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Optimizing a robust capital-constrained dual-channel supply chain under demand distribution uncertainty

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ABSTRACT

Dual-channel supply chain (DCSC) is one type of patterns that combines offline channel and online channel in a common market. It is known that demand plays an important role in DCSC. In the literature, however, the demand is assumed to be deterministic or stochastic with completely known probability distribution. In contrast, this paper addresses the uncertain demand from a new perspective by considering the distribution uncertainty, and the uncertainty source comes from subjectivity. When only partial demand distribution information is available, this paper introduces a new uncertainty distribution set to characterize the ambiguous demand distribution. Based on the proposed uncertainty distribution set, a novel distributionally robust bilevel optimization modeling framework is developed for our capital-constrained DCSC. In order to address the upstream manufacturer's capital constraint, three financing strategies, i.e., bank financing, trade credit and hybrid financing (a combination of bank and equity financing) are considered. An analytically tractable method is developed to obtain the corresponding robust equilibrium solutions under the three financing strategies. Numerical analysis are conducted to demonstrate how the demand ambiguity and the equity ratio affect the manufacturer's equilibrium financing strategy. The numerical results show that the change of the uncertainty perturbation parameters can change the manufacturer's financing strategy. When the values of uncertainty perturbation parameters are relatively small, the equilibrium financing strategy is either trade credit or hybrid financing. When the values of uncertainty perturbation parameters are medium or large, bank financing is the equilibrium financing strategy. Also, the manufacturer's equilibrium financing strategy is affected by the equity financing ratio. When the equity financing ratio is small, the equilibrium financing strategy is either trade credit or hybrid financing. When the equity financing ratio is medium or large, the equilibrium financing strategy is always hybrid financing. The demand uncertainty can affect the manufacturer's financing strategy. As a result, the capital-constrained manufacturer should take into account the demand uncertainty to make her informed financing decisions.

1. Introduction

1.1. Motivation

Capital is fundamental and lacking of funding is an eternal challenge in a supply chain. During the world financial crisis in 2008, thousands of firms faced a shortage of funds and were unable to turn around their business. For example, Circuit City, a former electronic retail giant, declared bankruptcy in 2009 partially due to insufficient cash flow (Cai et al., 2014). Shortage of funds has also been aggravated by the COVID-19 pandemic. According to the survey by the National Federation of Independent Business (NFIB), more than 75% of small firms in the United States have been affected by COVID-19. There are about 300,000 small businesses in the NFIB database, and the NFIB conducted a survey on random businesses and found that most businesses are negotiating with banks to seek loan assistance (Zhang et al., 2021). Specifically, Zhen et al. (2020) indicated that DCSC often faces a shortage of fund to support production, and thus, it is very critical to select an appropriate financing scheme for the manufacturer to make informed operational decisions.

A single financing strategy can no longer satisfy the needs of enterprise financing and many companies begin to seek hybrid financing. It is known that the main financing options are trade credit financing and bank loan financing. However, it has been investigated that when only choosing bank loan financing, the capital-constrained supply chain

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member can neither improve the efficiency of the supply chain nor achieve perfect coordination of the supply chain (Deng et al., 2018). Although trade credit emerges as an effective financing strategy and is widely used in the supply chain financing, the potential risk of trade credit may affect the company's operation (Yang & Birge, 2018). Based on the above discussions, we also consider a hybrid financing option to combine bank loan financing and equity financing.

It is well known that pricing decisions are often affected by uncertain market demand due to the innovation and market turbulence. However, the exact possibility distribution or membership function of market demand is usually assumed to be known in the existing literature. In fact, it is very difficult to obtain the exact possibility distribution due to lack of data. In these cases, the demand information can be approximately estimated based on the experiences or subjective judgments of experts.

1.2. Research questions and methodology

Motivated by the above observations, when only partial demand distribution information is available, how to characterize the uncertain demand distribution becomes an important issue for the DCSC members. In this paper, we assume the capital-constrained manufacturer doesn't know the exact distribution of the market demand and assume that the distribution belongs to an uncertainty distribution set. In the presence of distribution ambiguity, we study the pricing problem in a DCSC consisting of a capital-constrained manufacturer and a retailer aiming to address the following questions. First, how to characterize the ambiguous distribution of the uncertain demand without adequate historical data? Second, what are the equilibrium solutions under each financing strategy when demand distribution is ambiguous? Third, how do the uncertain parameters affect the capital-constrained manufacturer's financing strategy? Finally, how does the equity financing ratio affect the manufacturer's financing strategy?

To answer the above questions, this paper extends the current literature by examining pricing and financing in a manufacturer-retailer type DCSC, in which the demand distribution is assumed to be uncertain. The estimation of the exact demand distribution is very difficult because limited historical data are available. In order to overcome this difficulty, this paper considers only partial demand distribution information is available based on the limited historical data. To depict the distribution perturbation, a new uncertainty distribution set is proposed. On the basis of the uncertainty distribution set, a novel distributionally robust bilevel optimization modeling framework is developed for a DCSC. For the capital constraint of the manufacturer, we consider bank loan financing, trade credit financing and hybrid financing that is a combination of bank loan financing and equity financing. We develop robust bilevel optimization methods to model the interactions between the manufacturer and the retailer and derive the robust equilibrium decisions under the three strategies. The effects of the uncertainty perturbation parameters and the equity financing ratio on the manufacturer's financing strategies are discussed.

1.3. Contribution

The contributions of the paper can be summarized as follows. First, when only partial demand distribution information is available, a new uncertainty distribution set is introduced to characterize the ambiguous uncertain demand. Second, a novel distributionally robust bilevel optimization modeling framework is developed for a capital-constrained DCSC based on the proposed uncertainty set. An analytically tractable method is presented to derive the robust equilibrium solutions of the proposed robust models. Third, we investigate via numerical analysis how the uncertainty perturbation parameters and the equity financing ratio affect the manufacturer's robust equilibrium financing strategy. The numerical results show that the change of the uncertainty perturbation parameters can affect the manufacturer's financing strategy. Also, the manufacturer's robust equilibrium financing strategy is affected by the equity financing ratio and the equilibrium financing strategy is either trade credit or hybrid financing while bank loan financing can never be the dominated strategy. Finally, some managerial insights are obtained for the capital-constrained manufacturer in selecting financing strategy.

The rest of this paper is organized as follows. Section 2 presents a literature review of related studies. Section 3 introduces the robust bilevel DCSC model formulation. Section 4 presents the robust equilibrium analyses under the three strategies and the comparative results of the robust models under various strategies are given in Section 5. The numerical experiments are conducted in Section 6. Section 7 concludes the paper. All proofs are presented in Appendix.

2. Literature review

Literature related to our work can be categorized into the following streams: pricing problem in DCSC, uncertain bilevel supply chain, and financing environment in supply chain.

2.1. Pricing problem in DCSC

It is an important issue to determine the product prices of both channels in DCSC. Pricing problems of the DCSC have been considered by many researchers. For example, Zhou et al. (2019) examined the pricing problem in DCSC by developing a screening model where the base demand of the market is assumed to be a scenario-based random variable. Wang et al. (2019) considered the pricing problem in a fuzzy DCSC where the consumer demands for each product are characterized as fuzzy variables. Soleimani (2016) analyzed the pricing decisions of a DCSC considering both the manufacturing cost and the customer demand as fuzzy variables. Yan et al. (2020) analyzed the pricing problem in a DCSC consisting of one capital-constrained supplier and one e-retailer providing finance. Huang et al. (2021) considered pricing problem for a DCSC with one manufacturer and one retailer under stochastic demand. Dai et al. (2019) studied pricing strategies in DCSC when the retailer has fairness concerns. Meng et al. (2021) explored products collaborative pricing policies in DCSC considering government subsidies and consumers' dual preferences. Javadi et al. (2019) studied the optimal pricing decisions in a DCSC under different government intervention policies.

The interesting studies mentioned above mostly assumed that the DCSC has sufficient capital. In practice, however, capital constraint, especially short-term liquidity shortage, is a common phenomenon in practice. Also, the mentioned literature mostly assumed the demand is deterministic or stochastic(fuzzy) with known distribution. In fact, the demand is usually uncertain and the exact distribution is difficult to obtain due to the lack of data. In contrast with the existing literature, we study the pricing problem in DCSC by considering upstream manufacturer's capital constraint under uncertain demand distribution.

2.2. Uncertain bilevel supply chain

In the existing bilevel supply chain research, the general uncertainty types mainly include stochastic uncertainty and subjective uncertainty. When the uncertain parameters possess stochastic nature and the exact probability distributions can be obtained, stochastic optimization is usually used to deal with this kind of uncertainty. For instances, Wang et al. (2011) developed a bi-level stochastic programming for a facility location and task allocation problem of a two-echelon supply chain against stochastic demand. Setak et al. (2019) proposed a bi-level programming two-stage stochastic approach to design a reliable supply chain. Muneeb et al. (2020) presented a decentralized bi-level vendor selection problem where demand and supply are normal random variables. Rezapour and Farahani (2014) presented a bi-level model for designing the network structure of a competitive supply chain under stochastic price and service level dependent elastic demands. However, the exact probability distributions of the uncertain parameters are sometimes unavailable due to the lack of historical data. In these cases, the distribution information should be approximately estimated based on the experiences or subjective judgments of experts. Some researchers addressed the subjective uncertainty in bilevel supply chain based on fuzzy optimization theory. Ji and Zhen (2006) formulated a bilevel programming model for the newsboy problem with fuzzy demands. Zhou et al. (2017) proposed a bi-level programming model for the plant selection and production allocation problem where demands are described as type-2 triangular fuzzy variables. Ghomi-Avili et al. (2021) proposed a robust bi-level optimization model of the single-product multi-period supply chain network design problem characterizing the demand as a fuzzy variable. Ghomi-Avili et al. (2018) presented a fuzzy bi-objective bi-level model for a closed-loop supply chain network design in the presence of random disruptions at suppliers. In the above studies, the exact possibility distribution or membership function of market demand is assumed to be known in advance. In this paper, we study the pricing problem for a DCSC from a new perspective and characterize the ambiguous distribution of demand via an uncertain distribution set.

2.3. Financing environment in supply chain

Sufficient capital is very important to ensure normal operations of the supply chain, and the shortage of funds will lead to the interruption risk of supply chain and even the bankruptcy of small and medium-sized enterprises. To deal with supply chain capital constraints, two main financing strategies of bank loan and trade credit are considered in the rapidly growing literature. For example, Dada and Hu (2008) considered a capital-constrained retailer's optimal ordering quantity and designed coordination mechanisms when facing a profit-maximizing bank. Yan and Sun (2013) designed a supply chain financing system with a manufacturer, a retailer and a commercial bank where the retailer is capital-constrained under demand uncertainties. Kouvelis and Zhao (2015) studied contract design and coordination of a supply chain under bank loan financing when both the supplier and the retailer are capital-constrained. The listed literature investigated firms' optimal operational decisions when bank loan is adopted. Shi et al. (2017) investigated the purchase timing, quantity and financing decisions of a capital-constrained retailer towards seasonal product under a random price-dependent demand.

In addition to bank loan financing, trade credit is another important and frequently used way of financing. Studies on trade credit financing mainly focus on operational decisions, contract design and coordination, and credit risk sharing. Peura et al. (2017) analyzed price decisions of two competing firms with and without trade credit. Yang and Birge (2018) investigated the risk-sharing role of trade credit by allowing the retailer to partially share the demand risk with the supplier. Xiao et al. (2017) considered a financially constrained supply chain where the retailer finances his operations through trade credit from the supplier. Lee and Rhee (2011) shed light on trade credit from a supplier's perspective, and presented it as a tool for supply chain coordination. Some other papers showed that trade credit can play a role as a strategic tool in a competitive environment. For instance, Peura et al. (2017) examined that trade credit can soften the horizontal price competition. Wu et al. (2019) indicated that manufacturers can use trade credit as a strategic response to the bargaining power.

The above literature compares the supply chain performance from the viewpoint of single financing strategy. With the intensification of market competition, capital-constrained supply chain members begin to seek hybrid financing. Shen et al. (2020) found that the mix use of bank financing and trade credit financing provides a win-win situation for the supply chain members. Yang et al. (2021) considered a three-echelon supply chain consisting of a commercial bank, an e-commerce platform, and a capital-constrained retailer and focused on a mixed financing scheme by combining bank credit financing with e-commerce platform financing. Zhang et al. (2021) designed three financing strategies including bank loan financing, equity financing, and hybrid financing and revealed the financing preference of the supply chain members. However, the current research mainly focuses on financing under a deterministic or stochastic environment. Our study differs from the existing literature and investigates the effects of the hybrid financing (a combination of bank loan financing and equity financing) on the supply chain performance in an uncertain environment.

3. Formulation of robust bilevel DCSC model

3.1. Robust bilevel DCSC model without capital constraint

We consider a DCSC consisting of a manufacturer (she) and a retailer (he). The manufacturer produces and sells her products to the retailer at a wholesale price w, which is referred as a retail channel and simultaneously sells the products to the end consumers at a selling price P_d, which is referred as a direct-selling channel. The retailer purchases the products from the manufacturer and sells to the customers at a retail price P_r. Customers can purchase products through either of the two channels according to their preferences. To avoid the trivial case, we assume $P_r > w > c$. In order to prevent the retailer from purchasing the products from the direct channel, we assume $P_d > w$.

We assume a linear price-dependent demand structure, which is widely used in the literature (Qin et al., 2020; Shen et al., 2019; Soleimani, 2016; Xu et al., 2014; Yan et al., 2021; Zhou et al., 2019). The demand functions of the retail channel and the direct channel are assumed as $D_r = \gamma a - \beta_1 P_r + \beta_2 P_d$, $D_d = (1-\gamma)a - \beta_1 P_d + \beta_2 P_r$, where a represents the market size, γ is the degree of customer loyalty to the retail channel, and correspondingly, $1-\gamma$ represents the degree of customer loyalty to the direct channel. β_1 is the ownership price sensitivity, which means that a unit of price reduction increases the demand by β_1. β_2 is the cross-price sensitivity, that is, a larger value of β_2 means switching customers who are sensitive to the difference between the direct-selling price P_d and the retail price P_r. In fact, β_2 captures the degree of competition between the two channels. $\beta_1 > \beta_2$ signifies that the effect of ownership price is greater than that of cross-price.

Usually the market size a is assumed to be deterministic in the existing literature. However, the market size is uncertain due to the impacts of the economic environment and business conditions. In our model, we consider the distribution μ_ξ of the uncertain market size ξ is partially available and varies in an uncertainty distribution set $\mathcal{F}$. We assume that both parties in the bilevel optimization model are risk-neutral.

The manufacturer, as the upper level decision maker, desires to maximize her own expected profit by determining the wholesale price w and the direct-selling price P_d and taking into account the actions of the retailer (follower). For any given $\mu_\xi \in \mathcal{F}$, the risk-neutral criterion is adopted to construct the upper level objective function

$$\max_{P_d, w} \quad \mathrm{E}[\pi_m(P_d, P_r, w; \xi)]. \tag{1}$$

After observing the actions of the manufacturer, the retailer sets his retail price P_r to maximize his expected profit. The lower level objective is represented as follows

$$\max_{P_r} \quad \mathrm{E}[\pi_r(P_r; \xi)], \tag{2}$$

where $\pi_m(P_d, P_r, w; \xi) = P_d D_d + w D_r - c(D_d + D_r)$ represents the manufacturer's profit, and $\pi_r(P_r; \xi) = (P_r - w)D_r$ is the profit of the retailer. E is the expected value operator of fuzzy variables (Liu & Liu, 2003).

Thus the uncertain bilevel DCSC model without capital constraint can be represented as follows

$$\left\{\begin{array}{ll} \max\limits_{P_d,w,U_m} & U_m, \\ \text{s.t.} & \mathrm{E}[\pi_m(P_d,P_r,w;\xi)] \geq U_m, \\ & \max\limits_{P_r,U_r} \quad U_r, \\ & \text{s.t.} \quad \mathrm{E}[\pi_r(P_r;\xi)] \geq U_r. \end{array}\right\}_{\mu_\xi \in \mathcal{F}.} \tag{3}$$

Evidently, model (3) is a family of fuzzy expected value models when μ_ξ varies in the uncertainty distribution set $\mathcal{F}$. The critical difficulty is that a collection of expected value models is not associated with the concepts of optimal solution and optimal value. Therefore, how to define these concepts to model (3) depends on the underlying decision environment. Here we focus on the following decision making environment:

(A1) The upper level decisions and the lower level decision in model (3) represent "here and now" decisions;

(A2) The upper level decision maker and lower level decision maker must be fully responsible for the consequences of the decisions to be made when and only when μ_ξ belongs to the corresponding uncertainty distribution set $\mathcal{F}$.

Based on the assumptions above, we can determine a meaningful feasible solution to the uncertain model (3) based on the worst-case criterion, which is called a distributionally robust feasible solution. Thus, the robust counterpart of model (3) is formally written as

$$\begin{array}{ll} \max\limits_{P_d,w,\hat{U}_m} & \hat{U}_m, \\ \text{s.t.} & \inf\limits_{\mu_\xi\in\mathcal{F}} \mathrm{E}[\pi_m(P_d,P_r,w;\xi)] \geq \hat{U}_m, \\ & \max\limits_{P_r,\hat{U}_r} \quad \hat{U}_r, \\ & \text{s.t.} \quad \inf\limits_{\mu_\xi\in\mathcal{F}} \mathrm{E}[\pi_r(P_r;\xi)] \geq \hat{U}_r. \end{array} \tag{4}$$

It is evident that model (4) includes infinitely many integrals. In order to derive the computationally tractable formulation, we will give the uncertainty distribution set $\mathcal{F}$ in Section 4.

3.2. Robust bilevel DCSC model with capital constraint

In this section, we consider a capital-constrained DCSC. The manufacturer with limited working-capital may face a capital constraint problem when producing products at unit production cost c. Similar with the literature (Tang et al., 2017), the manufacturer is assumed to have no initial capital and is dependent on external sources to finance her operations. We assume that the manufacturer does not face bankruptcy risk because she has limited liability. Three financing strategies are considered, namely, trade credit financing, bank loan financing and hybrid financing. The symbols $D_r^t(D_d^t)$, $D_r^b(D_d^b)$ and $D_r^h(D_d^h)$ represent the demand of the retail channel (direct-selling channel) in trade credit, bank financing and hybrid financing, respectively.

▲ Trade credit financing: The retailer provides trade credit to the manufacturer at an interest rate r_t. That is, the manufacturer borrows $c(D_r^t + D_d^t)$ from the retailer for production, and repays the retailer $c(D_r^t + D_d^t)(1 + r_t)$ at the end of the selling period. The retailer earns a profit of $c(D_r^t + D_d^t)r_t$ through providing finance to the manufacturer.

▲ Bank loan financing: A bank provides loans to the manufacturer at an interest rate r_b. The manufacturer obtains loan $c(D_r^b + D_d^b)$ from the bank for producing adequate products at time zero and should repay the principal plus the interest of the loan $c(D_r^b + D_d^b)(1 + r_b)$ at the end of the time period.

▲ Hybrid financing: A hybrid financing strategy is a combination of bank loan financing and equity financing. In contrast to bank loan, the manufacturer opting for equity financing doesn't need to repay the principal and interest at the end of the period but has to pay dividends to the creditors. We assume that the equity financing ratio is κ, while the remaining ratio $1-\kappa$ is the bank loan. That is, the manufacturer borrows $(1-\kappa)c(D_r^h + D_d^h)$ from bank and the remaining capital comes from the investors.

For the remainder of the study, we add subscript or superscript k to a symbol to distinguish it among different strategies, with $k = t, b, h$ corresponding to trade credit, bank loan, and hybrid financing, respectively.

Based on the descriptions above, the uncertain bilevel DCSC model with capital constraint can be represented as

$$\left\{\begin{array}{ll} \max\limits_{P_d^k,w_k,U_m^k} & U_m^k, \\ \text{s.t.} & \mathrm{E}[\pi_m^k(P_d^k,P_r^k,w_k;\xi)] \geq U_m^k \\ & \max\limits_{P_r^k,U_r^k} \quad U_r^k, \\ & \text{s.t.} \quad \mathrm{E}[\pi_r^k(P_r^k;\xi)] \geq U_r^k. \end{array}\right\}_{\mu_\xi \in \mathcal{F},} \tag{5}$$

where $k = t, b, h$.

Based on the assumptions A1, A2 and worst-case criterion, the robust counterpart of model (5) is represented as

$$\begin{array}{ll} \max\limits_{P_d^k,w_k,\hat{U}_m^k} & \hat{U}_m^k, \\ \text{s.t.} & \inf\limits_{\mu_\xi\in\mathcal{F}} \mathrm{E}[\pi_m^k(P_d^k,P_r^k,w_k;\xi)] \geq \hat{U}_m^k \\ & \max\limits_{P_r^k,\hat{U}_r^k} \quad \hat{U}_r^k, \\ & \text{s.t.} \quad \inf\limits_{\mu_\xi\in\mathcal{F}} \mathrm{E}[\pi_r^k(P_r^k;\xi)] \geq \hat{U}_r^k. \end{array} \tag{6}$$

where $k = t, b, h$.

4. Model analysis

4.1. Analysis of robust bilevel DCSC model without capital constraint

Usually, the market size a is assumed to be deterministic in the existing literature. Actually, the market size a is uncertain due to the innovation and the impacts of the economic environment, and the distribution information is often unavailable, but from the available historical data, the uncertain market size varies on a bounded interval. In these cases, the demand information can be approximately estimated based on the experiences or subjective judgments of experts. In our model, we consider the distribution μ_ξ of the uncertain market size ξ is partially available and varies in an uncertainty distribution set $\mathcal{F}$ which will be introduced as follows.

In order to describe the distribution perturbation of the uncertain market size ξ, we assume the uncertain market size ξ is represented by a parametric level interval type-2 trapezoidal fuzzy variable $\widetilde{\mathrm{Tra}}[r_1,r_2,r_3,r_4;\theta], \theta = (\theta_l^1,\theta_r^1,\theta_l^2,\theta_r^2)$ and let ξ^λ be its λ selection variable. For any given $\lambda \in [0,1]$, the distribution of ξ^λ is denoted by $\mu_{\xi^\lambda}(x;\theta)$ which is determined by the nested sets $\{[x_u^L, x_u^R] : x_u^L \in J_u^L, x_u^R \in J_u^R, u \in [0,1]\}$, where x_u^L, x_u^R are represented as

$$\begin{aligned} x_u^L &= \lambda(r_1 + u(r_2-r_1) - \theta_l^1\min\{u,1-u\}) \\ &\quad + (1-\lambda)(r_1 + u(r_2-r_1) + \theta_r^1\min\{u,1-u\}), \\ x_u^R &= \lambda(r_4 - u(r_4-r_3) - \theta_l^2\min\{u,1-u\}) \\ &\quad + (1-\lambda)(r_4 - u(r_4-r_3) + \theta_r^2\min\{u,1-u\}). \end{aligned}$$

Then an uncertainty distribution set $\mathcal{F}$ associated with ξ is defined as follows

$$\mathcal{F} = \begin{array}{l} \{\mu_{\xi^\lambda}(x;\theta) | \mu_{\xi^\lambda}(x;\theta) \text{ is determined by the nested sets} \\ \{[x_u^L, x_u^R] : x_u^L \in J_u^L, x_u^R \in J_u^R, u \in [0,1]\}, \text{where } \lambda \in [0,1]\}. \end{array} \tag{7}$$

Based on the uncertainty set $\mathcal{F}$ introduced above, Theorem 1 gives the robust equilibrium solutions without capital constraint.

Theorem 1. *Consider model (4), without capital constraint, when the distribution μ_ξ of the uncertain market size ξ varies in the uncertainty set $\mathcal{F}$, the robust equilibrium wholesale price w^* and the robust equilibrium selling prices P_d^*, P_r^* of model (4) are represented as*

$$w^* = \frac{\eta[\gamma\beta_1 + (1-\gamma)\beta_2] + c(\beta_1^2 - \beta_2^2)}{2(\beta_1^2 - \beta_2^2)},$$

$$P_d^* = \frac{\eta[\gamma\beta_2 + (1-\gamma)\beta_1] + c(\beta_1^2 - \beta_2^2)}{2(\beta_1^2 - \beta_2^2)},$$

$$P_r^* = \frac{\eta[2(1-\gamma)\beta_1\beta_2 + \gamma(3\beta_1^2 - \beta_2^2)] + c(\beta_1+\beta_2)(\beta_1^2 - \beta_2^2)}{4\beta_1(\beta_1^2 - \beta_2^2)},$$

where $\eta = -\frac{\theta_l^1+\theta_l^2}{8} + \frac{r_1+r_2+r_3+r_4}{4}$.

Proof. The proof of Theorem 1 is in Appendix. □

Based on Theorem 1, the following proposition is given to show the impact of the uncertain perturbation parameters on the robust equilibrium decisions without capital constraint.

Proposition 1. *Without capital constraint, the robust equilibrium wholesale price w^* and robust equilibrium selling prices P_d^*, P_r^* decrease with the parameters $\theta_l^i, i = 1, 2$ increasing, that is, $\frac{\partial w^*}{\partial \theta_l^i} < 0$, $\frac{\partial P_d^*}{\partial \theta_l^i} < 0$, and $\frac{\partial P_r^*}{\partial \theta_l^i} < 0$.*

Proof. The proof of Proposition 1 is in Appendix. □

Proposition 1 indicates that the uncertain perturbation parameters θ_l^1, θ_l^2 can affect the pricing decisions of the manufacturer and the retailer. Parameters θ_l^1, θ_l^2 describe the demand distribution uncertainty degree. The bigger θ_l^1, θ_l^2, the bigger the uncertainty degree. From Proposition 1, we find that the robust equilibrium wholesale price w^* and robust equilibrium selling prices P_d^*, P_r^* decrease as θ_l^1, θ_l^2 increases. That is to say, the bigger the demand distribution uncertainty degree is, the more conservative the pricing decisions are.

4.2. Analysis of robust bilevel DCSC model with capital constraint

In this section, we analyze the robust equilibrium decisions where the capital-constrained manufacturer finances her operations by trade credit, bank loan or hybrid financing.

Trade credit financing

In this strategy as shown in Fig. 1, the retailer provides credit to the manufacturer. First, the manufacturer sets her wholesale price w_t and her direct selling price P_d^t simultaneously. Then the retailer determines his retail price P_r^t. Finally, the manufacturer borrows $c(D_r^t + D_d^t)$ from the retailer and the products are produced, where $D_r^t = \gamma a - \beta_1 P_r^t + \beta_2 P_d^t$, $D_d^t = (1-\gamma)a - \beta_1 P_d^t + \beta_2 P_r^t$ are the demand functions of the retailer and the manufacturer, respectively. At the end of the selling period, the manufacturer repays the retailer. Therefore, the profits of the retailer and the manufacturer in trade credit strategy are $\pi_r^t = (P_r^t - w_t)D_r^t + c(D_r^t + D_d^t)r_t$, $\pi_m^t = P_d^t D_d^t + w_t D_r^t - c(D_r^t + D_d^t)(1 + r_t)$.

The uncertain bilevel DCSC model in trade credit financing can be represented as

$$\left\{\begin{array}{lll} \max\limits_{P_d^t, w_t, U_m^t} & U_m^t, & \\ \text{s.t.} & \mathrm{E}[\pi_m^t(P_d^t, P_r^t, w_t; \xi)] \geq U_m^t & \\ & \max\limits_{P_r^t, U_r^t} & U_r^t, \\ & \text{s.t.} & \mathrm{E}[\pi_r^t(P_r^t; \xi)] \geq U_r^t. \end{array}\right\}_{\mu_\xi \in \mathcal{F}} \tag{8}$$

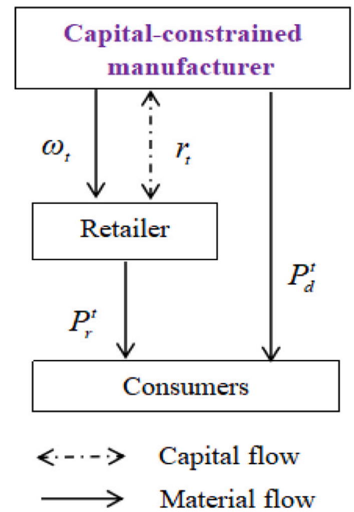

Fig. 1. The dual-channel structure in trade credit financing.

Based on the assumptions A1, A2 and worst-case criterion, the robust counterpart of model (8) is represented as

$$\begin{array}{lll} \max\limits_{P_d^t, w_t, \hat{U}_m^t} & \hat{U}_m^t, & \\ \text{s.t.} & \inf\limits_{\mu_\xi \in \mathcal{F}} \mathrm{E}[\pi_m^t(P_d^t, P_r^t, w_t; \xi)] \geq \hat{U}_m^t & \\ & \max\limits_{P_r^t, \hat{U}_r^t} & \hat{U}_r^t, \\ & \text{s.t.} & \inf\limits_{\mu_\xi \in \mathcal{F}} \mathrm{E}[\pi_r^t(P_r^t; \xi)] \geq \hat{U}_r^t. \end{array} \tag{9}$$

Solving model (9), Theorem 2 gives the robust equilibrium decisions under trade credit financing.

Theorem 2. *Considering model (9), when the distribution μ_ξ of the uncertain market size ξ varies in the uncertainty set $\mathcal{F}$, the robust equilibrium wholesale price w_t^* and robust equilibrium selling prices P_d^{t*}, P_r^{t*} in trade credit strategy are*

$$w_t^* = \frac{\eta\beta_1[\gamma\beta_1 + (1-\gamma)\beta_2] + c[\beta_1 + (2\beta_1 - \beta_2)r_t](\beta_1^2 - \beta_2^2)}{2\beta_1(\beta_1^2 - \beta_2^2)},$$

$$P_d^{t*} = \frac{\eta[\gamma\beta_2 + (1-\gamma)\beta_1] + c(1+r_t)(\beta_1^2 - \beta_2^2)}{2(\beta_1^2 - \beta_2^2)},$$

$$P_r^{t*} = \frac{\eta[2(1-\gamma)\beta_1\beta_2 + \gamma(3\beta_1^2 - \beta_2^2)] + c(\beta_1 + \beta_2 + 2\beta_2 r_t)(\beta_1^2 - \beta_2^2)}{4\beta_1(\beta_1^2 - \beta_2^2)},$$

where $\eta = -\frac{\theta_l^1+\theta_l^2}{8} + \frac{r_1+r_2+r_3+r_4}{4}$.

Proof. The proof of Theorem 2 is in Appendix. □

Based on Theorem 2, the following proposition is given to investigate the sensitivity analysis about the trade credit interest rate r_t.

Proposition 2. *Under trade credit strategy, the robust equilibrium wholesale price w_t^* and robust equilibrium selling prices P_d^{t*}, P_r^{t*} increase in trade credit rate r_t, that is, $\frac{\partial w_t^*}{\partial r_t} > 0$, $\frac{\partial P_d^{t*}}{\partial r_t} > 0$, and $\frac{\partial P_r^{t*}}{\partial r_t} > 0$.*

Proof. The proof of Proposition 2 is in Appendix. □

Proposition 2 indicates that the trade credit rate plays an important role on the manufacturer's robust equilibrium wholesale price and direct selling price decisions. The robust equilibrium wholesale price

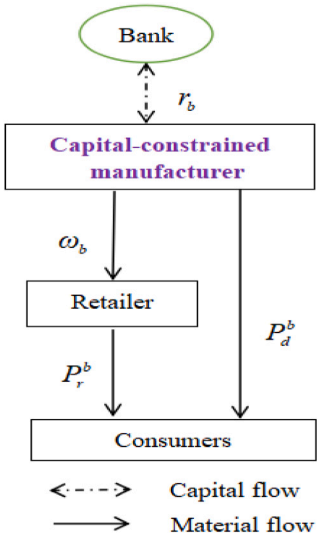

Fig. 2. The dual-channel structure in bank loan financing.

and robust equilibrium direct selling price increase in trade credit rate. That is to say, the manufacturer's trade credit financing results in higher robust equilibrium wholesale price and direct selling price. Also, the retailer's robust equilibrium retail price increases with the trade credit rate increasing. The higher robust equilibrium direct selling price and retail price will directly hurt the consumers' welfare.

Bank loan financing

In the bank loan strategy as shown in Fig. 2, the manufacturer borrows a loan from the bank. First, the manufacturer reports her wholesale price w_b and her direct-selling price P_d^b. Then, the retailer sets his retail price P_r^b. Finally, the manufacturer borrows $c(D_r^b + D_d^b)$ from the bank and produces the products. The demand functions of the retailer and the manufacturer are $D_r^b = \gamma a - \beta_1 P_r^b + \beta_2 P_d^b$, $D_d^b = (1-\gamma)a - \beta_1 P_d^b + \beta_2 P_r^b$, respectively. Therefore, the profits of the retailer and the manufacturer in bank loan strategy are $\pi_r^b(P_r^b) = (P_r^b - w_b)D_r^b$, $\pi_m^b(P_d^b, P_r^b, w_b) = P_d^b D_d^b + w_b D_r^b - c(D_r^b + D_d^b)(1+r_b)$.

The uncertain bilevel DCSC model in bank loan financing can be represented as

$$\left\{\begin{array}{lll} \max\limits_{P_d^b, w_b, U_m^b} & U_m^b, & \\ \text{s.t.} & \mathrm{E}[\pi_m^b(P_d^b, P_r^b, w_b;\xi)] \geq U_m^b & \\ & \max\limits_{P_r^b, U_r^b} \quad U_r^b, & \\ & \text{s.t.} \quad \mathrm{E}[\pi_r^b(P_r^b;\xi)] \geq U_r^b. & \end{array}\right\}_{\mu_\xi \in \mathcal{F}} \tag{10}$$

Based on the assumptions A1, A2 and worst-case criterion, the robust counterpart of model (10) is represented as

$$\begin{array}{ll} \max\limits_{P_d^b, w_b, \hat{U}_m^b} & \hat{U}_m^b, \\ \text{s.t.} & \inf\limits_{\mu_\xi \in F} \mathrm{E}[\pi_m^b(P_d^b, P_r^b, w_b;\xi)] \geq \hat{U}_m^b \\ & \max\limits_{P_r^b, \hat{U}_r^b} \quad \hat{U}_r^b, \\ & \text{s.t.} \quad \inf\limits_{\mu_\xi \in F} \mathrm{E}[\pi_r^b(P_r^b;\xi)] \geq \hat{U}_r^b. \end{array} \tag{11}$$

The robust equilibrium solutions under bank loan financing are derived by solving model (11) and are given in Theorem 3.

Theorem 3. *Under bank loan financing, the robust equilibrium wholesale price w_b^* and the robust equilibrium selling prices P_d^{b*}, P_r^{b*} of model (11) with the distribution μ_ξ of the uncertain market size ξ varying in the uncertainty set $\mathcal{F}$ are*

$$w_b^* = \frac{\eta[\gamma\beta_1 + (1-\gamma)\beta_2] + c(\beta_1^2 - \beta_2^2)(1+r_b)}{2(\beta_1^2 - \beta_2^2)},$$

$$P_d^{b*} = \frac{\eta[\gamma\beta_2 + (1-\gamma)\beta_1] + c(\beta_1^2 - \beta_2^2)(1+r_b)}{2(\beta_1^2 - \beta_2^2)},$$

$$P_r^{b*} = \frac{\eta[2\beta_1\beta_2(1-\gamma) + \gamma(3\beta_1^2 - \beta_2^2)] + c(\beta_1+\beta_2)(\beta_1^2 - \beta_2^2)(1+r_b)}{4\beta_1(\beta_1^2 - \beta_2^2)},$$

where $\eta = -\frac{\theta_l^1 + \theta_l^2}{8} + \frac{r_1 + r_2 + r_3 + r_4}{4}$.

Proof. The proof of Theorem 3 is in Appendix. □

Based on Theorem 3, Proposition 3 is given to show the sensitivity analysis about the bank financing interest rate r_b.

Proposition 3. *Under bank financing strategy, the robust equilibrium wholesale price w_b^* and robust equilibrium selling prices P_d^{b*}, P_r^{b*} increase in bank financing rate r_b, that is,* $\frac{\partial w_b^*}{\partial r_b} > 0$, $\frac{\partial P_d^{b*}}{\partial r_b} > 0$, *and* $\frac{\partial P_r^{b*}}{\partial r_b} > 0$.

Proof. The proof of Proposition 3 is in Appendix. □

Proposition 3 shows that the bank financing interest rate plays an important role on the manufacturer's robust equilibrium wholesale price and direct selling price decisions in bank financing strategy. The robust equilibrium wholesale price and robust equilibrium direct selling price increase with the increasing of bank financing interest rate. That is to say, the manufacturer's bank financing leads to higher robust equilibrium wholesale price and direct selling price. The retailer's robust equilibrium retail price also increases as the bank financing rate increases. The consumers' welfare can be hurt due to the higher robust equilibrium direct selling price and retail price.

Hybrid financing

In the hybrid financing strategy as shown in Fig. 3, the manufacturer simultaneously uses bank loan and equity financing. We assume that the equity financing ratio is κ, while the remaining ratio $1-\kappa$ is the bank loan. At the end of the selling period, the manufacturer first pays the bank loan principal plus interest $(1-\kappa)c(D_r^h + D_d^h)(1+r_b)$, and then transfers the κ portion of the remaining profit to the equity investors. Therefore, the profits of the retailer and the manufacturer in hybrid financing strategy are $\pi_r^h = (P_r^h - w_h)D_r^h$, $\pi_m^h = (1-\kappa)[P_d^h D_d^h + w_h D_r^h - c(1-\kappa)(D_r^h + D_d^h)(1+r_b)]$.

The uncertain bilevel DCSC model in hybrid financing can be represented as

$$\left\{\begin{array}{lll} \max\limits_{P_d^h, w_h, U_m^h} & U_m^h, & \\ \text{s.t.} & \mathrm{E}[\pi_m^h(P_d^h, P_r^h, w_h;\xi)] \geq U_m^h & \\ & \max\limits_{P_r^h, U_r^h} \quad U_r^h, & \\ & \text{s.t.} \quad \mathrm{E}[\pi_r^h(P_r^h;\xi)] \geq U_r^h. & \end{array}\right\}_{\mu_\xi \in \mathcal{F}.} \tag{12}$$

Based on the assumptions A1, A2 and worst-case criterion, the robust counterpart of model (12) is represented as

$$\begin{array}{ll} \max\limits_{P_d^h, w_h, \hat{U}_m^h} & \hat{U}_m^h, \\ \text{s.t.} & \inf\limits_{\mu_\xi \in F} \mathrm{E}[\pi_m^h(P_d^h, P_r^h, w_h;\xi)] \geq \hat{U}_m^h \\ & \max\limits_{P_r^h, \hat{U}_r^h} \quad \hat{U}_r^h, \\ & \text{s.t.} \quad \inf\limits_{\mu_\xi \in F} \mathrm{E}[\pi_r^h(P_r^h;\xi)] \geq \hat{U}_r^h. \end{array} \tag{13}$$

Through solving model (13), the robust equilibrium decisions under hybrid financing are given in Theorem 4.

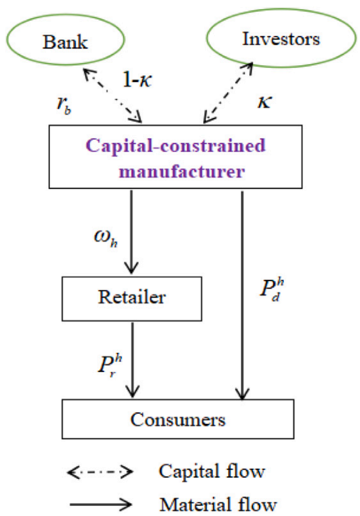

Fig. 3. The dual-channel structure in hybrid financing.

Theorem 4. *When the distribution μ_ξ of the uncertain market size ξ varies in the uncertainty set $\mathcal{F}$, the robust equilibrium wholesale price w_h^* and the robust equilibrium selling prices P_d^{h*}, P_r^{h*} of model (13) are*

$$w_h^* = \frac{\eta[\gamma\beta_1 + (1-\gamma)\beta_2] + c(1-\kappa)(1+r_b)(\beta_1^2-\beta_2^2)}{2(\beta_1^2-\beta_2^2)},$$

$$P_d^{h*} = \frac{\eta[\gamma\beta_2 + (1-\gamma)\beta_1] + c(1-\kappa)(1+r_b)(\beta_1^2-\beta_2^2)}{2(\beta_1^2-\beta_2^2)},$$

$$P_r^{h*} = \frac{\eta[2\beta_1\beta_2(1-\gamma) + \gamma(3\beta_1^2-\beta_2^2)] + c(1-\kappa)(1+r_b)(\beta_1+\beta_2)(\beta_1^2-\beta_2^2)}{4\beta_1(\beta_1^2-\beta_2^2)}$$

where $\eta = -\frac{\theta_l^1+\theta_l^2}{8} + \frac{r_1+r_2+r_3+r_4}{4}$.

Proof. The proof of Theorem 4 is in Appendix. □

Based on Theorem 4, the following proposition is given to investigate the sensitivity analysis about the equity financing ratio κ.

Proposition 4. *Under hybrid financing strategy, the robust equilibrium wholesale price w_h^* and robust equilibrium selling prices P_d^{h*}, P_r^{h*} decrease in equity financing ratio κ, that is, $\frac{\partial w_h^*}{\partial\kappa} < 0$, $\frac{\partial P_d^{h*}}{\partial\kappa} < 0$, and $\frac{\partial P_r^{h*}}{\partial\kappa} < 0$.*

Proof. The proof of Proposition 4 is in Appendix. □

Under the hybrid financing strategy, the manufacturer's unit marginal financing cost is $(1-\kappa)(1+r_b)$. Therefore, a relatively high equity financing ratio means that a lower financing cost for the manufacturer and the manufacturer borrows less from the bank and less interest needs to be paid to the bank, which reduces the financial pressure of the manufacturer to repay due debts and interest and improves the competitiveness of the manufacturer. The manufacturer can lower the wholesale price and the direct selling price slightly in order to win higher consumer demand and higher profit. The lower wholesale price reduces the purchasing cost of the retailer, so the retailer can reduce the retail price to attract more consumers and get higher retail profit. These observations clarify why the robust equilibrium wholesale price and the robust equilibrium selling prices are negatively correlated with the equity financing ratio.

5. Results about robust DCSC model under various strategies

In this section, we will give the comparison results on the robust equilibrium wholesale prices w_k^* and the robust equilibrium selling prices $P_d^{k*}, P_r^{k*}, k = t, b, h$ derived from models (9), (11) and (13).

Theorem 5. *By comparing the robust equilibrium wholesale prices $w_k^*, k = t, b, h$ derived from models (9), (11) and (13), we obtain the following results. There exist thresholds κ_0, r_t^1, r_t^2 such that*

(a) when $0 < \kappa < \kappa_0$, we have

(1) if $0 < r_t < r_t^1$, then $w_b^* > w_h^* > w_t^*$, (2) if $r_t^1 < r_t < r_t^2$, then $w_b^* > w_t^* > w_h^*$,

(3) if $r_t^2 < r_t < 1$, then $w_t^* > w_b^* > w_h^*$;

(b) when $\kappa_0 < \kappa < 1$, we have

(1) if $0 < r_t < r_t^2$, then $w_b^* > w_t^* > w_h^*$, (2) if $r_t^2 < r_t < 1$, then $w_t^* > w_b^* > w_h^*$, where $r_t^1 = \frac{\beta_1(r_b-\kappa-\kappa r_b)}{2\beta_1-\beta_2}$ and $r_t^2 = \frac{\beta_1 r_b}{2\beta_1-\beta_2}, \kappa_0 = \frac{r_b}{1+r_b}$.

Proof. The proof of Theorem 5 is in Appendix. □

Theorem 5 presents the comparative results on robust equilibrium wholesale price under three financing strategies. From Theorem 5, we can find that the relationship between w_h^* and w_b^* is independent of κ and r_t and w_h^* is always less than w_b^*. The reason is that w_h^* decreases in the equity financing ratio κ which can be obtained from the expression of w_h^* in Theorem 4, and the hybrid financing strategy includes the bank loan financing strategy as a special case with $\kappa = 0$. The relationship between w_t^* and w_b^* depends on the trade credit rate r_t. Based on Theorem 2, w_t^* increases when r_t increases. That is, the manufacturer will raise the wholesale price w_t to offset her financing cost when trade credit is expensive. Thus, there exists a threshold of trade credit rate r_t^2 such that w_t^* is higher than w_b^* if $r_t^2 < r_t < 1$, otherwise, w_b^* is higher. The relationship between w_t^* and w_h^* is jointly determined by κ and r_t. Specifically, when the equity financing ratio κ is small, a threshold r_t^1 exists such that $w_t^* < w_h^*$ if $0 < r_t < r_t^1$, otherwise $w_t^* > w_h^*$; When the equity financing ratio κ is large, w_t^* is consistently higher than w_h^* independent of r_t.

Theorem 6. *The following results are obtained through comparing the robust equilibrium direct selling prices $P_d^{k*}, k = t, b, h$ derived from models (9), (11) and (13). Thresholds κ_0, r_t^3, r_t^4 exist such that*

(a) when $0 < \kappa < \kappa_0$, we have

(1) if $0 < r_t < r_t^3$, then $P_d^{b*} > P_d^{h*} > P_d^{t*}$, (2) if $r_t^3 < r_t < r_t^4$, then $P_d^{b*} > P_d^{t*} > P_d^{h*}$,

(3) if $r_t^4 < r_t < 1$, then $P_d^{t*} > P_d^{b*} > P_d^{h*}$;

(b) when $\kappa_0 < \kappa < 1$, we have

(1) if $0 < r_t < r_t^4$, then $P_d^{b*} > P_d^{t*} > P_d^{h*}$, (2) if $r_t^4 < r_t < 1$, then $P_d^{t*} > P_d^{b*} > P_d^{h*}$, where $r_t^3 = r_b - \kappa - \kappa r_b$ and $r_t^4 = r_b, \kappa_0 = \frac{r_b}{1+r_b}$.

Proof. The proof of Theorem 6 is in Appendix. □

Theorem 6 presents the comparative results for the manufacturer's robust equilibrium direct-selling prices under the three different financing strategies. First, P_d^{h*} is always less than P_d^{b*} because the hybrid financing includes the bank financing as a special case when $\kappa = 0$. From Theorem 6, we find that the size of r_t and κ is the main factor to affect the relationships between P_d^{h*} and P_d^{t*} and between P_d^{b*} and P_d^{t*}. According to Theorem 2, P_d^{t*} is increasing as r_t increases. That is, the manufacturer will raise her direct-selling price in order to retain the profits obtained from her direct-selling channel when trade credit is expensive. Thus, a threshold r_t^4 exists such that $P_d^{b*} > P_d^{t*}$ if $0 < r_t < r_t^4$, otherwise P_d^{t*} is higher. The relationship of P_d^{h*} and P_d^{t*} depends on not only the credit rate r_t but also the equity ratio κ. For small equity financing ratio κ, a threshold r_t^3 exists such that $P_d^{h*} > P_d^{t*}$ if $0 < r_t < r_t^3$, otherwise P_d^{t*} is higher. However, for a larger equity financing ratio κ, P_d^{t*} is consistently higher than P_d^{h*}.

Theorem 7. *The following results are obtained through comparing the robust equilibrium retail prices $P_r^{k*}, k = t, b, h$ derived from models (9), (11) and (13). There exist thresholds κ_0, r_t^5, r_t^6 such that*

(a) when $0 < \kappa < \kappa_0$, we have

(1) if $0 < r_t < r_t^5$, then $P_r^{b*} > P_r^{h*} > P_r^{t*}$, (2) if $r_t^5 < r_t < r_t^6$, then $P_r^{b*} > P_r^{t*} > P_r^{h*}$,

(3) if $r_t^6 < r_t < 1$, then $P_r^{t*} > P_r^{b*} > P_r^{h*}$;
(b) when $\kappa_0 < \kappa < 1$, we have
(1) if $0 < r_t < r_t^6$, then $P_r^{b*} > P_r^{t*} > P_r^{h*}$, (2) if $r_t^6 < r_t < 1$, then $P_r^{t*} > P_r^{b*} > P_r^{h*}$,
where $r_t^5 = \frac{(\beta_1+\beta_2)(r_b-\kappa-\kappa r_b)}{2\beta_2}$ and $r_t^6 = \frac{(\beta_1+\beta_2)r_b}{2\beta_2}, \kappa_0 = \frac{r_b}{1+r_b}$.

Proof. The proof of Theorem 7 is in Appendix. □

Theorem 7 presents the comparative results for the retailer's robust equilibrium retail prices under three different financing strategies. As indicated in Theorem 5, the robust equilibrium wholesale price in hybrid financing strategy is always less than that in bank loan financing whenever the equity ratio is large or small. That is, the retailer has a lower purchase cost under hybrid financing strategy than that under bank loan financing. Thus, the retailer sets lower retail price under hybrid financing strategy than that under bank loan financing. So, P_r^{h*} is always less than P_r^{b*}, which is independent of κ. From Theorem 5, when r_t is large, the manufacturer raises her wholesale price w_t to transfer her financing cost to the retailer. In order to retain the obtained profits, the retailer must raise his retail price P_r, which gives an explanation about the relationship between P_r^{t*} and P_r^{h*}. P_r^{t*} increases as r_t increases. Therefore, there exists a threshold of trade credit rate r_t^6 such that P_r^{t*} is larger than P_r^{b*} if $r_t^6 < r_t < 1$, otherwise, P_r^{b*} is larger. The relationship of P_r^{t*} and P_r^{h*} depends on the equity financing ratio κ, when the equity financing ratio κ is small, a threshold r_t^5 exists such that $P_r^{h*} > P_r^{t*}$ when $0 < r_t < r_t^5$, otherwise P_r^{t*} is larger. When the equity financing ratio κ is large, $P_r^{t*} > P_r^{h*}$ always holds.

Theorem 8. *Through comparing the robust equilibrium solutions derived from models (9), (11) and (13) with that of model (4), we get $w_k^* > w^*, P_r^{k*} > P_r^*, P_d^{k*} > P_d^*$, where $k = t, b, h$ and $0 < \kappa < \kappa_0$.*

Proof. The proof of Theorem 8 is in Appendix. □

Theorem 8 shows that the manufacturer's robust equilibrium wholesale price and robust equilibrium selling prices under capital constraint are greater than that without capital constraint. That is, the manufacturer transfers her financing cost to the retailer by increasing the wholesale price and to the consumers by increasing the direct-selling price. In order to retain the obtained profits, the retailer also sets higher retail price.

6. Numerical experiment

We have derived the analytical robust equilibrium solutions for each financing strategy in Section 4. However, it is a challenge to analytically compare the equilibrium profits of each financing strategy. Therefore, we resort to a numerical analysis to get more observations. To prevent the retailer purchasing from the direct channel, the direct selling price P_d should be equal to or greater than the wholesale price w, which requires that $\gamma \le 0.5$. Specifically, we set $\gamma = 0.4$. Referring to some literature on DCSC (Xu et al., 2014), we use the following parameters in all instances unless otherwise stated: $\beta_1 = 2, \beta_2 = 1, c = 105$. We employ Maple to carry out the computations. The numerical experiments are executed on a personal computer (Lenovo with Intel(R) Core(TM) 3.00 GHz CPU and RAM 8.00 GB) by using the Microsoft Windows 10 operating system.

6.1. Computational results without capital constraint

In this subsection, we fix the parameters $\theta_r^1 = \theta_r^2 = 10$ and set the values of parameter $\theta_l^2 = 10, 20$ and 25, respectively, and discuss the relation between the robust equilibrium profit of the manufacturer and the parameter θ_l^1. The computational results are shown in Fig. 4, from which we find that the robust equilibrium profit of the manufacturer is a monotone decreasing function with respect to parameters θ_l^1 and θ_l^2.

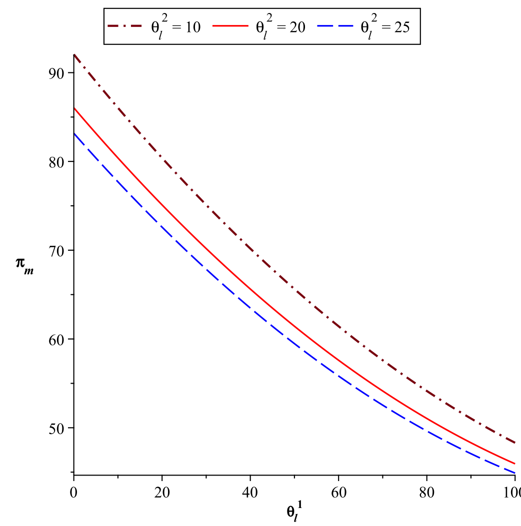

Fig. 4. Comparison of manufacturer's profits without capital constraint under different θ_l.

Table 1
Price of robustness with respect to θ_l^1.

θ_l^1	θ_l^2	$\theta_r^1 = \theta_r^2$	π_m	Price of robustness
10	20	10	80.39	18.05
20	20	10	75.10	23.34
30	20	10	70.18	28.26
40	20	10	65.63	32.81
50	20	10	61.43	37.01
60	20	10	57.60	40.84

In order to compare with the optimization method in which the possibility distribution of the demand is known, we take the nominal possibility distribution (corresponding to $\theta_l^1 = \theta_r^1 = \theta_l^2 = \theta_r^2 = 0$) as the exact possibility distribution of the demand. Using Maple, the obtained nominal equilibrium profit of the manufacturer is 98.44. Using the proposed distributionally robust optimization method, the price of robustness with various values of θ_l^1 is reported in Table 1.

6.2. Computational results with capital constraint

The impact of the perturbation parameters on manufacturer's financing strategy

In this subsection, we will discuss the effects of the perturbation parameters on the financing choice of the manufacturer. We want to identify which financing strategy the manufacturer should select by comparing the manufacturer's and retailer's profits under the three financing strategies with different values of the perturbation parameters. We set $r_1 = 100, r_2 = 200, r_3 = 250, r_4 = 350, \kappa = 0.05, \theta_l^2 = 60, \theta_r^1 = \theta_r^2 = 10$. We assume that $\pi_m^t(\pi_r^t), \pi_m^b(\pi_r^b)$ and $\pi_m^h(\pi_r^h)$ correspond to the manufacturer's (retailer's) profit in trade credit, bank loan and hybrid financing, respectively.

From Fig. 5, we can get the following observations.

(I) If $r_t \ge r_b$, (1) when θ_l^1 is relatively small, the retailer's profit in trade credit dominates that of the other two, however, as the Stackelberg leader, the manufacturer prefers hybrid financing. Thus, hybrid financing is the equilibrium financing strategy. (2) when θ_l^1 is relatively large, both the manufacturer and the retailer prefer bank loan financing. Therefore, bank financing is the equilibrium financing strategy.

(II) If $r_t < r_b$, (1) when θ_l^1 is relatively small, the manufacturer prefers trade credit, while the retailer prefers bank financing. That is, the retailer doesn't want to offer credit to the manufacturer, so, the manufacturer can only choose hybrid financing which is the worst

Table 2
Computational results of the manufacturer's and retailer's profits under deterministic demand.

	π_m^t	π_m^b	π_m^h	π_r^t	π_r^b	π_r^h	Equilibrium strategy
$r_t = r_b = 0.06$	42.4464	56.7394	87.1941	48.1388	14.1788	7.7372	Hybrid financing
$r_t = 0.01 < r_b = 0.08$	87.0384	50.5575	72.2992	18.0169	17.1112	9.8235	Trade credit
$r_t = 0.06 > r_b = 0.04$	42.4462	66.7800	105.3973	48.1388	11.5199	5.8996	Hybrid financing

selection for the retailer. Therefore, the equilibrium financing strategy should be trade credit. (2) when θ_l^1 is medium, the manufacturer prefers trade credit, while the retailer prefers bank financing. Also the retailer doesn't want to offer credit to the manufacturer, so, the manufacturer can only choose bank financing which is the best selection for the retailer. Thus, bank financing is the equilibrium financing strategy. (3) when θ_l^1 is large, both the manufacturer and the retailer prefer bank financing.

(III) Furthermore, by comparing with the equilibrium financing strategy under deterministic demand, we obtain the following unexpected results. When the demand is deterministic, from Table 2, we find (1) If $r_t < r_b$, both the manufacturer and the retailer prefer trade credit financing. As a result, the equilibrium financing strategy is trade credit financing. (2) If $r_t \geq r_b$, the manufacturer prefers hybrid financing while the retailer prefers trade credit. In this case, the retailer is willing to offer credit to the manufacturer, while, the manufacturer, as the leader, prefers hybrid financing instead of trade credit. As a result, hybrid financing is the equilibrium financing strategy. In contrast, when the demand is uncertain, from the observations (I) and (II), we know that bank financing is the robust equilibrium financing strategy when the values of uncertainty perturbation parameters are medium or large.

In summary, the uncertainty perturbation parameters can affect the manufacturer's financing strategies. Specifically, when the values of uncertainty perturbation parameters are relatively small, the robust equilibrium financing strategy is either hybrid financing or trade credit financing dependent on the relationship between the bank rate and the credit rate, which is consistent with the results under deterministic demand. When the values of uncertainty perturbation parameters are medium or large, bank financing is the robust equilibrium financing strategy, which is different from the results under deterministic demand.

The impact of the equity ratio on manufacturer's financing strategy

In this subsection, we will explore how the equity financing ratio κ influences the manufacturer's financing strategy under uncertain market demand. We set $r_1 = 100, r_2 = 200, r_3 = 250, r_4 = 350, \theta_l^1 = \theta_l^2 = \theta_r^1 = \theta_r^1 = 20$, and the parameter κ varies from 0 to 1. Fig. 6(a) shows the comparison of the manufacturer's equilibrium profits under different financing strategies with different financing rates r_t and r_b. As is shown in Fig. 6(a), when $r_t \geq r_b$, $\pi_m^t < \pi_m^b$; when $r_t < r_b$, $\pi_m^t > \pi_m^b$. This is because the manufacturer's unit procurement cost is lower under trade credit than that under bank loan when $r_t < r_b$. About the relationship between π_m^b and π_m^h, whether equity financing can bring value to the manufacturer depends on various business factors. According to Fig. 6(a), when $\kappa = 0$, $\pi_m^h = \pi_m^b$, because in this case, the hybrid financing reduces to the bank loan financing. For relatively small κ, the manufacturer's profit first decreases, and then increases in the equity financing ratio κ. So $\pi_m^b > \pi_m^h$ when κ is relatively small. For relatively large equity financing ratio κ, the manufacturer's profit first increases, and then decreases in κ. Thus, $\pi_m^b < \pi_m^h$ when κ is relatively large which can be observed from Fig. 6(a). Fig. 6(a) also shows that π_m^h is always greater than π_m^t when κ is medium or large.

Fig. 6(b) illustrates the effect of the equity ratio κ on the retailer's profits under the three financing strategies. Fig. 6(b) shows that π_r^t is always greater than π_r^b. The retailer's profit in hybrid financing first decreases, then increases as κ increases. Therefore, for relatively small κ, the retailer's profit in hybrid financing is dominated by the other two strategies, otherwise, for relatively large κ, the profit in hybrid financing is higher.

Through comparing the observations obtained from Figs. 6(a) and (b), we give the final equilibrium financing strategy based on different relationships of r_t and r_b and different κ.

• If $r_t < r_b$, (1) when κ is relatively small, trade credit is a dominated strategy for both the manufacturer and the retailer; (2) when κ is medium, the retailer prefers trade credit, however, as the Stackelberg leader, the manufacturer prefers hybrid financing, that is, the manufacturer doesn't select trade credit even though the retailer is willing to provide trade credit. Therefore, hybrid financing is the equilibrium financing strategy in this case. (3)when κ is relatively large, both the manufacturer and the retailer prefer hybrid financing. Thus, the equilibrium financing strategy is hybrid financing.

• If $r_t \geq r_b$, (1) when κ is relatively small or medium, the retailer prefers trade credit, however, as the Stackelberg leader, the manufacturer prefers hybrid financing, that is, the manufacturer doesn't select trade credit even though the retailer is willing to provide trade credit. Therefore, hybrid financing is the equilibrium financing strategy in this case; (2) when κ is relatively large, both the manufacturer and the retailer prefer hybrid financing. As a result, hybrid financing is a equilibrium financing strategy.

From the discussion above, the equilibrium financing strategy is either trade credit or hybrid financing while bank loan financing can never be the dominated strategy. Specifically, when the equity financing ratio is small, the equilibrium financing strategy is either trade credit financing or hybrid financing dependent on the relationship of the credit rate and the bank rate. When the equity financing ratio is medium or large, the equilibrium financing strategy is independent of the financing rates and hybrid financing always dominates the other two financing strategies.

6.3. Managerial insights

From the above numerical analysis, we obtain the following managerial insights.

(1) From Table 1, we find that the robust equilibrium profits of the manufacturer under different values of perturbation parameters are different from the nominal equilibrium profit of the manufacturer. That is, the equilibrium profit of the manufacturer depends heavily on the distribution of the demand. When the exact possibility distribution of the demand is unavailable, the decision maker should not adopt the nominal equilibrium solutions, because a very small perturbation of the nominal possibility distribution can make a significant impact on the quality of nominal equilibrium solutions. In this case, the decision maker should apply the proposed distributionally robust optimization method to the pricing problem. The obtained robust equilibrium solutions are the uncertainty-immunized solutions under distribution uncertainty.

(2) By comparing the profits of the manufacturer and the retailer under the three financing strategies with different values of the equity ratio, we find that the manufacturer's equilibrium financing strategy is affected by both the equity ratio κ and the relationship of the rates r_t and r_b. That is, when the capital-constrained decision maker selects his appropriate financing strategy, he should take the equity ratio κ and the relationship of the rates r_t and r_b into account.

(3) From the comparison of the manufacturer's profits under the three financing strategies with different values of the perturbation parameters, the manufacturer's preferred financing strategy changes as the values of the perturbation parameters change, which is suggested that the manufacturer should not ignore the uncertainty of the demand when she makes her financing decisions.

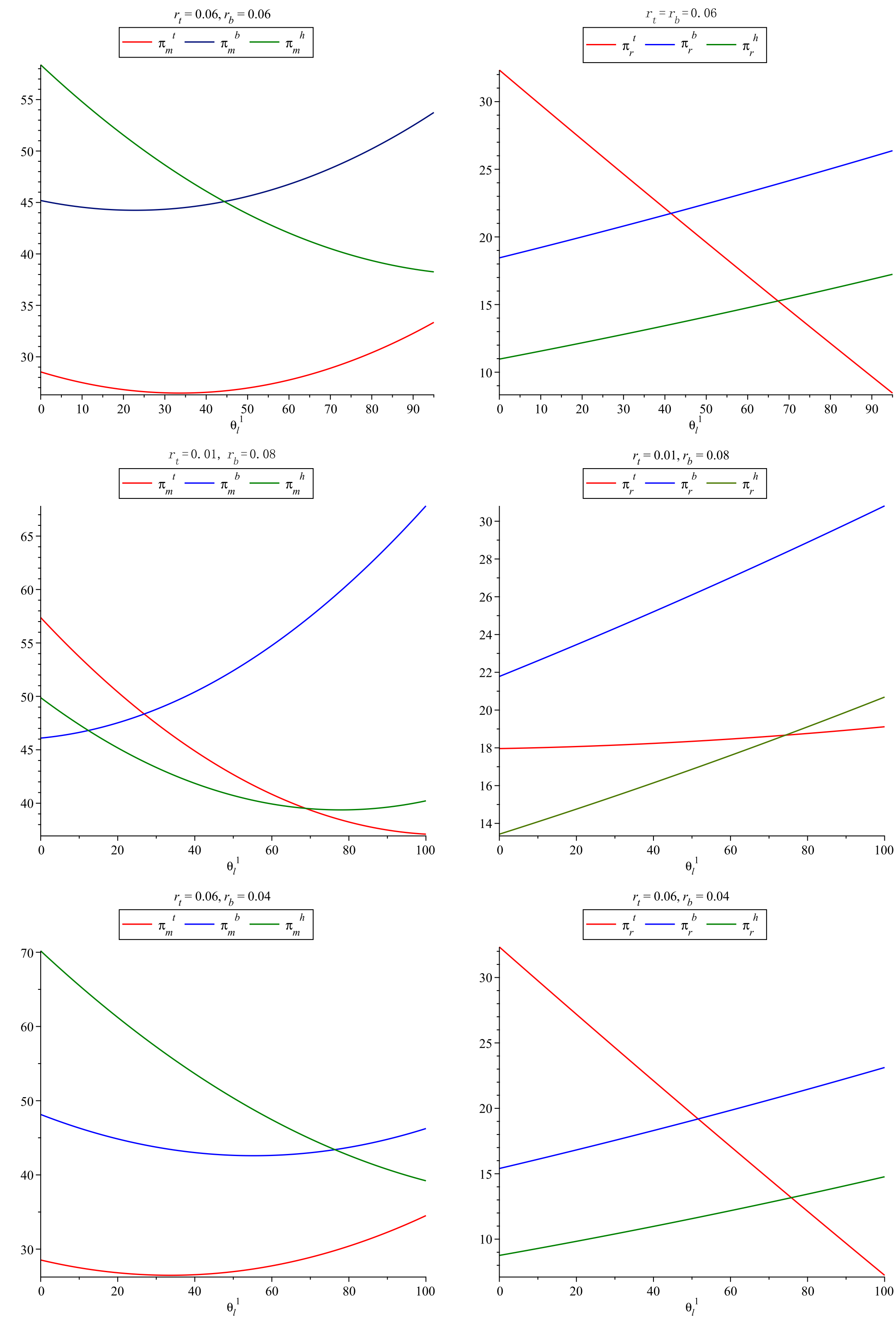

Fig. 5. Comparisons of manufacturer's and retailer's profits under different θ_l^1.

7. Conclusion

In this paper, a pricing and financing decision problem was studied under uncertain market demand in a capital-constrained DCSC. The major findings of the work can be summarized as the following three aspects.

• We studied the pricing and financing in a manufacturer-retailer DCSC where the upstream manufacturer is capital-constrained which is

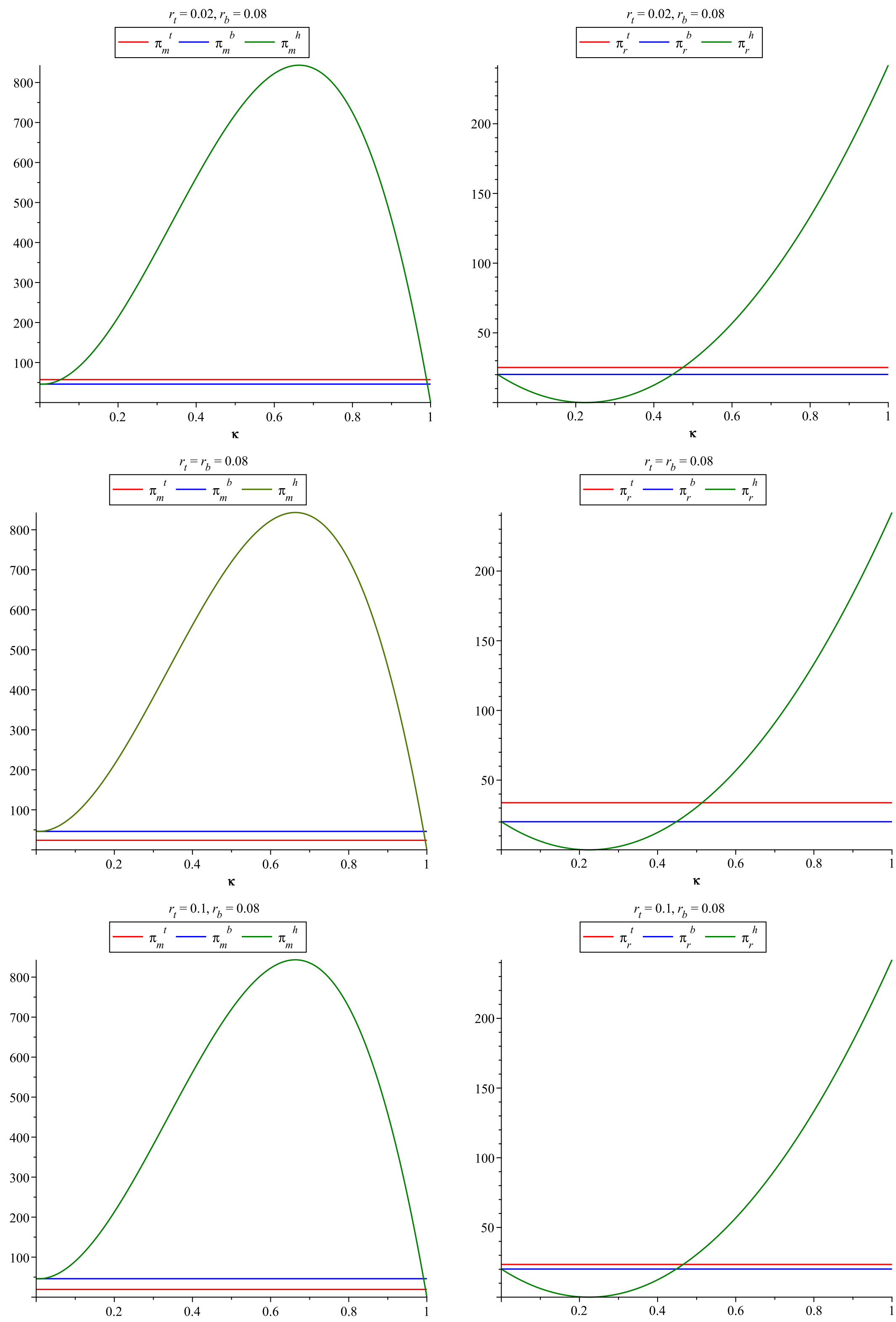

Fig. 6. Comparisons of manufacturer's and retailer's profits under different κ.

different from the existing literature. Three financing strategies including trade credit, bank loan and hybrid financing which is a combination of bank financing and equity financing were considered.

• We developed a novel distributionally robust bilevel optimization modeling framework for a capital-constrained DCSC under uncertain demand. The uncertain demand is characterized by a parametric level interval type-2 fuzzy variable. The variable possibility distribution of the uncertain demand varies on a bounded interval. In order to depict the perturbations of the distribution of the uncertain demand, a new uncertainty distribution set is introduced based on the type-2 fuzzy theory.

• The impacts of the equity financing ratio and the uncertainty perturbation parameters on the manufacturer's equilibrium financing strategy were investigated through numerical analysis. The analysis results show that the equilibrium financing strategy is either trade credit or hybrid financing while bank loan financing can never be the dominated strategy. The numerical results also indicate that the capital-constrained manufacturer may change her financing strategy when the values of the uncertainty perturbation parameters change. Some managerial insights were given to help the capital-constrained manufacturer make her informed financing decision.

CRediT authorship contribution statement

Huili Pei: Methodology, Data curation, Writing – original draft, Software. **Hongliang Li:** Conceptualization, Methodology, Validation. **Yankui Liu:** Visualization, Supervision, Validation, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix

Proof of Theorem 1

Proof. Based on the definition of the uncertainty distribution set $\mathcal{F}$ defined in (7), the expected profits of the manufacturer and the retailer can be calculated as follows

$$\begin{aligned}&\mathrm{E}[\pi_m(P_d,P_r,w;\xi^\lambda)]\\&=\left(\tfrac{\theta_r^1+\theta_r^2}{8}-\tfrac{\lambda(\theta_l^1+\theta_r^1+\theta_l^2+\theta_r^2)}{8}+\tfrac{r_1+r_2+r_3+r_4}{4}\right)[P_d(1-\gamma)+w\gamma-c]\\&\quad-\beta_1(P_d^2+wP_r)+\beta_2(P_rP_d+wP_d)+c(\beta_1-\beta_2)(P_r+P_d).\end{aligned} \tag{14}$$

$$\begin{aligned}\mathrm{E}[\pi_r(P_r;\xi^\lambda)]&=\gamma\left(\tfrac{\theta_r^1+\theta_r^2}{8}-\tfrac{\lambda(\theta_l^1+\theta_r^1+\theta_l^2+\theta_r^2)}{8}+\tfrac{r_1+r_2+r_3+r_4}{4}\right)(P_r-w)\\&\quad+(\beta_2P_d-\beta_1P_r)(P_r-w).\end{aligned} \tag{15}$$

Let

$$\begin{aligned}&L_1(P_d,P_r,w)\\&=\left(-\tfrac{\theta_l^1+\theta_l^2}{8}+\tfrac{r_1+r_2+r_3+r_4}{4}\right)[P_d(1-\gamma)+w\gamma-c]-\beta_1(P_d^2+wP_r)\\&\quad+\beta_2(P_rP_d+wP_d)+c(\beta_1-\beta_2)(P_r+P_d).\end{aligned}$$

$$L_2(P_r)=\gamma\left(-\tfrac{\theta_l^1+\theta_l^2}{8}+\tfrac{r_1+r_2+r_3+r_4}{4}\right)(P_r-w)+(\beta_2P_d-\beta_1P_r)(P_r-w).$$

The robust values of (14) and (15) can be represented as

$$\inf_{\mu_\xi\in\mathcal{F}}\mathrm{E}[\pi_m(P_d,P_r,w;\xi^\lambda)]=L_1(P_d,P_r,w).$$

$$\inf_{\mu_\xi\in\mathcal{F}}\mathrm{E}[\pi_r(P_r;\xi^\lambda)]=L_2(P_r).$$

The robust counterpart model (4) can be equivalently represented as follows

$$\begin{aligned}\max_{P_d,w,\hat U_m}\quad&\hat U_m,\\ \text{s.t.}\quad&L_1(P_d,P_r,w)\geq\hat U_m,\\&\max_{P_r,\hat U_r}\quad\hat U_r,\\&\text{s.t.}\quad L_2(P_r)\geq\hat U_r.\end{aligned} \tag{16}$$

From the expression of $L_2(P_r)$, we have the second order sufficient condition $\frac{\partial^2L_2}{\partial P_r^2}=-2\beta_1<0$, which ensures that unique optimal solution exists. For given P_d,w, the retailer's response function is derived from the first-order condition of $L_2(P_r)$.

$$\frac{\partial L_2}{\partial P_r}=0\Rightarrow P_r=\frac{\gamma\eta+w\beta_1+\beta_2P_d}{2\beta_1}.$$

Getting the response of the retailer, the manufacturer maximizes her profit and determines the optimal decisions P_d,w. The Hessian matrix associated with $L_1(P_d,P_r,w)$ is given by

$$H=\begin{pmatrix}\frac{\partial^2L_1}{\partial w^2}&\frac{\partial^2L_1}{\partial w\partial P_d}\\\frac{\partial^2L_1}{\partial P_d\partial w}&\frac{\partial^2L_1}{\partial P_d^2}\end{pmatrix}=\begin{pmatrix}-\beta_1&\beta_2\\\beta_2&-2\beta_1+\frac{\beta_2^2}{\beta_1}\end{pmatrix}.$$

Then, $|H|=2(\beta_1^2-\beta_2^2)>0$, H is negative definite. Thus, $L_1(P_d,P_r,w)$ is jointly concave in P_d and w. Using the first-order optimality condition

$$\begin{cases}\frac{\partial L_1}{\partial w}=0\\\frac{\partial L_1}{\partial P_d}=0\end{cases}$$

We get $w^*=\frac{\eta[\gamma\beta_1+(1-\gamma)\beta_2]+c(\beta_1^2-\beta_2^2)}{2(\beta_1^2-\beta_2^2)}$, $P_d^*=\frac{\eta[\gamma\beta_2+(1-\gamma)\beta_1]+c(\beta_1^2-\beta_2^2)}{2(\beta_1^2-\beta_2^2)}$. Substituting the values of P_d^* and w^* into the value of P_r, we get

$$P_r^*=\frac{\eta[2(1-\gamma)\beta_1\beta_2+\gamma(3\beta_1^2-\beta_2^2)]+c(\beta_1+\beta_2)(\beta_1^2-\beta_2^2)}{4\beta_1(\beta_1^2-\beta_2^2)}.\quad\square$$

Proof of Proposition 1

Proof. Taking the first-order derivatives of w^* and P_d^*,P_r^* with respect to $\theta_l^i, i=1,2$, we have

$$\frac{\partial w^*}{\partial\theta_l^i}=-\frac{\gamma\beta_1+(1-\gamma)\beta_2}{16(\beta_1^2-\beta_2^2)},$$

$$\frac{\partial P_d^*}{\partial\theta_l^i}=-\frac{\gamma\beta_2+(1-\gamma)\beta_1}{16(\beta_1^2-\beta_2^2)},$$

$$\frac{\partial P_r^*}{\partial\theta_l^i}=-\frac{2(1-\gamma)\beta_1\beta_2+\gamma(3\beta_1^2-\beta_2^2)}{32\beta_1(\beta_1^2-\beta_2^2)}.$$

By the assumptions $\beta_1>\beta_2, 0<\gamma<1$, we can easily derive that $\frac{\partial w^*}{\partial\theta_l^i}<0$, $\frac{\partial P_d^*}{\partial\theta_l^i}<0$ and $\frac{\partial P_r^*}{\partial\theta_l^i}<0$. $\square$

Proof of Theorem 2

Proof. According to the definition of the uncertainty distribution set $\mathcal{F}$ defined in (7), the expected profits of the manufacturer and the retailer in trade credit financing can be calculated as follows

$$
\begin{aligned}
&\mathrm{E}[\pi_m^t(P_d^t, P_r^t, w_t; \xi^\lambda)] \\
&= [\tfrac{\theta_r^1+\theta_r^2}{8} - \tfrac{\lambda(\theta_l^1+\theta_r^1+\theta_l^2+\theta_r^2)}{8} + \tfrac{r_1+r_2+r_3+r_4}{4}][P_d^t(1-\gamma) + w_t\gamma - c(1+r_t)] \\
&\quad - \beta_1[(P_d^t)^2 + w_t P_r^t] + \beta_2(P_r^t P_d^t + w_t P_d^t) \\
&\quad + c(\beta_1 - \beta_2)(P_r^t + P_d^t)(1+r_t).
\end{aligned} \tag{17}
$$

$$
\begin{aligned}
\mathrm{E}[\pi_r^t(P_r^t; \xi^\lambda)] \;=\; & [\gamma(P_r^t - w_t) + c r_t][\tfrac{\theta_r^1+\theta_r^2}{8} - \tfrac{\lambda(\theta_l^1+\theta_r^1+\theta_l^2+\theta_r^2)}{8} + \tfrac{r_1+r_2+r_3+r_4}{4}] \\
& + (\beta_2 P_d^t - \beta_1 P_r^t)(P_r^t - w_t) + c(\beta_2 - \beta_1)(P_r^t + P_d^t) r_t.
\end{aligned} \tag{18}
$$

Let

$$
\begin{aligned}
&L_1^t(P_d^t, P_r^t, w_t) \\
&= (-\tfrac{\theta_l^1+\theta_l^2}{8} + \tfrac{r_1+r_2+r_3+r_4}{4})[P_d^t(1-\gamma) + w_t\gamma - c(1+r_t)] \\
&\quad - \beta_1[(P_d^t)^2 + w_t P_r^t] \\
&\quad + \beta_2(P_r^t P_d^t + w_t P_d^t) + c(\beta_1 - \beta_2)(P_r^t + P_d^t)(1+r_t).
\end{aligned}
$$

$$
\begin{aligned}
L_2^t(P_r^t) \;=\; & [\gamma(P_r^t - w_t) + c r_t](-\tfrac{\theta_l^1+\theta_l^2}{8} + \tfrac{r_1+r_2+r_3+r_4}{4}) \\
& + (\beta_2 P_d^t - \beta_1 P_r^t)(P_r^t - w_t) + c(\beta_2 - \beta_1)(P_r^t + P_d^t) r_t.
\end{aligned}
$$

The robust values of (17) and (18) can be represented as

$$\inf_{\mu_\xi \in \mathcal{F}} \mathrm{E}[\pi_m^t(P_d^t, P_r^t, w_t; \xi^\lambda)] \;=\; L_1^t(P_d^t, P_r^t, \omega_t).$$

$$\inf_{\mu_\xi \in \mathcal{F}} \mathrm{E}[\pi_r^t(P_r^t; \xi^\lambda)] \;=\; L_2^t(P_r^t).$$

The robust counterpart model (9) can be equivalently represented as follows

$$
\begin{aligned}
\max_{P_d^t, w_t, \hat{U}_m^t} \quad & \hat{U}_m^t, \\
\text{s.t.} \quad & L_1^t(P_d^t, P_r^t, w_t) \ge \hat{U}_m^t \\
& \max_{P_r^t, \hat{U}_r^t} \quad \hat{U}_r^t, \\
& \text{s.t.} \quad L_2^t(P_r^t) \ge \hat{U}_r^t.
\end{aligned} \tag{19}
$$

The second order sufficient condition of $L_2^t(P_r^t)$ is $\frac{\partial^2 L_2^t}{\partial (P_r^t)^2} = -2\beta_1 < 0$, which ensures that unique optimal solution exists. For given P_d^t, w_t, the retailer's response function is derived from the first-order condition of $L_2^t(P_r^t)$.

$$\frac{\partial L_2^t}{\partial P_r^t} = 0 \Rightarrow P_r^t = \frac{\gamma\eta + w_t\beta_1 + \beta_2 P_d^t + c(\beta_2 - \beta_1) r_t}{2\beta_1}.$$

Getting the response of the retailer, the manufacturer maximizes her profit and determines the optimal decisions P_d^t, w_t. The Hessian matrix associated with $L_1^t(P_d^t, P_r^t, w_t)$ is given by

$$H = \begin{pmatrix} \frac{\partial^2 L_1^t}{\partial w_t^2} & \frac{\partial^2 L_1^t}{\partial w_t \partial P_d^t} \\ \frac{\partial^2 L_1^t}{\partial P_d^t \partial w_t} & \frac{\partial^2 L_1^t}{\partial (P_d^t)^2} \end{pmatrix} = \begin{pmatrix} -\beta_1 & \beta_2 \\ \beta_2 & -2\beta_1 + \frac{\beta_2^2}{\beta_1} \end{pmatrix}.$$

Then, $|H| = 2(\beta_1^2 - \beta_2^2) > 0$, H is negative definite. Thus, $L_1^t(P_d^t, P_r^t, w_t)$ is jointly concave in P_d^t and w_t. Using the first-order optimality condition

$$\begin{cases} \frac{\partial L_1^t}{\partial w_t} = 0 \\ \frac{\partial L_1^t}{\partial P_d^t} = 0 \end{cases}$$

We get $w_t^* = \frac{\eta\beta_1[\gamma\beta_1 + (1-\gamma)\beta_2] + c[\beta_1 + (2\beta_1 - \beta_2) r_t](\beta_1^2 - \beta_2^2)}{2\beta_1(\beta_1^2 - \beta_2^2)}$, $P_d^{t*} = \frac{\eta[\gamma\beta_2 + (1-\gamma)\beta_1] + c(1+r_t)(\beta_1^2 - \beta_2^2)}{2(\beta_1^2 - \beta_2^2)}$. Substituting the values of P_d^{t*} and w_t^* into the value of P_r^t, we get

$$P_r^{t*} = \frac{\eta[2(1-\gamma)\beta_1\beta_2 + \gamma(3\beta_1^2 - \beta_2^2)] + c(\beta_1 + \beta_2 + 2\beta_2 r_t)(\beta_1^2 - \beta_2^2)}{4\beta_1(\beta_1^2 - \beta_2^2)}.$$

where $\eta = -\frac{\theta_l^1+\theta_l^2}{8} + \frac{r_1+r_2+r_3+r_4}{4}$. □

Proof of Proposition 2

Proof. Taking the first-order derivatives of w_t^*, P_d^{t*} and P_r^{t*}, we get

$$\frac{\partial w_t^*}{\partial r_t} = \frac{c(2\beta_1 - \beta_2)}{2\beta_1}, \frac{\partial P_d^{t*}}{\partial r_t} = \frac{c}{2}, \frac{\partial P_r^{t*}}{\partial r_t} = \frac{c\beta_2}{2\beta_1}.$$

Based on the assumption $\beta_1 > \beta_2$, $\frac{\partial w_t^*}{\partial r_t} > 0$, $\frac{\partial P_d^{t*}}{\partial r_t} > 0$, $\frac{\partial P_r^{t*}}{\partial r_t} > 0$ can be obtained easily. □

Proof of Theorem 3

Proof. According to the definition of the uncertainty distribution set $\mathcal{F}$ defined in (7), the expected profits of the manufacturer and the retailer in bank loan financing can be calculated as follows

$$
\begin{aligned}
&\mathrm{E}[\pi_m^b(P_d^b, P_r^b, w_b; \xi^\lambda)] \\
&= [\tfrac{\theta_r^1+\theta_r^2}{8} - \tfrac{\lambda(\theta_l^1+\theta_r^1+\theta_l^2+\theta_r^2)}{8} + \tfrac{r_1+r_2+r_3+r_4}{4}][P_d^b(1-\gamma) + \omega_b\gamma - c(1+r_b)] \\
&\quad - \beta_1[(P_d^b)^2 + w_b P_r^b] + \beta_2(P_r^b P_d^b + w_b P_d^b) \\
&\quad + c(\beta_1 - \beta_2)(P_r^b + P_d^b)(1+r_b).
\end{aligned} \tag{20}
$$

$$
\begin{aligned}
&\mathrm{E}[\pi_r^b(P_r^b; \xi^\lambda)] \\
&= \gamma(P_r^b - w_b)[\tfrac{\theta_r^1+\theta_r^2}{8} - \tfrac{\lambda(\theta_l^1+\theta_r^1+\theta_l^2+\theta_r^2)}{8} + \tfrac{r_1+r_2+r_3+r_4}{4}] \\
&\quad + (\beta_2 P_d^b - \beta_1 P_r^b)(P_r^b - w_b).
\end{aligned} \tag{21}
$$

Let

$$
\begin{aligned}
&L_1^b(P_d^b, P_r^b, w_b) \\
&= (-\tfrac{\theta_l^1+\theta_l^2}{8} + \tfrac{r_1+r_2+r_3+r_4}{4})[P_d^b(1-\gamma) + w_b\gamma - c(1+r_b)] \\
&\quad - \beta_1[(P_d^b)^2 + w_b P_r^b] + \beta_2(P_r^b P_d^b + w_b P_d^b) \\
&\quad + c(\beta_1 - \beta_2)(P_r^b + P_d^b)(1+r_b).
\end{aligned}
$$

$$
\begin{aligned}
L_2^b(P_r^b) \;=\; & \gamma(P_r^b - w_b)(-\tfrac{\theta_l^1+\theta_l^2}{8} \\
& + \tfrac{r_1+r_2+r_3+r_4}{4}) + (\beta_2 P_d^b - \beta_1 P_r^b)(P_r^b - w_b).
\end{aligned}
$$

The robust values of (20) and (21) can be represented as

$$\inf_{\mu_\xi \in \mathcal{F}} \mathrm{E}[\pi_m^b(P_d^b, P_r^b, w_b; \xi^\lambda)] \;=\; L_1^b(P_d^b, P_r^b, w_b).$$

$$\inf_{\mu_\xi \in \mathcal{F}} \mathrm{E}[\pi_r^b(P_r^b; \xi^\lambda)] \;=\; L_2^b(P_r^b).$$

The robust counterpart model (11) can be equivalently represented as follows

$$
\begin{aligned}
\max_{P_d^b, w_b, \hat{U}_m^b} \quad & \hat{U}_m^b, \\
\text{s.t.} \quad & L_1^b(P_d^b, P_r^b, w_b) \ge \hat{U}_m^b \\
& \max_{P_r^b, \hat{U}_r^b} \quad \hat{U}_r^b, \\
& \text{s.t.} \quad L_2^b(P_r^b) \ge \hat{U}_r^b.
\end{aligned} \tag{22}
$$

The second order sufficient condition of $L_2^b(P_r^b)$ is $\frac{\partial^2 L_2^b}{\partial (P_r^b)^2} = -2\beta_1 < 0$, which ensures that unique optimal solution exists. For given P_d^b, w_b, the retailer's response function is derived from the first-order condition of $L_2^b(P_r^b)$.

$$\frac{\partial L_2^b}{\partial P_r^b} = 0 \Rightarrow P_r^b = \frac{\gamma\eta + w_b\beta_1 + \beta_2 P_d^b}{2\beta_1}.$$

Getting the response of the retailer, the manufacturer maximizes her profit and determines the optimal decisions P_d^b, w_b. The Hessian matrix associated with $L_1^b(P_d^b, P_r^b, w_b)$ is given by

$$H = \begin{pmatrix} \frac{\partial^2 L_1^b}{\partial w_b^2} & \frac{\partial^2 L_1^b}{\partial w_b \partial P_d^b} \\ \frac{\partial^2 L_1^b}{\partial P_d^b \partial w_b} & \frac{\partial^2 L_1^b}{\partial (P_d^b)^2} \end{pmatrix} = \begin{pmatrix} -\beta_1 & \beta_2 \\ \beta_2 & -2\beta_1 + \frac{\beta_2^2}{\beta_1} \end{pmatrix}.$$

Then, $|H| = 2(\beta_1^2 - \beta_2^2) > 0$, H is negative definite. Thus, $L_1^b(P_d^b, P_r^b, w_b)$ is jointly concave in P_d^b and w_b. Using the first-order optimality condition

$$\begin{cases} \frac{\partial L_1^b}{\partial w_b} = 0 \\ \frac{\partial L_1^b}{\partial P_d^b} = 0 \end{cases}$$

We get $w_b^* = \frac{\eta[\gamma\beta_1 + (1-\gamma)\beta_2] + c(\beta_1^2 - \beta_2^2)(1+r_b)}{2(\beta_1^2 - \beta_2^2)}$, $P_d^{b*} = \frac{\eta[\gamma\beta_2 + (1-\gamma)\beta_1] + c(\beta_1^2 - \beta_2^2)(1+r_b)}{2(\beta_1^2 - \beta_2^2)}$. Substituting the values of P_d^{b*} and w_b^* into the value of P_r^b, we get

$$P_r^{b*} = \frac{\eta[2\beta_1\beta_2(1-\gamma) + \gamma(3\beta_1^2 - \beta_2^2)] + c(\beta_1 + \beta_2)(\beta_1^2 - \beta_2^2)(1+r_b)}{4\beta_1(\beta_1^2 - \beta_2^2)}.$$

where $\eta = -\frac{\theta_l^1 + \theta_l^2}{8} + \frac{r_1 + r_2 + r_3 + r_4}{4}$. □

Proof of Proposition 3

Proof. Taking the first-order derivatives of w_b^*, P_d^{b*} and P_r^{b*}, we get

$$\frac{\partial w_b^*}{\partial r_b} = \frac{c}{2}, \frac{\partial P_d^{b*}}{\partial r_b} = \frac{c}{2}, \frac{\partial P_r^{b*}}{\partial r_b} = \frac{c(\beta_1 + \beta_2)}{4\beta_1},$$

and $\frac{\partial w_b^*}{\partial r_b} > 0, \frac{\partial P_d^{b*}}{\partial r_b} > 0, \frac{\partial P_r^{b*}}{\partial r_b} > 0$ can be obtained easily. □

Proof of Theorem 4

Proof. According to the definition of the uncertainty distribution set $\mathcal{F}$ defined in (7), the expected profits of the manufacturer and the retailer in hybrid financing can be calculated as follows

$$\begin{aligned} &\mathrm{E}[\pi_m^h(P_d^h, P_r^h, w_h; \xi^\lambda)] \\ &= [\tfrac{\theta_r^1 + \theta_r^2}{8} - \tfrac{\lambda(\theta_l^1 + \theta_r^1 + \theta_l^2 + \theta_r^2)}{8} + \tfrac{r_1 + r_2 + r_3 + r_4}{4}](1-\kappa)[P_d^h(1-\gamma) \\ &\quad + w_h\gamma - c(1+r_b)(1-\kappa)] - \beta_1(1-\kappa)[(P_d^h)^2 + w_h P_r^h] \\ &\quad + \beta_2(1-\kappa)(P_r^h P_d^h + w_h P_d^h) \\ &\quad + c(1-\kappa)^2(\beta_1 - \beta_2)(P_r^h + P_d^h)(1+r_b). \end{aligned} \tag{23}$$

$$\begin{aligned} \mathrm{E}[\pi_r^h(P_r^h; \xi^\lambda)] = \; & \gamma(P_r^h - w_h)[\tfrac{\theta_r^1 + \theta_r^2}{8} - \tfrac{\lambda(\theta_l^1 + \theta_r^1 + \theta_l^2 + \theta_r^2)}{8} + \tfrac{r_1 + r_2 + r_3 + r_4}{4}] \\ & + (\beta_2 P_d^h - \beta_1 P_r^h)(P_r^h - w_h). \end{aligned} \tag{24}$$

Let

$$\begin{aligned} &L_1^h(P_d^h, P_r^h, w_h) \\ &= (-\tfrac{\theta_l^1 + \theta_l^2}{8} + \tfrac{r_1 + r_2 + r_3 + r_4}{4})(1-\kappa)[P_d^h(1-\gamma) + w_h\gamma - c(1+r_b) \\ &\quad (1-\kappa)] - \beta_1(1-\kappa)[(P_d^h)^2 + w_h P_r^h] + \beta_2(1-\kappa)(P_r^h P_d^h + w_h P_d^h) \\ &\quad + c(1-\kappa)^2(\beta_1 - \beta_2)(P_r^h + P_d^h)(1+r_b). \end{aligned}$$

$$L_2^h(P_r^h) = \gamma(P_r^h - w_h)(-\tfrac{\theta_l^1 + \theta_l^2}{8} + \tfrac{r_1 + r_2 + r_3 + r_4}{4}) + (\beta_2 P_d^h - \beta_1 P_r^h)(P_r^h - w_h).$$

The robust values of (23) and (24) can be represented as

$$\inf_{\mu_\xi \in \mathcal{F}} \mathrm{E}[\pi_m^h(P_d^h, P_r^h, w_h; \xi^\lambda)] = L_1^h(P_d^h, P_r^h, w_h).$$

$$\inf_{\mu_\xi \in \mathcal{F}} \mathrm{E}[\pi_r^h(P_r^h; \xi^\lambda)] = L_2^h(P_r^h).$$

The robust counterpart model (13) can be equivalently represented as follows

$$\begin{aligned} \max_{P_d^h, w_h, \hat{U}_m^h} \quad & \hat{U}_m^h, \\ \text{s.t.} \quad & L_1^h(P_d^h, P_r^h, w_h) \geq \hat{U}_m^h \\ & \max_{P_r^h, \hat{U}_r^h} \quad \hat{U}_r^h, \\ & \text{s.t.} \quad L_2^h(P_r^h) \geq \hat{U}_r^h. \end{aligned} \tag{25}$$

The second order sufficient condition of $L_2^h(P_r^h)$ is $\frac{\partial^2 L_2^h}{\partial (P_r^h)^2} = -2\beta_1 < 0$, which ensures that unique optimal solution exists. For given P_d^h, w_h, the retailer's response function is derived from the first-order condition of $L_2^h(P_r^h)$.

$$\frac{\partial L_2^h}{\partial P_r^h} = 0 \Rightarrow P_r^h = \frac{\gamma\eta + w_h\beta_1 + \beta_2 P_d^h}{2\beta_1}.$$

Getting the response of the retailer, the manufacturer maximizes her profit and determines the optimal decisions P_d^{h*}, w_h. The Hessian matrix associated with $L_1^h(P_d^h, P_r^h, w_h)$ is given by

$$H = \begin{pmatrix} \frac{\partial^2 L_1^h}{\partial w_h^2} & \frac{\partial^2 L_1^h}{\partial w_h \partial P_d^h} \\ \frac{\partial^2 L_1^h}{\partial P_d^h \partial w_h} & \frac{\partial^2 L_1^h}{\partial (P_d^h)^2} \end{pmatrix} = \begin{pmatrix} -\beta_1 & \beta_2 \\ \beta_2 & -2\beta_1 + \frac{\beta_2^2}{\beta_1} \end{pmatrix}.$$

Then, $|H| = 2(\beta_1^2 - \beta_2^2) > 0$, H is negative definite. Thus, $L_1^h(P_d^h, P_r^h, w_h)$ is jointly concave in P_d^h and w_h. Using the first-order optimality condition

$$\begin{cases} \frac{\partial L_1^h}{\partial w_h} = 0 \\ \frac{\partial L_1^h}{\partial P_d^h} = 0 \end{cases}$$

We get $w_h^* = \frac{\eta[\gamma\beta_1 + (1-\gamma)\beta_2] + c(1-\kappa)(1+r_b)(\beta_1^2 - \beta_2^2)}{2(\beta_1^2 - \beta_2^2)}$, $P_d^{h*} = \frac{\eta[\gamma\beta_2 + (1-\gamma)\beta_1] + c(1-\kappa)(1+r_b)(\beta_1^2 - \beta_2^2)}{2(\beta_1^2 - \beta_2^2)}$,. Substituting the values of P_d^{h*} and w_h^* into the value of P_r^h, we get

$$P_r^{h*} = \frac{\eta[2\beta_1\beta_2(1-\gamma) + \gamma(3\beta_1^2 - \beta_2^2)] + c(1-\kappa)(1+r_b)(\beta_1 + \beta_2)(\beta_1^2 - \beta_2^2)}{4\beta_1(\beta_1^2 - \beta_2^2)},$$

where $\eta = -\frac{\theta_l^1 + \theta_l^2}{8} + \frac{r_1 + r_2 + r_3 + r_4}{4}$. □

Proof of Proposition 4

Proof. Taking the first-order derivatives of w_h^*, P_d^{h*} and P_r^{h*}, we get

$$\frac{\partial w_h^*}{\partial \kappa} = -\frac{c(1+r_b)}{2}, \frac{\partial P_d^{h*}}{\partial \kappa} = -\frac{c(1+r_b)}{2}, \frac{\partial P_r^{h*}}{\partial \kappa} = -\frac{c(1+r_b)(\beta_1 + \beta_2)}{4\beta_1},$$

and $\frac{\partial w_h^*}{\partial \kappa} < 0, \frac{\partial P_d^{h*}}{\partial \kappa} < 0, \frac{\partial P_r^{h*}}{\partial \kappa} < 0$ can be obtained easily. □

Proof of Theorem 5

Proof. In accordance with Theorem 3 and Theorem 4, we have $w_b^* - w_h^* = \frac{\kappa c(1+r_b)}{2} > 0$. Therefore, $w_b^* > w_h^*$.

In accordance with Theorem 2 and Theorem 3, we have $w_b^* - w_t^* = \frac{c[\beta_1 r_b - (2\beta_1 - \beta_2)r_t]}{2\beta_1}$, when $0 < r_t < \frac{\beta_1 r_b}{2\beta_1 - \beta_2} = r_t^2$, $w_b^* > w_t^*$; when $r_t^2 < r_t < 1$, $w_b^* < w_t^*$.

In accordance with Theorem 2 and Theorem 4, we have $w_t^* - w_h^* = \frac{(2\beta_1 - \beta_2)r_t - \beta_1(r_b - \kappa - \kappa r_b)}{2\beta_1}$, when $\kappa_0 = \frac{r_b}{1+r_b} < \kappa < 1$, the numerator is greater than 0 which results in $\frac{(2\beta_1 - \beta_2)r_t - \beta_1(r_b - \kappa - \kappa r_b)}{2\beta_1} > 0$, thus, $w_t^* > w_h^*$. When $0 < \kappa < \kappa_0 = \frac{r_b}{1+r_b}$, (1) if $0 < r_t < r_t^1 = \frac{\beta_1(r_b - \kappa - \kappa r_b)}{2\beta_1 - \beta_2}$, then $w_t^* < w_h^*$; (2) if $r_t^1 = \frac{\beta_1(r_b - \kappa - \kappa r_b)}{2\beta_1 - \beta_2} < r_t < 1$, then $w_t^* > w_h^*$. By comparing r_t^2 with r_t^1, we can easily derive $r_t^2 > r_t^1$. Based on the analysis above, the comparing results about $w_k^*, k = t, b, h$ can be obtained. □

Proof of Theorem 6

Proof. In accordance with Theorem 3 and Theorem 4, we have $P_d^{b*} - P_d^{h*} = \frac{\kappa c(1+r_b)}{2} > 0$. Therefore, $P_d^{b*} > P_d^{h*}$.

In accordance with Theorem 2 and Theorem 3, we have $P_d^{b*} - P_d^{t*} = \frac{c[r_b - r_t]}{2}$, when $0 < r_t < r_b = r_t^4$, $P_d^{b*} > P_d^{t*}$; when $r_t^4 < r_t < 1$, $P_d^{b*} < P_d^{t*}$.

In accordance with Theorem 2 and Theorem 4, we have $P_d^{t*} - P_d^{h*} = \frac{c(r_t - (r_b - \kappa - \kappa r_b))}{2}$, when $\kappa_0 = \frac{r_b}{1+r_b} < \kappa < 1$, the numerator is greater than 0 which results in $\frac{c(r_t - (r_b - \kappa - \kappa r_b))}{2} > 0$, thus, $P_d^{t*} > P_d^{h*}$. When $0 < \kappa < \kappa_0 = \frac{r_b}{1+r_b}$, (1) if $0 < r_t < r_t^3 = r_b - \kappa - \kappa r_b$, then $P_d^{t*} < P_d^{h*}$; (2) if $r_t^3 < r_t < 1$, then $P_d^{t*} > P_d^{h*}$. By comparing r_t^4 with r_t^3, we can easily derive $r_t^4 > r_t^3$. Based on the analysis above, the comparing results about $P_d^{k*}, k = t, b, h$ can be obtained. □

Proof of Theorem 7

Proof. In accordance with Theorem 3 and Theorem 4, we have $P_r^{b*} - P_r^{h*} = \frac{\kappa c(1+r_b)(\beta_1+\beta_2)}{4\beta_1} > 0$. Therefore, $P_r^{b*} > P_r^{h*}$.

In accordance with Theorem 2 and Theorem 3, we have $P_r^{b*} - P_r^{t*} = \frac{c[(\beta_1+\beta_2)r_b - 2\beta_2 r_t]}{4\beta_1}$, when $0 < r_t < \frac{(\beta_1+\beta_2)r_b}{2\beta_2} = r_t^6$, $P_r^{b*} > P_r^{t*}$; when $r_t^6 < r_t < 1$, $P_r^{b*} < P_r^{t*}$.

In accordance with Theorem 2 and Theorem 4, we have $P_r^{t*} - P_r^{h*} = \frac{(2\beta_2)r_t - (\beta_1+\beta_2)(r_b - \kappa - \kappa r_b)}{4\beta_1}$, when $\kappa_0 = \frac{r_b}{1+r_b} < \kappa < 1$, the numerator is greater than 0 which results in $\frac{(2\beta_2)r_t - (\beta_1+\beta_2)(r_b - \kappa - \kappa r_b)}{4\beta_1} > 0$, thus, $P_r^{t*} > P_r^{h*}$. When $0 < \kappa < \kappa_0 = \frac{r_b}{1+r_b}$, (1) if $0 < r_t < r_t^5 = \frac{(\beta_1+\beta_2)(r_b - \kappa - \kappa r_b)}{2\beta_2}$, then $P_r^{t*} < P_r^{h*}$; (2) if $r_t^5 < r_t < 1$, then $P_r^{t*} > P_r^{h*}$. By comparing r_t^6 with r_t^5, we can easily derive $r_t^6 > r_t^5$. Based on the analysis above, the comparing results about $P_r^{k*}, k = t, b, h$ can be obtained. □

Proof of Theorem 8

Proof. Based on Theorem 1–Theorem 4, we have $w_t^* - w^* = \frac{c r_t(2\beta_1 - \beta_2)}{2\beta_1} > 0$, $w_b^* - w^* = \frac{c r_b}{2} > 0$, and $w_h^* - w^* = \frac{c(r_b - \kappa - \kappa r_b)}{2} > 0$ when $0 < \kappa < \frac{r_b}{1+r_b}$. Thus, $w_k^* > w^*$, where $k = t, b, h$ and $0 < \kappa < \frac{r_b}{1+r_b}$.

Also, $P_d^{t*} - P_d^* = \frac{c r_t}{2} > 0$, $P_d^{b*} - P_d^* = \frac{c r_b}{2} > 0$, and $P_d^{h*} - P_d^* = \frac{c(r_b - \kappa - \kappa r_b)}{2} > 0$ when $0 < \kappa < \frac{r_b}{1+r_b}$. Thus, $P_d^{k*} > P_d^*$, where $k = t, b, h$ and $0 < \kappa < \frac{r_b}{1+r_b}$.

Finally, $P_r^{t*} - P_r^* = \frac{c\beta_2 r_t}{2\beta_1} > 0$, $P_r^{b*} - P_r^* = \frac{c r_b(\beta_1+\beta_2)}{4\beta_1} > 0$, and $P_r^{h*} - P_r^* = \frac{c(\beta_1+\beta_2)(r_b - \kappa - \kappa r_b)}{4\beta_1} > 0$ when $0 < \kappa < \frac{r_b}{1+r_b}$. Thus, $P_r^{k*} > P_r^*$, where $k = t, b, h$ and $0 < \kappa < \frac{r_b}{1+r_b}$. □

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