limits_{p_r} f_2(p_r). \end{array} \tag{18}$$

Using backward induction, Proposition 4 gives the equilibrium outcomes.

Proposition 4: If $\nu > \frac{(\beta_2^2 + \beta_1^2)\delta_r^2 + 2\beta_1^2\delta_d^2 + 4\beta_1\beta_2\delta_r\delta_d}{8\beta_1(\beta_1^2 - \beta_2^2)}$, we have the following robust equilibrium decisions:

(a) The manufacturer's robust equilibrium decisions are

$$\begin{aligned} p_d^R \ = \ &\tfrac{1}{2\phi}\{m\beta_1[(\delta_r^2 - 8\nu\beta_1)(1-\gamma) - (8\nu\beta_2 + \delta_r\delta_d)\gamma] \\ &+ c[(2\beta_2^2 + \beta_1\beta_2 + \beta_1^2)\delta_r^2 + (\beta_1^2 + 7\beta_1\beta_2)\delta_r\delta_d \\ &+ 8\beta_1\beta_2^2\nu - 8\beta_1^3\nu + 4\beta_1^2\delta_d^2]\}, \\ \omega^R \ = \ &\tfrac{1}{2\phi}\{m[(2\beta_1\delta_d^2 + \beta_2\delta_r\delta_d - 8\beta_1^2\nu)\gamma - (\beta_2\delta_r^2 + 8\beta_1 \\ &\beta_2\nu + 2\beta_1\delta_r\delta_d)(1-\gamma)] + c[(\beta_2^2 + \beta_1\beta_2 + 2\beta_1^2)\delta_r^2 \\ &+ (\beta_2^2 + 5\beta_1\beta_2 + 2\beta_1^2)\delta_r\delta_d + (2\beta_1\beta_2 + 2\beta_1^2)\delta_d^2 \\ &+ 8\nu\beta_1\beta_2^2 - 8\nu\beta_1^3]\}, \\ \beta^R \ = \ &\tfrac{1}{\phi}\{c(\beta_1^2 - \beta_2^2)(\beta_2\delta_r + \beta_1\delta_r + 2\beta_1\delta_d) \\ &- m[(\beta_2^2\delta_r + \beta_1^2\delta_r + 2\beta_1\beta_2\delta_d)\gamma \\ &+ 2\beta_1(\beta_1\delta_d + \beta_2\delta_r)(1-\gamma)]\}. \end{aligned}$$

(b) The retailer's robust equilibrium retail price is

$$\begin{aligned} p_r^R \ = \ &\tfrac{1}{2\phi}\{m[\gamma(2\beta_2\delta_r\delta_d + 3\beta_1\delta_d^2 + 4\nu\beta_2^2 - 12\nu\beta_1^2) \\ &- (1-\gamma)(2\beta_2\delta_r^2 + 3\beta_1\delta_r\delta_d + 8\beta_1\beta_2\nu)] \\ &+ c[(2\beta_1\beta_2 + 2\beta_1^2)\delta_r^2 + (2\beta_2^2 + 3\beta_1^2 + 3\beta_1\beta_2)\delta_r\delta_d \\ &+ (3\beta_1\beta_2 + \beta_1^2)\delta_d^2 + 4\nu(\beta_2^3 - \beta_1^2\beta_2 + \beta_1\beta_2^2 - \beta_1^3)]\}, \end{aligned}$$

where $\phi = (\beta_1^2 - \beta_2^2)(\delta_r^2 - 8\beta_1\nu) + 2(\beta_1\delta_d + \beta_2\delta_r)^2, m = -\frac{\theta_l^1 + \theta_l^2}{8} + \frac{r_1 + 2r_2 + r_3}{4}$.

Proposition 5: The following results show how the robust equilibrium decisions vary with respect to γ, $\frac{\partial p_d^R}{\partial \gamma} < 0$, $\frac{\partial p_r^R}{\partial \gamma} > 0$, $\frac{\partial \omega^R}{\partial \gamma} > 0$ and $\frac{\partial \beta^R}{\partial \gamma} < 0$.

Proposition 5 shows that, as the value of γ increases, the selling price p_d^R in the direct channel and the greening level β^R decrease, and the wholesale price ω^R and the selling price p_r^R in the retail channel increase. This is because, with the value of γ decreasing, the market size of the direct channel increases, which leads to the market demand in the direct channel also increases. The manufacturer gains more profits from dual channel green supply chain, which makes the manufacturer to invest more in green innovation. The price of the product is proportional to the greening degree. Therefore, the manufacturer must set higher sale price in the direct channel. On the other hand, the higher value of γ, the greater the market demand of the retail channel. Therefore, the manufacturer set higher wholesale price without affecting the greening degree of the product, which enforce the retailer to set higher retail price. However, the market demand of the direct channel decreases with the value of γ increases. To attract customers and maintain profit, the manufacturer must reduce the selling price of the direct channel.

Proposition 6: From Proposition 4, we can obtain $\frac{\partial p_d^R}{\partial \beta} > 0$, $\frac{\partial p_r^R}{\partial \beta} > 0$ and $\frac{\partial \omega^R}{\partial \beta} > 0$.

Proposition 6 implies that ω^R increases as β increases and then p_r^R also increases. This is because the production cost also increases when the greening degree β increases. Therefore, the manufacturer sets higher wholesale price and the retailer charges higher retail price and a higher sale price in the direct channel. Although the retail price p_r^R increases, the customers want to purchase more products because the products are more environmental friendly.

Compared with the results under the consistent pricing s-trategy, we find that the impacts of the perturbation parameters on the channel choice of the manufacturer are the same.

Figure 6 shows the impacts of the perturbation parameter θ_l^1 on the manufacturer's profits in both single channel and dual-channel green supply chain. From Figure 6, we find that when θ_l^1 is relatively small, the manufacturer's profit in single-channel green supply chain is greater than that in dual-channel green supply chain, π_m^s decreases with θ_l^1 increasing and is less than π_m^R when $\theta_l^1 > \kappa$, which is the same with the results under the consistent pricing strategy.

Figure 7 shows the impacts of the perturbation parameter θ_l^1 on the retailer's profits in both single channel and dual-channel green supply chain. We find that the retailer's profit in dual-channel green supply chain is always greater than that in single retail channel green supply chain as θ_l^1 increases, which is also the same with the results under the consistent pricing strategy.

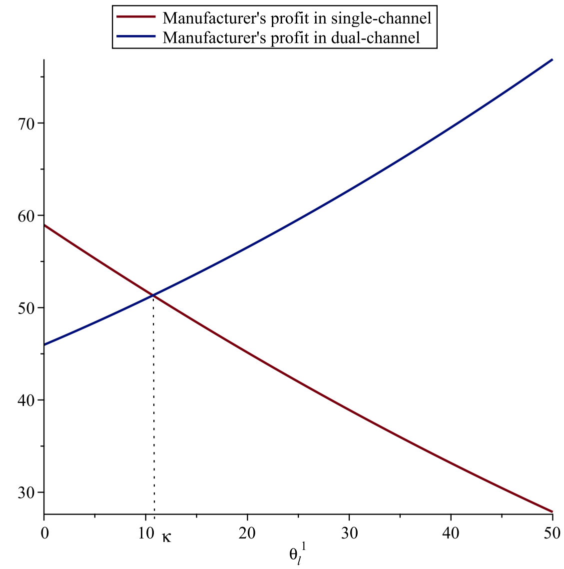

Fig. 6: Comparisons of manufacturer's profits between single and dual channel green supply chains in nonconsistent pricing strategy.

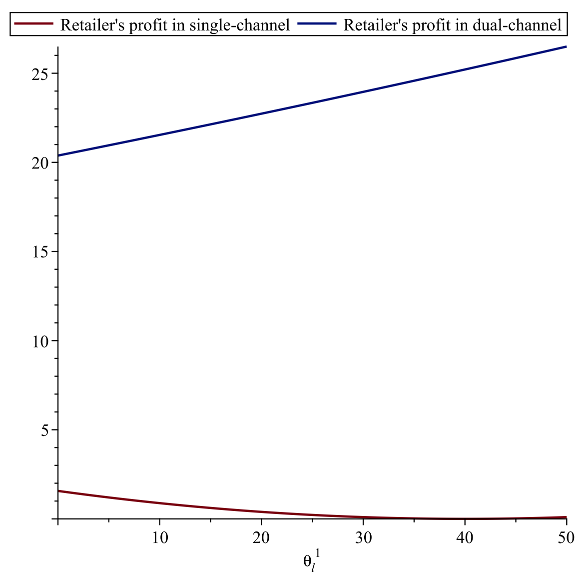

Fig. 7: Comparisons of retailer's profits between single and dual channel green supply chains in nonconsistent pricing strategy.

VII. Conclusions

In this paper, a pricing decision problem in a dual-channel green supply chain was considered with uncertain market size. The major findings of the work can be summarized below.

(1) This paper discussed the pricing decision problem for a dual-channel green supply chain in which the uncertain demand is characterized by a PLI type-2 fuzzy variable, whose possibility distribution is variable. In order to characterize the perturbations of the distribution of the uncertain market demand, a new uncertainty distribution set is introduced based on type-2 fuzzy theory.

(2) A novel distributionally robust model was proposed for the pricing decision problem of a two-echelon dual-channel green supply chain based on the manufacturer-led Stackelberg game framework. To the best of our knowledge, it is the first to combine robust optimization with game theory in the pricing decision problem of the dual-channel green supply chain.

(3) A new analytically tractable method was given to derive the robust equilibrium decisions of the robust pricing game model. Several numerical analysis were conducted to validate the behavior of our distributionally robust manufacturer-led

pricing model. The numerical results show that when the market size is uncertain, opening a direct channel is not always profitable for the manufacturer, but it is always beneficial to the retailer. We also demonstrate our results are robust in the case that the prices of the green products are not equal in the retail channel and the direct channel.

Several possible directions for future research follow this study. In this paper, we assume all information is known to all members in the supply chain. One could discuss the game results under asymmetric information. We assume that the demand is dependent on both prices and green level. In reality, the demand function is complex. One can consider demand of other types of demand functions. In addition, We assume that there is one player in each level of the supply chain. One can extend it to the case with competing manufacturers or competing retailers.

APPENDIX A

Proof of lemma 1

Proof: The manufacturer's profit: $\pi_m(p,\omega,\beta) = (p-c)((1-\gamma)a-\eta p+\delta_d\beta)+(\omega-c)(\gamma a-\eta p+\delta_r\beta)-\nu\beta^2$

The retailer's profit: $\pi_r(p) = (p-\omega)(\gamma a-\eta p+\delta_r\beta)$

From the expression of $\pi_r(p)$, we have the second order sufficient condition $\frac{\partial^2\pi_r}{\partial p^2} = -2\eta < 0$,which ensures that unique optimal solution exists. For given ω,β, the retailer's response function is derived from the first-order condition of the retailer's profit $\pi_r(p)$.

$$\frac{\partial \pi_r}{\partial p} = 0 \Rightarrow p = \frac{\delta_r\beta+\eta\omega+\gamma a}{2\eta}$$

After getting the reaction of the retailer, the manufacturer maximizes his profit and determines the optimal decisions ω,β. The Hessian matrix associated with the profit function $\pi_m(p,\omega,\beta)$ is given by

$$H = \begin{pmatrix} -\frac{3\eta}{2} & \frac{\delta_d}{2} \\ \frac{\delta_d}{2} & \frac{2\delta_r\delta_d-\delta_r^2}{2\eta}-2\nu \end{pmatrix}$$

Now, $|H| = 3\nu\eta+\frac{3\delta_r^2}{4}-\frac{\delta_d^2}{4}-\frac{3\delta_r\delta_d}{2} > 0$ if $\nu > \frac{\delta_d^2+6\delta_r\delta_d-3\delta_r^2}{12\eta}$, Therefore, H is negative definite if and only if $\nu > \frac{\delta_d^2+6\delta_r\delta_d-3\delta_r^2}{12\eta}$. Thus manufacturer's profit function $\pi_m(p,\omega,\beta)$ is jointly concave in ω and β. Equating the first order conditions to 0, that is,

$$\begin{cases} \frac{\partial \pi_m}{\partial\omega} = 0 \\ \frac{\partial \pi_m}{\partial\beta} = 0 \end{cases}$$

We get $\omega^b = \frac{1}{\eta\varphi}[a\gamma\psi+a(\delta_r\delta_d-\delta_d^2-\psi)+2\eta c(2\delta_r\delta_d-\psi)], \beta^b = \frac{1}{\varphi}[2a\gamma(\delta_d-3\delta_r)+a(\delta_d+3\delta_r)-4\eta c\delta_d]$.

Substituting the values of ω,β into the value of p we get, $p^b = \frac{1}{\varphi\eta}[2a(\gamma(2\eta\nu-\delta_r\delta_d+\delta_r^2)+\eta\nu+\delta_r^2)+\eta c(\psi-4\delta_r\delta_d)]$, where $\varphi = 12\eta\nu-\delta_d^2-6\delta_r\delta_d+3\delta_r^2, \psi = 4\eta\nu-\delta_d^2+\delta_r^2$. ■

Proof of Proposition 1

Proof:

$$\begin{aligned} \max_{\omega,\beta} \quad & g_1(p,\omega,\beta) \\ \text{s.t.} \quad & p \in \arg\max_p g_2(p). \end{aligned}$$

From the expression of $g_2(p)$, we have the second order sufficient condition $\frac{\partial^2 g_2}{\partial p^2} = -2\eta < 0$,which ensures that unique optimal solution exists. For given ω,β, the retailer's response function is derived from the first-order condition of the retailer's profit $g_2(p)$.

$$\frac{\partial g_2}{\partial p} = 0 \Rightarrow p = \frac{\delta_r\beta+\eta\omega+\gamma m}{2\eta}$$

After getting the reaction of the retailer, the manufacturer maximizes his profit and determines the optimal decisions ω,β. The Hessian matrix associated with the profit function $g_1(p,\omega,\beta)$ is given by

$$H = \begin{pmatrix} \frac{\partial^2 g_1}{\partial\omega^2} & \frac{\partial^2 g_1}{\partial\omega\partial\beta} \\ \frac{\partial^2 g_1}{\partial\beta\partial\omega} & \frac{\partial^2 g_1}{\partial\beta^2} \end{pmatrix} = \begin{pmatrix} -\frac{3\eta}{2} & \frac{\delta_d}{2} \\ \frac{\delta_d}{2} & \frac{2\delta_r\delta_d-\delta_r^2}{2\eta}-2\nu \end{pmatrix}$$

Now, $|H| = 3\nu\eta+\frac{3\delta_r^2}{4}-\frac{\delta_d^2}{4}-\frac{3\delta_r\delta_d}{2} > 0$ if $\nu > \frac{\delta_d^2+6\delta_r\delta_d-3\delta_r^2}{12\eta}$, Therefore, H is negative definite if and only if $\nu > \frac{\delta_d^2+6\delta_r\delta_d-3\delta_r^2}{12\eta}$. Thus manufacturer's profit function $g_1(p,\omega,\beta)$ is jointly concave in ω and β. Equating the first order conditions to 0, that is,

$$\begin{cases} \frac{\partial g_1}{\partial\omega} = 0 \\ \frac{\partial g_1}{\partial\beta} = 0 \end{cases}$$

We get $\omega^R = \omega^b+\frac{1}{\eta\varphi}(m-a)(\gamma\psi+\delta_r\delta_d-\delta_d^2-\psi), \beta^R = \beta^b+\frac{1}{\varphi}(m-a)[2\gamma(\delta_d-3\delta_r)+(\delta_d+3\delta_r)]$.

Substituting the values of ω,β into the value of p we get, $p^R = p^b+\frac{2}{\varphi\eta}(m-a)[\gamma(2\eta\nu-\delta_r\delta_d+\delta_r^2)+\eta\nu+\delta_r^2]$, where $\varphi = 12\eta\nu-\delta_d^2-6\delta_r\delta_d+3\delta_r^2, \psi = 4\eta\nu-\delta_d^2+\delta_r^2, m = \frac{-(\theta_l^1+\theta_l^2)}{8}+\frac{r_1+2r_2+r_3}{4}$. ■

Proof of Proposition 2

Proof: Taking the first-order derivatives of p^R,ω^R and β^R with respect to γ, we have

$$\frac{\partial p^R}{\partial\gamma} = \frac{2m}{\varphi\eta}(2\eta\nu-\delta_r\delta_d+\delta_r^2),$$

$$\frac{\partial \omega^R}{\partial\gamma} = \frac{\psi m}{\varphi\eta},$$

$$\frac{\partial \beta^R}{\partial\gamma} = \frac{2m}{\varphi}(\delta_d-3\delta_r).$$

We first prove $m > 0$. The construction of the uncertainty set ensure the following inequalities hold. $0 < \theta_l^1 < r_2-r_1 = a\Delta_l, 0 < \theta_r^1 < r_2-r_1 = a\Delta_l, 0 < \theta_l^2 < r_3-r_2 = a\Delta_r, 0 < \theta_r^2 < r_3-r_2 = a\Delta_r$. From the above four inequalities, we can obtain $0 < \frac{5a+a\Delta_r+3a(1-\Delta_l)}{8} < m = -\frac{\theta_l^1+\theta_l^2}{8}+\frac{r_1+2r_2+r_3}{4} < \frac{3a+a\Delta_r+a(1-\Delta_l)}{4}$

The condition $\nu > \frac{\delta_d^2+6\delta_r\delta_d-3\delta_r^2}{12\eta}$ ensures $\varphi > 0, \psi > 0, 2\eta\nu-\delta_r\delta_d+\delta_r^2 > 0$. Since $\delta_d < \delta_r$,$\delta_d-3\delta_r < 0$.

Based on the analysis above, Proposition 2 can be derived. ■

Proof of Proposition 3

Proof: Using the first order optimality condition of $g_2(p)$, we get

$p^R = \frac{\delta_r\beta+\eta\omega+\gamma m}{2\eta}$. Now $\frac{\partial p^R}{\partial\beta} = \frac{\delta_r}{2\eta} > 0$.

Using the first order optimality condition of $g_1(p,\omega,\beta)$, $\frac{\partial g_1}{\partial\beta}=0$, we have $\frac{\partial\omega^R}{\partial\beta}=\frac{1}{\eta\delta_d}(\delta_r^2-2\delta_r\delta_d+4\eta\beta\nu)>0$. ■

Proof of Proposition 4

Proof:

$$\begin{array}{ll}\max\limits_{p_d,\omega,\beta} & f_1(p_d,\omega,\beta)\\ \text{s.t.} & p_r\in\arg\max\limits_{p_r} f_2(p_r).\end{array}$$

From the expression of $f_2(p_r)$, we have the second order sufficient condition $\frac{\partial^2 f_2}{\partial p_r^2}=-2\beta_1<0$,which ensures that unique optimal solution exists. For given p_d,ω,β, the retailer's response function is derived from the first-order condition of the retailer's profit $f_2(p_r)$.

$$\frac{\partial f_2}{\partial p_r}=0\Rightarrow p_r=\frac{\delta_r\beta+\beta_1\omega+\gamma m+\beta_2 p_d}{2\beta_1}$$

After getting the reaction of the retailer, the manufacturer maximizes his profit and determines the optimal decisions p_d,ω,β. The Hessian matrix associated with the profit function $f_1(p_d,\omega,\beta)$ is given by

$$\begin{aligned} H &= \begin{pmatrix} \frac{\partial^2 f_1}{\partial\omega^2} & \frac{\partial^2 f_1}{\partial\omega\partial\beta} & \frac{\partial^2 f_1}{\partial\omega\partial p_d}\\ \frac{\partial^2 f_1}{\partial p_d\partial\omega} & \frac{\partial^2 f_1}{\partial p_d^2} & \frac{\partial^2 f_1}{\partial p_d\partial\beta}\\ \frac{\partial^2 f_1}{\partial\beta\partial\omega} & \frac{\partial^2 f_1}{\partial\beta\partial p_d} & \frac{\partial^2 f_1}{\partial\beta^2}\end{pmatrix}\\ &= \begin{pmatrix} -\beta_1 & \beta_2 & \frac{\delta_r}{2}\\ \beta_2 & -2\beta_1+\frac{\beta_2^2}{\beta_1} & \delta_d+\frac{\beta_2\delta_r}{2\beta_1}\\ \frac{\delta_r}{2} & \delta_d+\frac{\beta_2\delta_r}{2\beta_1} & -2\nu\end{pmatrix}\end{aligned}$$

Now, $|H_1|=-\beta_1<0$, $|H_2|=2\beta_1^2-\beta_2^2<0$,$|H|=4\nu(\beta_2^2-\beta_1^2)+\frac{\beta_1^2\delta_r^2+\beta_2^2\delta_r^2+2\beta_1^2\delta_d^2+4\beta_1\beta_2\delta_r\delta_d}{2\beta_1}<0$ if $\nu>\frac{(\beta_2^2+\beta_1^2)\delta_r^2+2\beta_1^2\delta_d^2+4\beta_1\beta_2\delta_r\delta_d}{8\beta_1(\beta_1^2-\beta_2^2)}$. Therefore, H is negative definite if and only if $\nu>\frac{(\beta_2^2+\beta_1^2)\delta_r^2+2\beta_1^2\delta_d^2+4\beta_1\beta_2\delta_r\delta_d}{8\beta_1(\beta_1^2-\beta_2^2)}$. Thus manufacturer's profit function $f_1(p_d,p_r\omega,\beta)$ is jointly concave in p_d,ω and β. Equating the first order conditions to 0, that is,

$$\begin{cases}\frac{\partial f_1}{\partial p_d}=0\\ \frac{\partial f_1}{\partial\omega}=0\\ \frac{\partial f_1}{\partial\beta}=0\end{cases}$$

We get the equilibrium solutions p_d^R,ω^R,β^R. Substituting the values of p_d,ω,β into the value of p_r we get the representation of p_r^R. ■

Proof of Proposition 5

Proof: Taking the first-order derivatives of p^R,ω^R and β^R with respect to γ, we have

$$\begin{aligned}\frac{\partial p_d^R}{\partial\gamma} &= \frac{m\beta_1}{2\phi}[8\nu(\beta_1-\beta_2)-\delta_r(\delta_r+\delta_d)],\\ \frac{\partial p_r^R}{\partial\gamma} &= \frac{m}{2\phi}[(3\beta_1+2\beta_2)\delta_r\delta_d+4\nu(\beta_2^2-3\beta_1^2+2\beta_1\beta_2)\\ &\quad +3\beta_1\delta_d^2+2\beta_2\delta_r^2],\\ \frac{\partial\omega^R}{\partial\gamma} &= \frac{m}{2\phi}[8\nu\beta_1(\beta_2-\beta_1)+(2\beta_1+\beta_2)\delta_r\delta_d+2\beta_1\delta_d^2+\beta_2\delta_r^2],\\ \frac{\partial\beta^R}{\partial\gamma} &= \frac{m}{\phi}[(\beta_1-\beta_2)^2\delta_r+2\beta_1\delta_d(\beta_2-\beta_1)]\end{aligned}$$

$m>0$ can be proved similar with that in Proposition 2. The condition $\nu>\frac{(\beta_2^2+\beta_1^2)\delta_r^2+2\beta_1^2\delta_d^2+4\beta_1\beta_2\delta_r\delta_d}{8\beta_1(\beta_1^2-\beta_2^2)}$ ensures $\phi<0$.

Based on the analysis above, Proposition 5 can be derived. ■

Proof of Proposition 6

Proof: Using the first order optimality condition of $f_2(p_r)$, we get $p_r^R=\frac{\delta_r\beta+\beta_1\omega+\gamma m+\beta_2 p_d}{2\beta_1}$. Now $\frac{\partial p_r^R}{\partial\beta}=\frac{\delta_r}{2\beta_1}>0$.

Using the first order optimality condition of $f_1(p_d,p_r,\omega,\beta)$, $\frac{\partial f_1}{\partial\beta}=0$, $\frac{\partial f_1}{\partial\omega}=0$, we have $\frac{\partial\omega^R}{\partial\beta}=\frac{8\nu\beta_1+\delta_r^2}{4\beta_1\delta_r}>0$,$\frac{\partial p_d^R}{\partial\beta}=\frac{8\nu\beta_1-\delta_r^2}{4\beta_2\delta_r}>0$. ■

Appendix B

Single-channel equilibruim

The manufacturer's profit: $\pi_m(\omega,\beta)=(\omega-c)(a-\eta p+\delta_r\beta)-\nu\beta^2$

The retailer's profit: $\pi_r(p)=(p-\omega)(a-\eta p+\delta_r\beta)$

Similar with the case of distributionally robust dual-channel model, we get

$$\inf_{\mu_{\xi^\lambda}\in\mathcal{U}}\mathrm{E}[\pi_m(\omega,\beta,\xi^\lambda)]=h_1(\omega,\beta).$$

$$\inf_{\mu_{\xi^\lambda}\in\mathcal{U}}\mathrm{E}[\pi_r(p,\xi^\lambda)]=h_2(p).$$

where $h_1(\omega,\beta)=(\omega-c)(m-\eta p+\delta_r\beta)-\nu\beta^2, h_2(p)=(p-\omega)(m-\eta p+\delta_r\beta), m=-\frac{\theta_l^1+\theta_l^2}{8}+\frac{r_1+2r_2+r_3}{4}$.

The robust counterpart model can be represented as follows

$$\begin{array}{ll}\max\limits_{\omega,\beta} & h_1(\omega,\beta)\\ \text{s.t.} & p\in\arg\max\limits_{p} h_2(p).\end{array}.$$

From the expression of $h_2(p)$, we have the second order sufficient condition $\frac{\partial^2 h_2}{\partial p^2}=-2\eta<0$, which ensures that unique optimal solution exists. For given ω,β, the retailer's response function is derived from the first-order condition of the retailer's profit $h_2(p)$.

$$\frac{\partial h_2}{\partial p}=0\Rightarrow p=\frac{\delta_r\beta+\eta\omega+m}{2\eta}$$

After getting the reaction of the retailer, the manufacturer maximizes his profit and determines the optimal decisions ω,β. The Hessian matrix associated with the profit function $h_1(\omega,\beta)$ is given by

$$H=\begin{pmatrix}-\eta & \frac{\delta_r}{2}\\ \frac{\delta_r}{2} & -2\nu\end{pmatrix}$$

Now, $|H|=2\nu\eta-\frac{\delta_r^2}{4}>0$ if $\nu>\frac{\delta_r^2}{8\eta}$, Therefore, H is negative definite if and only if $\nu>\frac{\delta_r^2}{8\eta}$. Thus manufacturer's profit function $h_1(\omega,\beta)$ is jointly concave in ω and β. Equating the first order conditions to 0, that is,

$$\begin{cases}\frac{\partial h_1}{\partial\omega}=\frac{1}{2}(m+\delta_r\beta-2\eta\omega+\eta c)=0\\ \frac{\partial h_1}{\partial\beta}=\frac{(\omega-c)\delta_r}{2}-2\nu\beta=0\end{cases}$$

We get $\omega=\frac{4\eta c\nu-c\delta_r^2+4m\nu}{8\eta\nu-\delta_r^2}$, $\beta=\frac{(m-\eta c)\delta_r}{8\eta\nu-\delta_r^2}$

Substituting the values of ω,β into the value of p we get, $p=\frac{2\eta c\nu-c\delta_r^2+6m\nu}{8\eta\nu-\delta_r^2}$.

From the above equilibrium values we derive the retailer's profit, manufacturer's profit.

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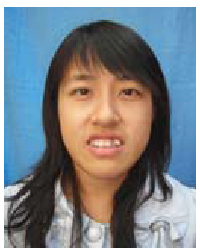

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