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Sustainable supplier selection and order allocation: Distributionally robust goal programming model and tractable approximation

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ABSTRACT

A challenge of planning real-life sustainable supplier selection and order allocation (SS/OA) problems for purchasing companies is to gather the extensive data and exact distributions of input data. Motivated by this challenge, this paper addresses imprecise probability distributions of uncertain per unit cost, CO_2 emissions, demand, supply capacity and the minimum order quality and characterizes these distribution uncertainty by ambiguity sets including the true distributions. Moreover, the purchasing company decisions to be optimized simultaneously include four conflicting objectives regarding cost, CO_2 emissions, society and suppliers' comprehensive value while considering risk measures incurred by cost and emissions to achieve company sustainability. To help decision makers formulate practicable policy, a novel distributionally robust sustainable SS/OA goal programming model is developed with expected constraints and joint chance constraints. The proposed optimization model can balance multiple conflicting objectives, and effectively solve our sustainable SS/OA problem. More importantly, we structure ambiguous distributions sets, and thus derive the computationally tractable approximation form of the proposed practical model. Finally, we illustrate our optimization method through a case study about a steel company, conduct a thorough inquiry into the effect of uncertainty and summarize the management implications of the results.

1. Introduction

The SS/OA problem, broadly speaking, refers to selecting the best number of suppliers while simultaneously finding the rational order allocation scheme among the selected suppliers based on a multiple-sourcing policy on the premise of meeting the specific requirements and limitations of suppliers and the purchasing company (Aissaoui, Haouari, & Hassini, 2007). In today's competitive global market, one of the essential requirements of companies is to make critical decisions in order to improve their performances and services (Nazari-Shirkouhi, Shakouri, Javadi, & Keramati, 2013; Chiu & Chiou, 2016; Ghorabaee, Amiri, Zavadskas, & Turskis, 2017). Selection of appropriate suppliers and allocation of orders among the assigned suppliers are strategic decisions in supply network management, that may affect successive decisions about the quality and price of company's final products (Ghadimi, Toosi, & Heavey, 2018). In this regard, addressing the SS/OA problem has become one of the most crucial activities for a company and can substantially impact other processes in managing the supply network (Nazari-Shirkouhi et al., 2013; Vahidi, Ali Torabi, & Ramezankhani, 2018; Kellner & Utz, 2019).

Recent increasing awareness and demands of a sustainable supply chain require that today's global business environment transforms the traditional SS/OA problem into a sustainable SS/OA problem by incorporating certain factors (e.g., CO_2 emissions (Coyle, Thomchick, & Ruamsook, 2015; Hamdan & Cheaitou, 2017; Hamdan & Cheaitou, 2017), society (Coyle et al., 2015), suppliers' comprehensive value (Ghadimi et al., 2018; Vahidi et al., 2018; Kellner & Utz, 2019; Xu, Qin, Liu, & Martínez, 2019)) related to sustainable development (Azadnia, Saman, & Wong, 2015; Mohammed, Harris, & Kannan, 2019). Nazari-Shirkouhi et al. (2013) noted that selecting sustainable suppliers and properly allocate orders among the contracted suppliers can significantly reduce costs, improve social satisfaction, reduce CO_2 emissions and enhance the competitiveness of the purchasing company. Under such policy requirements, companies committed to long-term and sustainable development must improve their comprehensive performance and quality as well as reduce costs and environmental pollution to actualize sustainability (Chiu & Chiou, 2016).

In respond to sustainability, companies need to consider some non-monetary criteria related to CO_2 emissions, society and the suppliers' comprehensive value in addition to the monetary criteria (Kellner &

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Utz, 2019). Hence, sustainable SS/OA usually presents a multi-objective programming problem. In this case, from the perspective of companies' sustainability and macro policy, this paper attempts to optimize four goals over cost, CO_2 emissions, social aspects and suppliers' comprehensive value. Multiple conflicting goals are almost impossible to achieve at the same time, which may lead to increased complexity of this problem. Accordingly, effective exploration of the balance between multiple goals represents an issue to be handled in this paper.

The optimization process in the sustainable multiple-objective programming problem is often challenged by various uncertain information (Ghorabaee et al., 2017), such as the per unit purchasing cost, CO_2 emissions and demand. Quite a few researches have considered that uncertain parameters were provided with known distributions (e.g., Liu, Gao, & Ma, 2019; Mari, Memon, Ramzan, Qureshi, & Iqbal, 2019; Shadkam & Bijari, 2017). In real-world sustainable SS/OA, decision makers are often unable to obtain accurate distributions of parameters due to various uncertainties. This paper is informed by the imprecise distributions of per unit cost, CO_2 emissions, demand, supply capacity and minimum order quality. Under these circumstances, the conventional methods with fixed distributions (e.g., fuzzy sets, probability distributions and stochastic optimization) cannot handle the problem with imprecise distributions. This will pose a challenge to be solved in this paper, because imprecise distributions may make the decision makers lose control in planning the problem.

The above discussions raise the motivation of this paper with regard to studying what approach decision makers should employ to optimize the sustainable SS/OA problem to enable sustainability of the purchasing company. The existence of multiple objectives and distribution uncertainty makes it difficult for decision makers to make optimal decisions. Therefore, for the long-term development of the company, it is necessary to provide a quantitative method for decision makers to formulate the appropriate scheme. To our best knowledge, there is no comprehensive method for solving the sustainable SS/OA problem that simultaneously integrating numerous imprecise probability distributions and multiple conflicting goals in previous works. To this end, this paper aims to address the following three questions: (Q1) How should imprecise distributions of uncertain parameters be depicted in the planning procedure? (Q2) How should a comprehensive and efficient model be built to integrate distribution uncertainty and balance multiple conflicting objectives? (Q3) How can a computationally tractable formulation of the proposed model be derived, and how does the proposed new approach address the case of sustainable SS/OA in practice? In the following sections, we will unpack the study by answering these three questions.

The remainder of this paper is organized as follows. Section 2 systematically reviews the relevant literature. Section 3 explains the problem in detail and puts forward a distributionally robust goal programming model. Section 4 derives a safe approximation model of the proposed model. Section 5 conducts a case study about a steel company to illustrate the effectiveness of the model. Section 6 draws some conclusions and future research directions.

2. Literature review

Sustainable supplier selection and proper order allocation have been key to each purchasing company's sustainable development (Ghorabaee et al., 2017). The sustainable SS/OA problem includes two distinct features, mainly multiple conflicting objectives and various uncertainties. Our work contributes to the reviewed relevant literature in three aspects, including multiple conflicting objectives, uncertainty and optimization method.

2.1. Multiple conflicting objectives in the sustainable SS/OA problem

Multiple conflicting objectives in sustainable SS/OA problems have been introduced in order to fulfil the sustainability of companies in recent years. The paper by Kannan, Khodaverdi, Olfat, Jafarian, and Diabat (2013) designated two conflicting objectives to select green suppliers and order allocation, in which the objectives are simultaneously maximization of the total purchasing value and minimization of the total purchasing cost. Subsequently, Vahidi et al. (2018) considered simultaneously optimizing the total sustainability and resilience scores of the selected suppliers and the total expected cost in planning sustainable SS/OA, and Ghadimi et al. (2018) and Hamdan and Cheaitou (2017); Hamdan and Cheaitou, 2017 considered the suppliers' sustainability and total cost. Both studies (Gören, 2018 & Mirzaee et al., Mirzaee, Naderi, & Pasandideh, 2018) included the total costs and the total purchasing value in the sustainable SS/OA model. Moreover, the environmental impact related to suppliers was considered as an objective in addition to the total cost in a previous study (Govindan, Jafarian, & Nourbakhsh, 2015). After that, Babbar and Amin (2018) took into account various objectives, including total costs, defect rate, environment, weights of suppliers and on-time delivery. Mohammed, Setchi, Filip, Harris, and Li (2018) simultaneously optimized the economy (cost), environmental impact, social impact of suppliers, travel time of all livestock and criteria weights of all selected suppliers in sustainable SS/OA problems, and Mohammed et al. (2019) considered optimizing the cost, carbon emission, social impact and suppliers' purchasing value. Moheb-Alizadeha and Handfield (2019) assessed three objective related to the total cost, CO_2 emissions and social responsibility.

Although the number of references that model sustainable multiple decisions has significantly increased in recent years, few papers addressed four multiple conflicting objectives, including cost, CO_2 emissions, society and suppliers' comprehensive value at the same time. This paper extends the reference (Mohammed et al., 2019) to optimizing total cost, CO_2 emissions, social impact and suppliers' comprehensive value under distribution uncertainty, which supports decision makers planning the sustainability of purchasing companies. Moreover, in this line of work, the distinguishing aspect of our paper is that we incorporate risk measures about cost and CO_2 emissions into our sustainable SS/OA model while considering four conflicting objectives, which can effectively resist significant deviation from expected goal levels and avoid high risks for decision makers.

2.2. Uncertainty in the sustainable SS/OA problem

In the process of planning actual sustainable SS/OA, decision makers often encounter vague and imprecise input data. Several references discussed the uncertain input data for sustainable SS/OA problems. Babbar and Amin (2018) posited that the per unit cost and demand are provided with stochastic uncertainty in a sustainable SS/OA problem, in which these uncertain parameters were characterized by stochastic scenarios, and Vahidi et al. (2018) assumed that the capacity of each supplier was based on stochastic scenarios. In reference (Mohammed et al., 2019), the per unit purchasing cost, transportation cost, supply capacity of suppliers, demand and CO_2 emissions were assumed as uncertain parameters and obeyed fuzzy triangular distributions. Mirzaee et al. (2018) set the aspiration levels for total costs and purchasing value of suppliers as fuzzy variables that obeyed triangular distributions in a sustainable SS/OA problem. In Kannan et al. (2013), the aspiration levels of total purchasing cost and total purchasing value were also considered as fuzzy variables and characterized by membership functions. Moghaddam (2015) described the uncertain supply and demand as fuzzy variables in a sustainable SS/OA problem and depicted the uncertainty by linear membership functions. Govindan et al. (2015) investigated SS/OA under stochastic demand, and the type of uncertainty was integrated into the model based on the scenario.

Although the above references considered the uncertainty in planning sustainable SS/OA, these documents characterized the uncertainty by fixed distributions, ignoring the imprecision of the distribution; that is, no references analysed the distribution uncertainty. The key

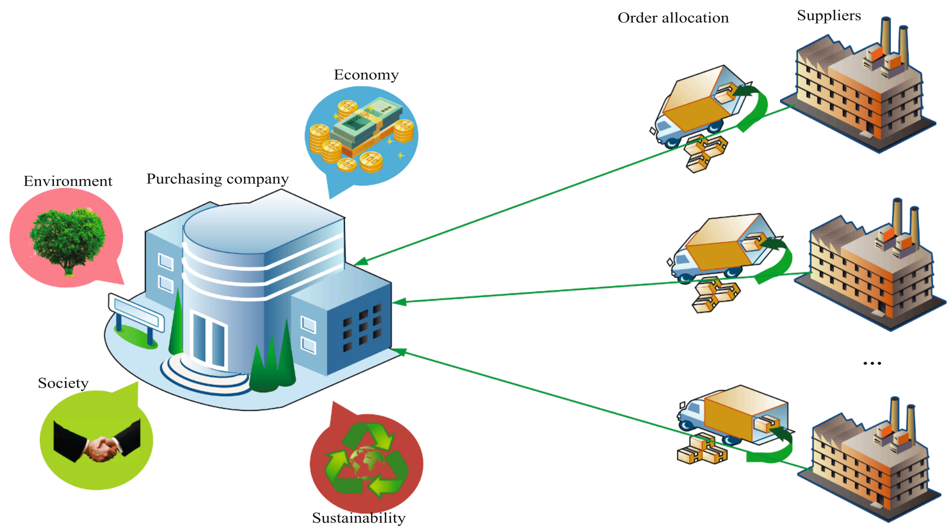


Fig. 1. Graphical representation of sustainable SS/OA problem.

difference is that this paper assumes that the available distribution information of uncertain input data is often partial rather than accurate. The advantage of our work is that it is well suited to situations where distribution information of uncertain input data is not fully available.

2.3. Optimization methods in the sustainable SS/OA problem

Because of the multiple conflicting objectives and uncertainty in sustainable SS/OA problems, choosing the appropriate optimization method is a challenging task. Our work can be classified as a study on handling uncertain distributions of input data and multiple conflicting objectives, which is different from the following references. Babbar and Amin (2018) proposed a two phase model, namely, a two-stage quality function deployment, and a stochastic multi-objective mathematical model, in which the stochastic (scenario) approach helped manage the uncertainty in the order allocation process, and trapezoidal fuzzy numbers were utilized to handle the vagueness of human thoughts. In reference (Mohammed et al., 2018), a multi-objective programming model was formulated to obtain the optimal order allocations in order to minimize the costs, the environmental impact (particularly CO_2 emissions), the travel time of products and to maximize the social impact and the total purchasing value, and the fuzzy optimization method was used to handle uncertainty. In Mohammed et al. (2019), a hybrid multi-criteria decision-making fuzzy multi-objective optimization was advanced for a sustainable SS/OA problem by considering economic, environmental and social aspects, and uncertain input data were characterized by fuzzy numbers. Bai (2015) formulated a credibility-based bi-objective fuzzy optimization model to address a supplier selection problem. Mirzaee et al. (2018) developed a preemptive fuzzy goal programming approach for supplier selection and order allocation under multiple objectives, and considered the uncertain aspiration levels to satisfy triangular fuzzy distributions. Moreover, Kannan et al. (2013) applied an integrated approach of fuzzy multi-attribute utility theory and multi-objective programming to plan SS/OA with multiple goals and uncertainty. Moghaddam (2015) presented a fuzzy multi-objective mathematical model to identify and rank the candidate suppliers and find the optimal numbers of new and refurbished parts and final products in a reverse logistics network configuration under uncertainty.

Based on a review of relevant literature available, fuzzy multi-objective programming or stochastic multi-objective programming is often adopted to study these kind of problems. However, these methods cannot address the distribution uncertainty in this paper. Under this observation, we intend to support decision makers by proposing a new distributionally robust goal programming model to plan the sustainable SS/OA problem involving multiple conflicting objectives and distribution uncertainty. Our performance method is based on the idea of goal programming (Charnes & Cooper, 1961) and distributionally robust optimization method (Scarf, 1958; Žáčková, 1966). Distributionally robust optimization can be considered as a combination of robust optimization (Ben-Tal & Hochman, 1972; Ben-Tal, Ghaoui, & Nemirovski, 2009) and stochastic optimization. For thorough coverage of developments and recent advances in robust optimization, the interested reader can refer to Gabrel, Murat, and Thiele (2014). This method is also utilized to study some other works such as supply chain network design (Ma & Du, 2018), sustainable development problem (Bai, Li, Jia, & Liu, 2019; Jia, Bai, Song, & Liu, 2019), transportation problems (Zhang & Yang, 2018) and portfolio optimization problems (Jia & Bai, 2018), thus demonstrating the advantages of the distributionally robust optimization method in solving uncertain problems. Accordingly, under the idea of distributionally robust optimization, this paper can provide a computationally tractable formulation for this distributionally robust goal programming model under ambiguity sets. Compared with the existing literature, our model supports the decision makers under distribution uncertainty to optimize cost, CO_2 emissions, society and suppliers' comprehensive value.

3. Distributionally robust sustainable SS/OA goal programming model

3.1. Problem statement

In this paper, a sustainable SS/OA problem is studied under an uncertain environment. The sustainable SS/OA problem, involving a centralized supply chain with a purchasing company and multiple suppliers, includes four conflicting goals related to sustainability in terms of cost, CO_2 emissions, society and suppliers' comprehensive value. Fig. 1 briefly states this problem. Before making a decision, the true distributions of per unit purchasing cost, unit transportation cost, CO_2 emissions, demand, supply capacity and acceptable minimum order quantity from suppliers are not exactly captured, which leads to more complicated decision making. To this end, this paper aims to formulate a comprehensive method–the distributionally robust goal programming model developed in Section 3.2.

3.2. Model development

The purpose of this paper is to solve a sustainable SS/OA problem involving multiple conflicting objectives under uncertain distribution. In the following, we build a distributionally robust goal programming model including four conflicting objectives: cost, CO_2 emissions, society and suppliers' comprehensive value.

We first provide the basic formulations related to the goals under uncertain distribution:

$$F_1(q, x, \zeta^p, \zeta^t) = \sum_{j \in J} C_j^{pur}(\zeta_j^p) q_j + \sum_{j \in J} C_j^{adm} x_j + \sum_{j \in J} C_j^{tran}(\zeta_j^t) \left\lceil \frac{q_j}{TC} \right\rceil Dis_j \tag{1}$$

represents the total cost, including the purchasing cost $\sum_{j \in J} C_j^{pur}(\zeta_j^p) q_j$, administration cost $\sum_{j \in J} C_j^{adm} x_j$ and transportation cost $\sum_{j \in J} C_j^{tran}(\zeta_j^t) \left\lceil \frac{q_j}{TC} \right\rceil Dis_j$, where $C_j^{pur}(\zeta_j^p)$ denotes the uncertain unit purchasing cost of the product order from supplier j, C_j^{adm} denotes the administration cost per order from supplier j, $C_j^{tran}(\zeta_j^t)$ denotes the uncertain unit transportation cost per mile from supplier j, TC represents the unit transportation capacity per lorry, Dis_j represents the transportation distance (in mile) of products from supplier j to the factory, decision variable q_j denotes the number of products ordered from supplier j, and decision variable $x_j = 1$ if supplier j is selected and $x_j = 0$ otherwise. Moreover, the uncertain unit purchasing cost $C_j^{pur}(\zeta_j^p)$ and unit transportation cost $C_j^{tran}(\zeta_j^t)$ are parameterized by random variables ζ_j^p and ζ_j^t. The specific parameterized forms are shown in Section 4.

$$F_2(q, \zeta^c) = \sum_{j \in J} CO_{2j}^{tran}(\zeta_j^c) \left\lceil \frac{q_j}{TC} \right\rceil Dis_j \tag{2}$$

represents the total CO_2 emissions in the process of transportation, where $CO_2^{tran}(\zeta_j^c)$ denotes the uncertain CO_2 emission in grams per mile for each lorry travelling from supplier j, and $CO_2^{tran}(\zeta_j^c) = (CO_2{}^{tran})^0 + (CO_2{}^{tran})^1 \zeta_j^c$.

$$F_3(q) = \sum_{j \in J} w_j^{soc} q_j \tag{3}$$

is an expression of social influence, where w_j^{soc} represents the performance coefficient of supplier j with respect to the social criteria.

$$F_4(q) = W^{eco}\left(\sum_{j \in J} w_j^{eco} q_j\right) + W^{env}\left(\sum_{j \in J} w_j^{env} q_j\right) + W^{soc}\left(\sum_{j \in J} w_j^{soc} q_j\right) \tag{4}$$

is a representation concerning the suppliers' comprehensive value as characterized by the comprehensive contribution of suppliers under three sets of criteria related to the economy, environment and society. The three sets of criteria will be provided in Section 5.1, so that the decision makers can select suppliers with better comprehensive value. Parameter w_j^{eco} represents the performance coefficient of supplier j with respect to the economic criteria, and w_j^{env} denotes the performance coefficient of supplier j with respect to the environmental criteria. Moreover, W^{eco}, W^{env} and W^{soc} represent weights of a set of economic criteria, a set of environmental criteria and a set of social criteria, respectively. The determinations of the weights and the performance coefficients are multi-criteria decision-making processes. They are calculated by using the three sets of criteria about economy, environment and society in Section 5.1. In the literature, a few researchers have described the comprehensive value in a similar way, e.g., Ghadimi et al. (2018),ohammed et al. (2019).

Based on the above basic formulations of the multiple conflicting goals, we build a new distributionally robust goal programming model with expected constraints and joint chance constraints for the sustainable SS/OA problem as follows:

$$\text{Objective} \quad \min P_1 d_1^+ + P_2 d_2^+ + P_3 d_3^- + P_4 d_4^- \tag{5}$$

s. t.

$$\textbf{Cost} \quad \mathbb{E}_{\zeta^p, \zeta^t \sim \mathbb{P}}(F_1(q, x, \zeta^p, \zeta^t)) - d_1^+ \leqslant g_1, \quad \forall\, \mathbb{P} \in \mathcal{P}_\zeta \tag{6}$$

$$\textbf{CO}_\mathbf{2}\ \textbf{emissions} \quad \mathbb{E}_{\zeta^c \sim \mathbb{P}}(F_2(q, \zeta^c)) - d_2^+ \leqslant g_2, \quad \forall\, \mathbb{P} \in \mathcal{P}_\zeta \tag{7}$$

$$\text{Society} \quad F_3(q) + d_3^- \geqslant g_3 \tag{8}$$

$$\text{Suppliers' comprehensive value} \quad F_4(q) + d_4^- \geqslant g_4 \tag{9}$$

$$\textbf{Risk on cost} \quad \mathbb{E}_{\zeta^p, \zeta^t \sim \mathbb{P}}[F_1(q, x, \zeta^p, \zeta^t) - \mathbb{E}(F_1(q, x, \zeta^p, \zeta^t))]^+ \leqslant \alpha, \quad \forall\, \mathbb{P} \in \mathcal{P}_\zeta \tag{10}$$

$$\textbf{Risk on CO}_\mathbf{2}\ \textbf{emissions} \quad \mathbb{E}_{\zeta^c \sim \mathbb{P}}[F_2(q, \zeta^c) - \mathbb{E}(F_2(q, \zeta^c))]^+ \leqslant \beta, \quad \forall\, \mathbb{P} \in \mathcal{P}_\zeta \tag{11}$$

$$\textbf{Demand satisfaction} \quad \Pr_{\xi \sim \mathbb{P}}\left\{\sum_{j \in J} q_j \geqslant D(\xi)\right\} \geqslant 1 - \epsilon_D, \quad \forall\, \mathbb{P} \in \mathcal{P}_\xi \tag{12}$$

$$\textbf{Quality assurance} \quad \Pr_{\xi \sim \mathbb{P}}\left\{\sum_{j \in J} \eta_j q_j \leqslant D(\xi)\theta\right\} \geqslant 1 - \epsilon_d, \quad \forall\, \mathbb{P} \in \mathcal{P}_\xi \tag{13}$$

$$\textbf{Supply capacity} \quad \Pr_{\eta^S \sim \mathbb{P}}\{q_j \leqslant S_j(\eta_j^S) x_j,\ \forall\, j \in J\} \geqslant 1 - \epsilon_s, \quad \forall\, \mathbb{P} \in \mathcal{P}_\eta \tag{14}$$

$$\textbf{Minimum order quantity} \quad \Pr_{\eta^Q \sim \mathbb{P}}\{q_j \geqslant Q_j^m(\eta_j^Q) x_j,\ \forall\, j \in J\} \geqslant 1 - \epsilon_Q, \quad \forall\, \mathbb{P} \in \mathcal{P}_\eta \tag{15}$$

$$\text{Maximum number of suppliers} \quad \sum_{j \in J} x_j \leqslant N_{max} \tag{16}$$

$$\text{Minimum number of suppliers} \quad \sum_{j \in J} x_j \geqslant N_{min} \tag{17}$$

$$\text{Decision variables} \quad x_j \in \{0, 1\},\ q_j \geqslant 0 \quad \forall\, j \tag{18}$$

$$\text{Non} - \text{negativity} \quad d_1^+,\ d_2^+,\ d_3^-\ d_4^- \geqslant 0 \quad \forall\, j. \tag{19}$$

In the above model, Eqs. (6), (7)(10)–(14) represent the inequalities, with bold title contain distribution uncertainty. The aspiration level g_i is the goal value of the ith objective ($i = 1,\ 2,\ 3,\ 4$), which is expected to attain by the decision maker. The values of aspiration levels g_i are set in advance by decision makers according to their wishes and the development requirement, which is one of the characteristics of the goal programming method. Deviations (d_1^+, d_2^+, d_3^-, d_4^-) in the model are variables that need to be solved. Under the idea of goal programming, if some of aspiration levels g_1, g_2, g_3 and g_4 are achieved, the corresponding deviations solved by the above model are zero. Otherwise, they show non-zero values. In the process of optimization, goals with high priority are usually achieved prior to those goals with low priority. Therefore, in general, if these deviations obtained do not destroy the priority structure, then they are acceptable deviations.

Eq. (5) aims to minimize the deviation, in which P_1, P_2, P_3 and P_4 abiding by $P_1 \gg P_2 \gg P_3 \gg P_4$ represent the relative importance of the goals over cost, CO_2 emissions, society and suppliers' comprehensive value.

Eq. (6) limits the total expected cost under ambiguous distribution set $\mathcal{P}_\zeta$ and, as much as possible, does not exceed a given aspiration level g_1. In this problem, we expect the cost to be as small as possible. We do not care if the expected cost is less than aspiration level g_1, but once the cost exceeds g_1, we expect the excess to be as small as possible. That is why only positive deviation d_1^+ is introduced in this constraint.

Eq. (7) ensures that the expected value of total CO_2 emissions dose not exceed aspiration level g_2, as far as possible. Similarly, we do not care whether the expected total CO_2 emissions are smaller than aspiration level g_2, but once the expected value exceeds g_2, we consider the excess to be as small as possible. Hence, we only need to introduce positive deviation d_2^+.

Eq. (8) insures that the social impact of suppliers is equal to the aspiration level g_3 as much as possible. In this problem, we expect the social impact to be as great as possible. Accordingly, as soon as the social impact falls below aspiration level g_3, we introduce the negative deviation d_3^- to make the gap between expected value and aspiration level g_3 to be as small as possible.

Eq. (9) ensures that the comprehensive value equals the aspiration level g_4 as much as possible. Similarly, we expect the comprehensive value to be as large as possible. Therefore, only negative deviation d_4^- is introduced in this expression.

In uncertain environments, it is necessary to consider risk measure, in which we characterize the risk by means of upper semi-deviation. Eq. (10) guarantees that the risk of the total cost exceeding the expected value is less than a given level α. Eq. (11) assures that the risk of the CO_2 emissions exceeding the expected value is smaller than a given level β.

Eq. (12) illustrates that the demand of the purchasing company under ambiguous distribution set $\mathcal{P}_\xi$ should be met at a certain probability level $1-\epsilon_D$, in which $D(\xi)$ is the uncertain demand. Moreover, uncertain demand $D(\xi)$ is parameterized by random variable ξ.

Eq. (13) assures the quantity under a certain probability level ϵ_d, where η_j denotes the jth supplier's product defect rates, and θ denotes the acceptable waste rate of the purchasing company.

Eq. (14), a joint chance constraint, states the probability level that the quantity of products ordered from each supplier ($\forall j \in J$) less than or equal to each supplier' supply capacity no more than $1-\epsilon_s$. $S_j(\eta_j^S)$ is the maximum capacity of supplier j.

Eq. (15), a joint chance constraint, indicates that the probability level, related to the quantity of product ordered from each supplier ($\forall j \in J$) satisfying the its minimum order quantity, is more than or equal to $1-\epsilon_Q$. In this constraint, $Q_j^m(\eta_j^Q)$ is the minimum order quantity of supplier j.

Eqs. (16) and (17) are constraints on the number of suppliers. Eq. (18) states the limit on decision variable x_j and q_j. This constraint ensures that the all quantities of products ordered from every supplier throughout the supply chain are non-negative. Finally, Eq. (19) determines that deviations (positive or negative) are non-negative.

The new model (5)–(19) is first used to study sustainable SS/OA problems. Due to the uncertain information involved in the problem, the model does not easily to obtain optimal solutions. Specifically, the distributions of uncertain parameters in expected constraints (6), (7), (10), (11), chance constraints (12) and (13) and joint chance constraints (14) and (15) are partially known and lie within ambiguity sets. Since ambiguity sets include infinite distributions, the model is provided with infinite constraints, which leads to a computationally intractable model. To obtain a tractable model, we must focus on a relatively novel approach, distributionally robust optimization, rather than traditional methods to find a tractable formulation. Moreover, the tractability of a distributionally robust model depends on the choice of ambiguity set. Accordingly, in Section 4, we provide specific ambiguity sets and derive the tractable formulation of model (5)–(19).

4. Tractable approximation

In this section, we derive a computationally tractable approximation formulation of model (5)–(19), the crux of which lies in the treatment of expected constraints (6) and (7), chance constraints (12) and (13) and joint chance constraints (14) and (15). In the following, we build the computationally tractable formulations of these constraints with distribution uncertainty via ambiguity sets.

First, we provide the computationally tractable formulations of inequalities (6), (7) and (10), (11) when random variables ζ^p, ζ^t and ζ^c satisfy the following ambiguity sets:

$$\mathcal{P}_\zeta = \{\mathbb{P}: supp(\zeta_j^k) \subseteq [-1,\ 1],\ \mathbb{E}_\mathbb{P}(\zeta_j^{kl}) = \mu_j^{kl},\ \mathbb{E}_\mathbb{P}[\zeta_j^{kl}-\mu_j^{kl}]^+ = (\bar{d}_j^{kl})^+,\ \forall\, l \in L,\ \forall\, j \in J\},\ k = p,\ t,\ c, \tag{20}$$

where, random variables ζ_j^{kl} ($\forall\, l \in L$) are mutually independent. μ_j^{kl} and $(\bar{d}_j^{kl})^+$ denote the mean value and mean upper semi-deviation of random variable ζ_j^{kl}, and $-1 \leqslant \mu_j^{kl} \leq 1$, $0 \leqslant (\bar{d}_j^{kl})^+ \leqslant \frac{(1-\mu_j^{kl})(\mu_j^{kl}+1)}{2}$ because of the mean absolute deviation $0 \leqslant \bar{d}_j^{kl} \leqslant \frac{2(1-\mu_j^{kl})(\mu_j^{kl}+1)}{1-(-1)}$ according to reference (Ben-Tal & Hochman, 1972) and $(\bar{d}_j^{kl})^+ = \frac{1}{2}\bar{d}_j^{kl}$.

Based on the ambiguity sets (20), expected constraints (6) and (7) can be easily transformed into explicitly tractable formulations. Now, we obtain a well-structured tractable equivalent forms for the expected constraints (10) and (11) by the following theorem.

Theorem 1. For uncertain purchasing cost $C_j^{pur}(\zeta_j^p)$, transportation cost $C_j^{tran}(\zeta_j^t)$ and CO_2 emissions $CO_{2j}(\zeta_j^c)$, they are parameterized by random variables ζ_j^p, ζ_j^t and ζ_j^c for all j (e.g. $C_j^{pur}(\zeta_j^p) = (C_j^{pur})^0 + \sum_{l\in L}(C_j^{pur})^l\zeta_j^{pl}$). Let the distributions of random variables ζ_j^p, ζ_j^t and ζ_j^c satisfy ambiguity sets (20). Then the following bounds hold:

$$\sup_{\mathbb{P}\in\mathcal{P}_\zeta}\mathbb{E}_{\zeta^p,\zeta^t\sim\mathbb{P}}[F_1(q,x,\zeta^p,\zeta^t)-\mathbb{E}(F_1(q,x,\zeta^p,\zeta^t))]^+ = \sum_{j\in J}\sum_{l\in L}\left(|(C_j^{pur})^l q_j|\,(\bar{d}_j^{pl})^+ + \left|(C_j^{tran})^l\left\lceil\frac{q_j}{TC}\right\rceil Dis_j\right|(\bar{d}_j^{tl})^+\right) \tag{21}$$

$$\sup_{\mathbb{P}\in\mathcal{P}_\zeta}\mathbb{E}_{\zeta^c\sim\mathbb{P}}[F_2(q,\zeta^c)-\mathbb{E}(F_2(q,\zeta^c))]^+ = \sum_{j\in J}\sum_{l\in L}\left|(CO_{2j})^l\left\lceil\frac{q_j}{TC}\right\rceil Dis_j\right|(\bar{d}_j^{cl})^+ \tag{22}$$

Proof. We only proof Eq. (21), and Eq. (22) can be proved similarly. For ease of exposure, we assume $y_j = C_j^{pur}(\zeta_j)q_j + C_j^{adm}x_j + C_j^{tran}(\zeta_j)\left\lceil\frac{q_j}{TC}\right\rceil Dis_j$, $y_j^0 = (C_j^{pur})^0 q_j + (C_j^{adm})x_j + (C_j^{tran})^0\left\lceil\frac{q_j}{TC}\right\rceil Dis_j$, $y_j^{pl} = (C_j^{pur})^l q_j$, and $y_j^{tl} = (C_j^{tran})^l\left\lceil\frac{q_j}{TC}\right\rceil Dis_j$. Since uncertain parameters $C_j^{pur}(\zeta_j^p)$ and $C_j^{tran}(\zeta_j^t)$ are parameterized by random variables ζ_j^p and ζ_j^t, then $y_j = y_j^0 + \sum_{l\in L}y_j^{pl}\zeta_j^{pl} + \sum_{l\in L}y_j^{tl}\zeta_j^{tl}$. Therefore, $F_1 = \sum_{j\in J}y_j = \sum_{j\in J}y_j^0 + \sum_{j\in J}\sum_{l\in L}(y_j^{pl}\zeta_j^{pl} + y_j^{tl}\zeta_j^{tl})$. Since the distributions of random variables ζ_j^p and ζ_j^t satisfy ambiguity sets (20), one has

$$\mathbb{E}_{\zeta^p,\zeta^t\sim\mathbb{P}}[F_1(q,x,\zeta^p,\zeta^t)] = \sum_{j\in J}y_j^0 + \sum_{j\in J}\sum_{l\in L}(y_j^{pl}\mu_j^{pl} + y_j^{tl}\mu_j^{tl}). \tag{23}$$

Then the following equation hold:

$$\begin{aligned}&\mathbb{E}_{\zeta^p,\zeta^t\sim\mathbb{P}}[F_1(q,x,\zeta^p,\zeta^t)-\mathbb{E}(F_1(q,x,\zeta^p,\zeta^t))]^+\\&=\mathbb{E}_{\zeta^p,\zeta^t\sim\mathbb{P}}\left[\sum_{j\in J}\sum_{l\in L}([y_j^{pl}\zeta_j^{pl}-y_j^{pl}\mu_j^{pl}]+[y_j^{tl}\zeta_j^{tl}-y_j^{tl}\mu_j^{tl}])\right]^+.\end{aligned}$$

Since $\mathbb{E}[\zeta-\mathbb{E}(\zeta)]^+ = \frac{1}{2}\mathbb{E}|\zeta-\mathbb{E}(\zeta)|$, one has

$$\begin{aligned}&\mathbb{E}_{\zeta^p,\zeta^t\sim\mathbb{P}}\left[\sum_{j\in J}\sum_{l\in L}([y_j^{pl}\zeta_j^{pl}-y_j^{pl}\mu_j^{pl}]+[y_j^{tl}\zeta_j^{tl}-y_j^{tl}\mu_j^{tl}])\right]^+\\
&=\frac{1}{2}\mathbb{E}_{\zeta^p,\zeta^t\sim\mathbb{P}}\left|\sum_{j\in J}\sum_{l\in L}([y_j^{pl}\zeta_j^{pl}-y_j^{pl}\mu_j^{pl}]+[y_j^{tl}\zeta_j^{tl}-y_j^{tl}\mu_j^{tl}])\right|\\
&\leqslant\frac{1}{2}\sum_{j\in J}\sum_{l\in L}\mathbb{E}_{\zeta^p\sim\mathbb{P}}|[y_j^{pl}\zeta_j^{pl}-y_j^{pl}\mu_j^{pl}]|\\
&\quad+\frac{1}{2}\sum_{j\in J}\sum_{l\in L}\mathbb{E}_{\zeta^t\mathbb{P}}|[y_j^{tl}\zeta_j^{tl}-y_j^{tl}\mu_j^{tl}]|\\
&=\sum_{j\in J}\sum_{l\in L}|y_j^{pl}|\left\{\frac{1}{2}\mathbb{E}_{\zeta^p\sim\mathbb{P}}|\zeta_j^{pl}-\mu_j^{pl}|\right\}+\sum_{j\in J}\sum_{l\in L}|y_j^{tl}|\left\{\frac{1}{2}\mathbb{E}_{\zeta^t\sim\mathbb{P}}|\zeta_j^{tl}\right.\\
&\quad\left.-\mu_j^{tl}|\right\}\\
&=\sum_{j\in J}\sum_{l\in L}(|y_j^{pl}|(\bar{d}_j^{pl})^+ + |y_j^{tl}|(\bar{d}_j^{tl})^+).\end{aligned}$$

That is,

$$\sup_{\mathbb{P}\in\mathcal{P}_\zeta}\mathbb{E}_{\zeta^p,\zeta^t\sim\mathbb{P}}[F_1(q,x,\zeta^p,\zeta^t)-\mathbb{E}(F_1(q,x,\zeta^p,\zeta^t))]^+ = \sum_{j\in J}\sum_{l\in L}\left(|(C_j^{pur})^l q_j|\,(\bar{d}_j^{pl})^+ + \left|(C_j^{tran})^l\left\lceil\frac{q_j}{TC}\right\rceil Dis_j\right|(\bar{d}_j^{tl})^+\right).$$

□

Next, we derive the tractable formulations of chance constraints

(12) and (13) and joint chance constraints (14) and (15). However, in the case of distribution uncertainty, it is difficult to find the equivalently tractable forms of the chance constraints. Therefore, an indirect method is used to seek out a tractable approximation form. This requires the appropriate perturbation sets and ambiguous distribution sets of random variables ξ, η_j^S and η_j^Q. In this paper, the random variable ξ satisfies the following ambiguous distribution set and perturbation set:

$$\mathcal{P}_\xi = \{\mathbb{P}: supp(\xi) \subseteq [-1,\ 1],\ \mathbb{E}_\mathbb{P}(\xi_l) = \mu_l,\ \mathbb{E}_\mathbb{P}[\xi^l - \mu_l]^+ = (\bar{d}_l^+),\ \forall\ l \in L\}, \tag{24}$$

$$\mathcal{Z}_\xi = \left\{\xi \in \mathbb{R}^L,\ -1 \leqslant \xi_l \leqslant 1,\ \sqrt{\sum_{l\in L} (\frac{\xi_l - \mu^l}{\sigma_l})^2} \leqslant \sqrt{2\ln(1/\epsilon)},\ l \in L\right\}, \tag{25}$$

and the random variables η_j^S and η_j^Q satisfy the following ambiguous distribution sets and the perturbation sets:

$$\mathcal{P}_\eta = \{\mathbb{P}: supp(\eta_j^a) \subseteq [-1,\ 1],\ \mathbb{E}_\mathbb{P}(\eta_j^{al}) = \mu_j^{al},\ \mathbb{E}_\mathbb{P}[\zeta_j^{al} - \mu_j^{al}]^+ = (\bar{d}_j^{al})^+,\ \forall\ l \in L,\ \forall\ j \in J\},\ a = S,\ Q, \tag{26}$$

$$\mathcal{Z}_\eta = \left\{\eta_j^a \in \mathbb{R}^L,\ -1 \leqslant \eta_j^{al} \leqslant 1,\ \sqrt{\sum_{l\in L} (\frac{\eta_j^{al} - \mu_j^l}{\sigma_j^{al}})^2} \leqslant \sqrt{2\ln(1/\epsilon)},\ l \in L\right\} a = S,\ Q, \tag{27}$$

where, parameter σ controls the size of perturbation sets.

Based on the above ambiguity sets and perturbation sets, we can deduce the computationally tractable formulations of chance constraints (12) and (13) and joint chance constraints (14) and (15) by the following theorems.

Theorem 2. For chance constraint (12) on demand satisfaction and chance constraint (13) on quality assurance, the uncertain demand $D(\xi)$ is parameterized by random variable ξ, i.e., $D(\xi) = D^0 + \sum_{l\in L} D^l\xi^l$. Let random variable ξ satisfies the ambiguity set (24) and perturbation set (25). Then vector $q \in \mathbb{R}^J$ respectively satisfies (12) and (13) if there exist (u, r) and $(f,\ h) \in \mathbb{R}^{L+1}$ such that (q, u, r) and (q, f, h) respectively satisfy the following constraint systems:

$$\begin{cases} D^0 - \sum_{j\in J} q_j = u_0 + r_0 \\ D^l = u_l + r_l,\ \ \forall\ l \in L \\ u_0 + \sum_{l\in L} |u_l| \leqslant 0 \\ r_0 + \sum_{l\in L} \mu_l r_l + \sqrt{2\ln(1/\epsilon_D)}\sqrt{\sum_{l\in L} (\sigma_l)^2(r_l)^2} \le 0 \end{cases} \tag{28}$$

and

$$\begin{cases} \sum_{j\in J} \eta_j q_j - D^0\theta = f_0 + h_0 \\ -D^l\theta = f_l + h_l,\ \ \forall\ l \in L \\ f_0 + \sum_{l\in L} |f_l| \leqslant 0 \\ h_0 + \sum_{l\in L} \mu_l h_l + \sqrt{2\ln(1/\epsilon_d)}\sqrt{\sum_{l\in L} (\sigma_l)^2(h_l)^2} \le 0, \end{cases} \tag{29}$$

where

$$\sigma_l = \sup_{m\in\mathbb{R}} \sqrt{\frac{2\ln(2\bar{d}_l^+\cosh(m) + (1 - 2\bar{d}_l^+)e^{\mu_l m}) - 2\mu_l m}{m^2}}. \tag{30}$$

That is, the constraint systems (28) and (29) are respectively safe approximations of chance constraints (12) and (13).

Proof. The steps for proving compliance with theorem 2.4.4 from Ben-Tal et al. (2009), then the chance constraint (12) can be approximated by the following system

$$\begin{cases} D^0 - \sum_{j\in J} q_j = u_0 + r_0 \\ D^l = u_l + r_l,\ \ \forall\ l \in L \\ u^0 + \sum_{l\in L} |u_l| \leqslant 0 \\ r^0 + \sum_{l\in L} \max[\mu_l^- r_l,\ \mu_l^+ r_l] + \sqrt{2\ln(1/\epsilon_D)}\sqrt{\sum_{l\in L} (\sigma_l)^2 r_l^2} \leqslant 0, \end{cases} \tag{31}$$

which is the robust counterpart of the inequality $\sum_{j\in J} q_j \geqslant D(\xi)$ under the following perturbation set

$$\mathcal{Z} = \left\{\xi \in \mathbb{R}^L,\ \exists\ z \in \mathbb{R}^L,\ \mu_l^- \leqslant \xi_l - z_l \leqslant \mu_l^+,\ -1 \leqslant \xi_l \leqslant 1,\ \sqrt{\sum_{l\in L} (\frac{z_l}{\sigma_l})^2} \leqslant \sqrt{2\ln(1/\epsilon_D)},\ l \in L\right\}.$$

Now, we need to determine parameters μ_l^-, μ_l^+ and σ_l such that the following property from Ben-Tal et al. (2009) holds:

$$\int \exp\{m\xi_l\}d\mathbb{P}(\xi_l) \leqslant \exp\left\{\max[\mu_l^- m,\ \mu_l^+ m] + \frac{1}{2}\sigma_l^2 m^2\right\},\quad \forall\ m \in \mathbb{R},\ \forall\ \mathbb{P} \in \mathcal{P}_\xi.$$

By a tight explicit bound on $\mathbb{E}_\mathbb{P}\exp(m^T\xi)$ from Postek, Ben-Tal, den Hertog, and Melenberg (2018), we can obtain

$$\sup_{\mathbb{P}\in\mathcal{P}_\xi} \left\{\int \exp\{m\xi_l\}d\mathbb{P}(\xi_l)\right\} = \sup_{\mathbb{P}\in\mathcal{P}_\xi} \mathbb{E}_{\xi\sim\mathbb{P}}\exp\{m\xi_l\} = \bar{d}_l\cosh(m) + (1 - \bar{d}_l)e^{\mu_l m},$$

where $\bar{d}_l$ is mean absolute deviation.

According to the relation between mean upper semi-deviation $\bar{d}_l^+$ and mean absolute deviation $\bar{d}_l$, we have $\sup_{\mathbb{P}\in\mathcal{P}_\xi}\{\int \exp\{m\xi_l\}d\mathbb{P}(\xi_l)\} = 2\bar{d}_l^+\cosh(m) + (1 - 2\bar{d}_l^+)e^{\mu_l m}$. Then, one has

$$2\bar{d}_l^+\cosh(m) + (1 - 2\bar{d}_l^+)e^{\mu_l m} \leqslant \left\{\max[\mu_l^+ m,\ \mu_l^- m] + \frac{1}{2}\sigma_l^2 m^2\right\}.$$

Since the distribution $\mathbb{P}$ of random variable satisfying ambiguity set (24) processes given expected value μ, it follows that $\mu_l^- = \mu_l^+ = \mu^l$. Thus,

$$r_0 + \sum_{l\in L} \max[\mu_l^- r_l,\ \mu_l^+ r_l] + \sqrt{2\ln(1/\epsilon_D)}\sqrt{\sum_{l\in L} (\sigma_l)^2(r_l)^2} = r_0 + \sum_{l\in L} \mu_l r_l + \sqrt{2\ln(1/\epsilon_D)}\sqrt{\sum_{l\in L} (\sigma_l)^2(r_l)^2}, \tag{32}$$

$$\mathcal{Z} = \left\{\xi \in \mathbb{R}^L,\ -1 \leqslant \xi_l \leqslant 1,\ \sqrt{\sum_{l\in L} (\frac{\xi_l - \mu_l}{\sigma_l})^2} \leqslant \sqrt{2\ln(1/\epsilon_D)},\ l \in L\right\},$$

and

$$2\bar{d}_l^+\cosh(m) + (1 - 2\bar{d}_l^+)e^{\mu_l m} \leqslant \left\{\mu_l m + \frac{1}{2}\sigma_l^2 m^2\right\}.$$

It follows that

$$\sigma_l = \sup_{m\in\mathbb{R}} \sqrt{\frac{2\ln(2\bar{d}_l^+\cosh(m) + (1 - 2\bar{d}_l^+)e^{\mu_l m}) - 2\mu_l m}{m^2}},$$

Combining (31) and (32), we arrive at (28).

The above process proves that the robust counterpart (28) of inequality $\sum_{j\in J} q_j \geqslant D(\xi)$ under perturbation set (25) is a safe approximation of chance constraint (12) under ambiguity set (24).

The proof of (29) is similar to the process of (28). □ □

Theorem 3. For joint chance constraint (14) on supply capacity, the uncertain independent supply capacity S_j is parameterized by random independent variable η_j^S, i.e., $S_j(\zeta_j) = S_j^0 + \sum_{l\in L} S_j^l\eta_j^{Sl}$. Let random independent variable η_j^{Sl} satisfies the ambiguity set (20). Then (q_j, x_j)

satisfy constraint (14) if there exist $s_j,\ v_j \in \mathbb{R}^{L+1}$ such that $(q_j, x_j, s_j,\ v_j)$ satisfy the following constraint system.

$$\begin{cases} q_j - S_j^0 x_j = s_j^0 + v_j^0 & \forall\, j \in J \\ -S_j^l x_j = s_j^l + v_j^l & \forall\, j \in J,\ \forall\, l \in L \\ s_j^0 + \sum_{l \in L} |s_j^l| \leqslant 0 & \forall\, j \in J,\ \forall\, l \in L \\ v_j^l + \sum_{l \in L} \mu_j^{Sl} v_j^l + \sqrt{2\ln(1/\bar{\epsilon}_s)} \sqrt{\sum_{l \in L} (\sigma_j^{Sl})^2 (v_j^l)^2} \le 0 & \forall\, j \in J,\ \forall\, l \in L, \end{cases} \tag{33}$$

where $\bar{\epsilon}_s = \frac{\epsilon_s}{|J|}$ and

$$\sigma_j^{Sl} = \sup_{m \in \mathbb{R}} \sqrt{\frac{2\ln(2(\bar{d}_j^{Sl})^{+}\cosh(m) + (1 - 2(\bar{d}_j^{Sl})^{+})e^{\mu_j^{Sl} m}) - 2\mu_j^{Sl} m}{m^2}}. \tag{34}$$

That is, the constraint system (33) is a safe tractable approximation of joint chance constraint (14).

Proof (*Proof*). A sufficient condition to joint chance constraint (14) is

$$\Pr_{\eta^S \sim \mathbb{P}}\{q_j \leqslant S_j(\eta_j^S) x_j\} \geqslant 1 - \frac{\epsilon_s}{|J|},\ \ \forall\, j \in J. \tag{35}$$

Similar to the derivation of Theorem 2, we transform the chance constraint (35) into the following approximation system.

$$\begin{cases} q_j - S_j^0 x_j = s_j^0 + v_j^0 & \forall\, j \in J \\ -S_j^l x_j = s_j^l + v_j^l & \forall\, j \in J,\ \forall\, l \in L \\ s_j^0 + \sum_{l \in L} |s_j^l| \leqslant 0 & \forall\, j \in J,\ \forall\, l \in L \\ v_j^l + \sum_{l \in L} \mu_j^{Sl} v_j^l + \sqrt{2\ln(1/\frac{\epsilon_s}{|J|})} \sqrt{\sum_{l \in L} (\sigma_j^{Sl})^2 (v_j^l)^2} \le 0 & \forall\, j \in J,\ \forall\, l \in L, \end{cases}$$

If for a given vector q_j there exist $(s_j,\ v_j) \in \mathbb{R}^{L+1}$ such that $(s_j,\ v_j)$ satisfy the above approximation system, then the vector q_j is feasible for chance constraint (35), which means that the vector q_j is feasible for joint constraint (14). This proves that the above approximation system is a safe approximation of joint chance constraint (14). □

Theorem 4. For joint chance constraint (15) on minimum order quantity, the uncertain independent minimum order quantity Q_j^m is parameterized by random independent variable η_j^Q, i.e., $Q_j^m(\eta_j^Q) = (Q_j^m)^0 + \sum_{l \in L} (Q_j^m)^l \eta_j^{Ql}$. Let the distribution $\mathbb{P}_\eta$ of random independent variable η_j^{Ql} satisfies the ambiguity set (20). Then (q_j, x_j) satisfy constraint (15) if there exist $g_j,\ \ k_j \in \mathbb{R}^{L+1}$ such that $(q_j, x_j, g_j,\ \ k_j)$ satisfy the following constraint system.

$$\begin{cases} ((Q_j^m)^0 x_j - q_j = g_j^0 + k_j^0 & \forall\, j \in J \\ (Q_j^m)^l x_j = g_j^l + k_j^l & \forall\, j \in J,\ \forall\, l \in L \\ g_j^0 + \sum_{l \in L} |g_j^l| \leqslant 0 & \forall\, j \in J,\ \forall\, l \in L \\ k_j^0 + \sum_{l \in L} \mu_j^{Ql} k_j^l + \sqrt{2\ln(1/\bar{\epsilon}_Q)} \sqrt{\sum_{l \in L} (\sigma_j^{Ql})^2 (k_j^l)^2} \le 0 & \forall\, j \in J,\ \forall\, l \in L, \end{cases} \tag{36}$$

where $\bar{\epsilon}_Q = \frac{\epsilon_Q}{|J|}$ and

$$\sigma_j^{Ql} = \sup_{m \in \mathbb{R}} \sqrt{\frac{2\ln(2(\bar{d}_j^{Ql})^{+}\cosh(m) + (1 - 2(\bar{d}_j^{Ql})^{+})e^{\mu_j^{Ql} m}) - 2\mu_j^{Ql} m}{m^2}}. \tag{37}$$

That is, the constraint system (36) is a tractable safe approximation of joint chance constraint (15).

Proof (*Proof*). The process of proof is similar to Theorem 3. □

As useful conclusions of Theorems 1–4, we derive tractable formulations of constraints (10)–(15) with uncertain parameters. Accordingly, these theorems can be utilized to deduce the following computationally tractable approximation model of the distributionally robust goal programming model (5)–(19).

$$\begin{aligned} \min \quad & P_1 d_1^{+} + P_2 d_2^{+} + P_3 d_3^{-} + P_4 d_4^{-} \\ \text{s. t.} \quad & \sum_{j \in J} \Bigg[(C_j^{pur})^0 q_j + (C_j^{adm}) x_j + (C_j^{tran})^0 \left\lceil \frac{q_j}{TC} \right\rceil Dis_j \\ & + \sum_{l \in L} \left((C_j^{pur})^l q_j \mu_j^{pl} + (C_j^{tran})^l \left\lceil \frac{q_j}{TC} \right\rceil Dis_j \mu_j^{tl} \right) \Bigg] - d_1^{+} \leqslant g_1 \\ & \sum_{j \in J} \left[CO_{2j}^0 \left\lceil \frac{q_j}{TC} \right\rceil Dis_j + \sum_{l \in L} \left(CO_{2j}^l \left\lceil \frac{q_j}{TC} \right\rceil Dis_j \right) \mu_j^{cl} \right] - d_2^{+} \leqslant g_2 \\ & F_3(q) + d_3^{-} \geqslant g_3 \\ & F_4(q) + d_4^{-} \geqslant g_4 \\ & \sum_{j \in J} \sum_{l \in L} \left(|(C_j^{pur})^l q_j|\, (\bar{d}_j^{pl})^{+} + \left| (C_j^{tran})^l \left\lceil \frac{q_j}{TC} \right\rceil Dis_j \right| (\bar{d}_j^{tl})^{+} \right) \leqslant \alpha \\ & \sum_{j \in J} \sum_{l \in L} \left| CO_{2j}^l \left\lceil \frac{q_j}{TC} \right\rceil Dis_j \right| (\bar{d}_j^{cl})^{+} \leqslant \beta \\ & \text{constraints } (16)-(19),\ \ (28)--(29),\ \ (33),\ \ \text{and } (36). \end{aligned} \tag{38}$$

Model (38) is the computationally tractable approximation system of model (5)–(19). If vector x and q satisfy model (38), then x and q are feasible for model (5)–(19).

5. Case study

In this section, we conduct a case study for a steel company to understand the performance of our distributionally robust goal programming method. All mathematical models are solved by CPLEX software studio 12.6.3 running on a personal computer.

5.1. Description of the case study

Sustainable supply chain design has attracted widespread attention in recent years, and the critical steps involve selecting sustainable supplier and appropriately allocating orders (Azadnia et al., 2015). However, environmental issues have become an inevitable challenge for achieving sustainable SS/OA. In responding to the demands for sustainable requirement, companies are confronted with many challenges (Ghorabaee et al., 2017). Many small steel companies have been forced to stop producing. The degree of the environmental pollution by steel companies exhibits a clear relationship with the raw materials. In the past few years, a steel company has purchased a raw material (limestone) from eight suppliers, but these suppliers may not be able to meet the requirements of sustainability over time. In response to sustainable development, the company is committed to transformation to pursue sustainability. Therefore, based on comprehensive considerations, the steel company plans to limit the number of suppliers and reselect the suppliers that satisfy cost, CO_2 emissions, society and comprehensive value goals to comply with sustainable development. To this end, the company initially locks in five suppliers, and the company's board of directors establishes a group of five experts (decision makers) from different sectors.

Three types of criteria from reference (Mohammed et al., 2019) are shown in Fig. 2. The three types of criteria are utilized by decision makers (DM1 to DM5) to evaluate the suppliers and thus compute the weights and suppliers' performance coefficients presented in Eq. (3) for the third goal F_3 and Eq. (4) for the fourth goal F_4. The significance of each criterion and the ratings of suppliers evaluated by decision makers are shown in Tables 1 and 2, respectively. Based on the evaluation in Tables 1 and 2, the weights of the criteria calculated by analytical hierarchy process and the performance values of the suppliers calculated by the technique of order preference by similarity to ideal solution

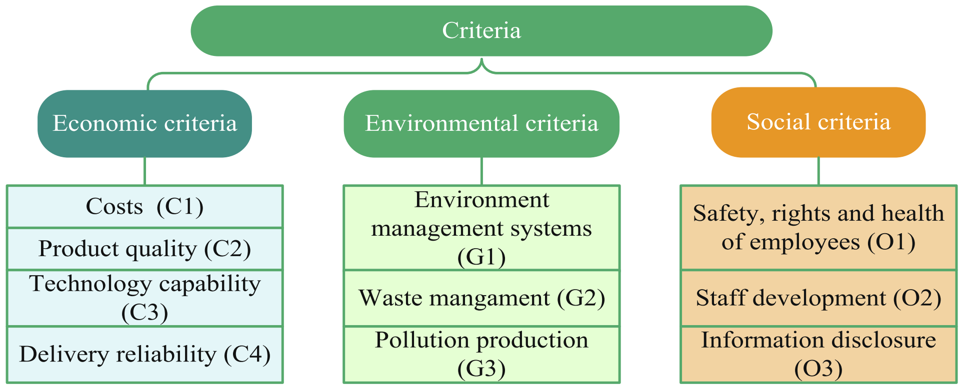


Fig. 2. Criteria used for evaluating suppliers.

are summarized in Fig. 3.

Input data are collected from the suppliers and the steel company. The steel company places an order with its suppliers on a weekly basis. There are five alternative suppliers, i.e., $N_{max} = 5$. The company needs to select at least two suppliers, i.e., $N_{min} = 2$. The demand for the limestone is approximately 7,000 tons per week; that is, the nominal value of demand is $D^0 = 7000$. According to the development requirements of the steel company, the decision makers decide that the aspiration levels are $g_1 = 600000$, $g_2 = 15000000$, $g_3 = 4000$ and $g_4 = 4500$. The capacity of each heavy truck is $TC = 50$ tons. Table 3 provides data for each supplier, in which the transportation distances between the purchasing company and the suppliers are obtained form Google map. In all ensuing experiments, we set the priority levels $P_1 = 10^7$, $P_2 = 10^5$, $P_3 = 10^3$ and $P_4 = 10$. In addition, we consider that each uncertain parameter in this problem is affected by three potential factors (i.e., $L = 3$). On the basis of the above data analysis, a series of experimental results and sensitivity analysis results can be obtained.

5.2. Computational results

Before proceeding with these experiments, some parameters used in the model need to be determined. We set mean values $\mu_j^{kl} = \mu_j^{al} = \mu_l = 0$ and semi-deviations $(\bar{d}_j^{kl})^+ = (\bar{d}_j^{al})^+ = (\bar{d}_j^{l})^+ = 0.05$ ($k = p, t, c$; $a = S, Q$). Then, the parameters $\sigma_j^{al} = \sigma_l = 0.4126421$ are solved using Eqs. (30), (34) and (37). We set the perturbation coefficients $D^l = 350$ ($\forall\, l \in L$). Moreover, the perturbation coefficients $(Q^m)^l = [7; 6; 5.5; 4; 2]$, $(C^{pur})^l = [0.5\%(C_1^{pur})^0; 0.5\%(C_2^{pur})^0; 0.5\%(C_3^{pur})^0; 0.5\%(C_4^{pur})^0; 0.05\%(C_5^{pur})^0]$, $(C_j^{tran})^l = 0.5\%(C_j^{tran})^0$, $CO_{2j}^l = 0.5\% CO_{2j}^0$ and $S_j^l = 1\% S_j^0$ ($\forall\, j \in J$, $\forall\, l \in L$). In addition, the probability levels are set as $\epsilon_D = \epsilon_d = \bar{\epsilon}_s = \bar{\epsilon}_Q = 0.1$. Basedonthe above data, we obtain the 9 sets of results with the different values of parameters θ, α and β. The resulting recommended supplier selection and the order allocation are shown in Tables 4, 5 and Fig. 4.

Table 4 summarizes the supplier selection, order allocation and fulfilment degree of each goal with respect to the company's acceptable waste rate θ. On the one hand, we can clearly observe that the fulfilment degree of each goal differs with varying θ. The first goal corresponding to $\theta = 0.04$ is unfulfilled, but the first goal is fulfilled when θ equals other values. The second goal is always achieved, and the third and fourth goal are always unfulfilled regardless of the value of the waste rate θ. Under these circumstances, acceptable waste rate $\theta = 0.04$ may not be a good selection for decision makers. Furthermore, we can observe that the unfulfilled degrees vary even if the goals are always unrealized. On the other hand, it is evident that the optimal numbers of contracted suppliers and corresponding order allocations are distinct under different waste rate θ. With $\theta = 0.04$, 0.06, supplier 2 and supplier 5 are selected. Suppliers 1, 2 and 5 are contracted for θ values of 0.08, 0.10 and 0.12, and when the waste rate θ is equal to 0.14, the resulting suppliers are suppliers 1, 2, 3, and 5. Although the resulting suppliers may be the same under different values of θ, the order allocations of contracted suppliers are different. For example, when $\theta = 0.04$, the order qualities q_2 and q_5 are 4550 and 3000, respectively, which are different from the values of 6250 and 1400 corresponding to $\theta = 0.06$. This means that decision makers can select a

Table 1
The importance of the criteria.

Decision makers	Economic				Environmental			Social		
	C1	C2	C3	C4	G1	G2	G3	O1	O2	O3
DM1	VH	VH	VH	H	VH	VH	M	H	M	VH
DM2	VH	H	VH	VH	VH	H	H	H	M	VH
DM3	VH	VH	VH	VH	M	VH	H	H	H	H
DM4	H	H	VH	VH	VH	M	H	M	H	VH
DM5	VH	VH	H	VH	H	H	VH	H	M	VH

Table 2
The evaluation of the suppliers.

Decision makers	Suppliers	Economic				Environmental			Social		
		C1	C2	C3	C4	G1	G2	G3	O1	O2	O3
DM1	S1	VH	M	M	H	VH	M	H	H	H	H
	S2	VH	H	H	H	M	H	VH	H	L	VH
	S3	H	VH	H	H	M	H	H	VH	L	VH
	S4	H	VH	H	M	H	VH	H	H	H	H
	S5	H	H	VH	H	M	H	H	M	H	H
DM2	S1	VH	H	H	M	H	M	VH	M	M	VH
	S2	VH	M	VH	H	M	VH	M	H	M	H
	S3	H	VH	M	H	H	VH	M	M	H	H
	S4	H	H	H	VH	M	VH	M	H	M	VH
	S5	VH	H	H	H	L	H	VH	M	H	VH
DM3	S1	VH	H	H	H	VH	H	H	VH	M	L
	S2	H	H	VH	H	H	VH	M	M	H	H
	S3	VH	H	M	L	H	H	H	H	L	VH
	S4	H	VH	H	H	M	H	H	H	M	VH
	S5	H	H	VH	M	H	H	L	VH	L	H
DM4	S1	VH	VH	H	VH	VH	M	L	M	H	VH
	S2	VH	H	H	H	VH	M	H	H	M	H
	S3	H	VH	VH	H	H	M	VH	VH	L	H
	S4	H	L	VH	VH	H	H	VH	VH	M	M
	S5	VH	H	H	VH	H	VH	H	L	VH	H
DM5	S1	VH	H	H	M	H	M	M	H	L	VH
	S2	VH	M	H	VH	H	M	VH	M	L	VH
	S3	VH	H	M	VH	M	M	H	H	H	H
	S4	H	L	VH	VH	M	H	H	H	L	VH
	S5	H	VH	VH	M	H	H	H	M	H	H

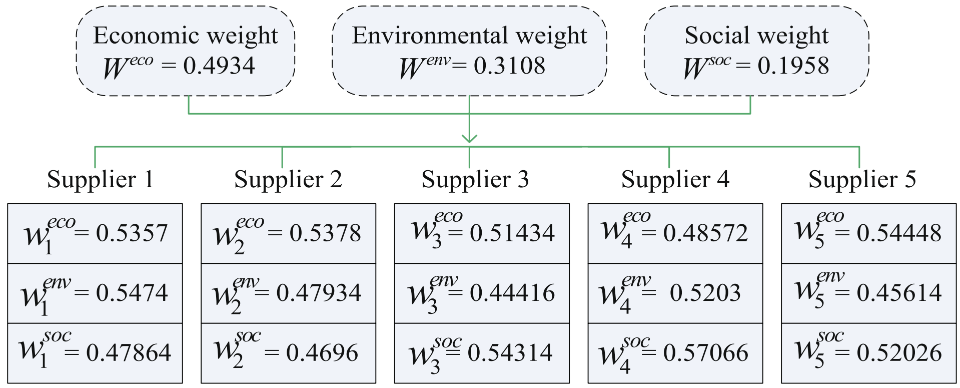


Fig. 3. The weights of criteria and the performance values of suppliers.

Table 3
Data table.

Data	S_1	S_2	S_3	S_4	S_5
$(C_j^{pur})^0$	50	60	70	80	140
C_j^{adm}	20	20	22	19	19
$(C_j^{tran})^0$	2	2	2	2	2
CO_{2j}^0	1100	1100	1100	1100	1100
Dis_j	82.4	74.3	76.2	78.5	51.4
S_j^0	10500	9660	7000	6300	11200
$(Q_j^m)^0$	1000	900	850	600	550
η_j	0.2	0.05	0.1	0.15	0.01

Table 4
Computation results with $(\alpha, \beta) = (6000, 10000)$ for different θ.

θ	0.04	0.06	0.08	0.10	0.12	0.14
d_1^+	117450	0	0	0	0	0
d_2^+	0	0	0	0	0	0
d_3^-	302.54	336.64	231.87	156.73	100	43.828
d_4^-	659.6	618.6	491.05	397.9	329.47	351.56
x_1	0	0	1	1	1	1
x_2	1	1	1	1	1	1
x_3	0	0	0	0	0	1
x_4	0	0	0	0	0	0
x_5	1	1	1	1	1	1
q_1	0	0	1200	2000	2800	3350
q_2	4550	6250	5250	4650	3900	2300
q_3	0	0	0	0	0	1050
q_4	0	0	0	0	0	0
q_5	3000	1400	1400	1350	1400	1350

Table 5
Computation results with $\theta = 0.11$ for different (α, β).

(α, β)	$\beta = 10000$			$\alpha = 6000$		
	$\alpha = 6000$	$\alpha = 5500$	$\alpha = 5000$	$\beta = 8500$	$\beta = 8000$	$\beta = 7500$
d_1^+	0	0	15979	42980	144890	246810
d_2^+	0	0	0	0	0	0
d_3^-	129.63	253.96	325.81	350.67	284.81	218.95
d_4^-	363.83	510.633	605.11	665.3	657.49	649.69
x_1	1	1	1	0	0	0
x_2	1	1	1	1	1	1
x_3	0	0	0	0	0	0
x_4	0	0	0	0	0	0
x_5	1	1	1	1	1	1
q_1	2400	2550	2750	0	0	0
q_2	4300	3550	2750	5500	4200	2900
q_3	0	0	0	0	0	0
q_4	0	0	0	0	0	0
q_5	1350	1650	2050	2050	3350	4650

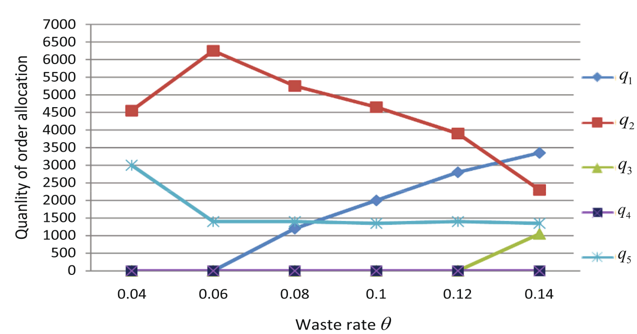


Fig. 4. Effect of changing the waste rate θ on the order allocation.

satisfactory supplier portfolio and allocate the orders by confirming the acceptable waste rate θ.

Fig. 4 depicts the effect of varying the company's acceptable waste rate θ on the order allocation. The horizontal axis corresponds to the waste rate θ and the vertical axis corresponds to the order allocation among the assigned suppliers. From Fig. 4, we can intuitively find that the resulting order allocations differ upon changing θ, especially the order qualities q_1, q_2, q_3 and q_5. When θ is equal to 0.04 or 0.06, the order quantity q_2 from the 2nd supplier is the largest, followed by q_5, and the order quantity of the other suppliers is 0. For $\theta > 0.06$, the order quantity q_1 increases with θ, the order quantity q_2 decreases, and the order quantity q_5 remains at an steady state. Under $\theta = 0.14$, the 3rd supplier begins to be allocated orders. In addition, it is worth noting that regardless of the value of θ, the order quantity q_4 from the fourth supplier is always 0.

Table 5 shows the supplier portfolios, the resulting order allocation and the fulfilled degree of each goal under different risk levels (α, β). On the one hand, it can be intuitively observed that the realization degrees of the goals are distinct under varying risk levels α and β. The first goal is fulfilled under $\alpha = 6000, 5500$ and $\beta = 8500, 8000, 7500$, but that corresponding to $\alpha = 5000$ is unfulfilled. The second goal is always realized, and the third and fourth goals are always unachieved. In this case, (α, β) values equal to (6000, 10000) and (5500, 10000) may become preferred choices than others for decision makers. On the other hand, different risk levels α and β may correspond to different supplier combinations and order assignments. The selected suppliers are suppliers 1, 2 and 5 when the risk level α changes, whereas the resulting suppliers are suppliers 2 and 5 when the risk level β are again altered. Even if different risk levels correspond to the same supplier portfolio, the order allocations from contracted suppliers are different. The order quality q_2 is equal to 5,500 as $\beta = 8500$, and those values corresponding to β values of 8,000 and 7,500 are 4,200 and 2,900, respectively. This indicates that supplier selection and order allocation are affected by the risk levels. Therefore, decision makers can make satisfactory decisions based on specific risk levels.

5.3. Sensitivity analysis

In this section, we conduct a series of sensitivity analyses with respect to parameter σ, probability level ϵ, uncertainty and weights. The probability level ϵ reflects the possibility of the event, and parameter σ controls the size of perturbation sets. We conduct the sensitivity analysis on ϵ and σ to explore their effects on decisions. For uncertain parameters in goals, we first probe the effects of cost uncertainty and CO_2 emissions uncertainty from high to low priority. Furthermore, we also study the impacts of joint cost and CO_2 emissions uncertainty on decision making. For the uncertainty in constraints, we take demand uncertainty as an example to discuss the impact of demand uncertainty on decision making. The method for exploring the impact of other uncertain parameters (e.g., supply capacity uncertainty) in constraints is similar to this method for the demand uncertainty. Finally, we explore the effects of weights and performance coefficients on decisions.

5.3.1. Effects of parameter σ and probability level ϵ on decisions

To demonstrate the effects of parameter σ and probability level ϵ on decision making, we carry out sensitivity analysis for different σ and ϵ values. In these experiments, we still set the mean value $\mu_j^{kl} = \mu_j^{al} = \mu_l = \mu$, semi-deviation $(\bar{d}_j^{kl})^+ = (\bar{d}_j^{al})^+ = (\bar{d}_j^{l})^+ = \bar{d}^+$ $(k = p,\ t,\ c;\ a = S,\ Q)$, and then parameters $\sigma_j^{al} = \sigma_l = \sigma$. By adjusting the parameters σ, we can obtain the series of solutions shown in Table 6. Moreover, we set $\epsilon_D = \epsilon_d = \bar{\epsilon}_s = \bar{\epsilon}_Q = \epsilon$. Based on the different probability levels, the obtained solutions are presented in Table 7.

Table 6 provides the 6 sets of solutions obtained by adjusting parameter σ, in which the value of parameter σ depends on $(\mu,\ \bar{d}^+)$. Therefore, six sets of $(\mu,\ \bar{d}^+)$ correspond to six different values of σ, and six sets of solutions can be solved. From Table 6, we can clearly observe that the different parameters σ correspond to different fulfilled degrees of the third and fourth goals, as well as different order qualities from designated suppliers. For example, the unrealized degree of the third goal is 204.77 under $\sigma = 0.4126421$, which is greater than the value of 183.82 under $\sigma = 0.3695402$. The order qualities q_1, q_2 and q_5 are 1,600, 4,900 and 1,400 under $\sigma = 0.4126421$ but 1,650, 4,950 and 1,350 under $\sigma = 0.3690423$, respectively. A similar case can be found for other values of σ.

Table 7 shows the sensitivity of the fulfilment degree of each goal and the order allocation to the probability level ϵ. We can easily observe that the achievement degrees of the third and fourth goals corresponding to different ϵ values exhibit slight differences. For example, the unrealized degrees of the third and the fourth goals are 157.18 and 399 when $\epsilon = 0.05$, slightly larger than the degrees of 156.73 and 397.9 when $\epsilon = 0.1$. Furthermore, it is easily observed that the resulting order allocations from suppliers 1, 2 and 5 are different for different probability levels. For example, the order qualities q_1, q_2 and q_5 are 1,950, 4,700 and 1,350 under $\epsilon = 0.05$, which are different from the values of 2,100, 4,550 and 1,350 corresponding to probability level ϵ as 0.3.

According to the sensitivity analyses for σ and ϵ, the different parameter σ and probability level ϵ exert a certain effects on the order allocation from contracted suppliers. That is, the order allocation is sensitive to parameter σ and probability level ϵ. In this case, decision maker may depend on personal experience and knowledge to confirm the parameter σ and probability level ϵ to reasonably allocate orders among the assigned suppliers and formulate a sustainable strategy.

Table 7
Sensitivity analysis with $\theta = 0.10$ and $\sigma = 0.4126421$ for different ϵ.

ϵ	d_1^+	d_2^+	d_3^-	d_4^-	q_1	q_2	q_3	q_4	q_5
0.05	0	0	157.18	399	1950	4700	0	0	1350
0.10	0	0	156.73	397.9	2000	4650	0	0	1350
0.20	0	0	156.28	396.81	2050	4600	0	0	1350
0.30	0	0	155.82	395.71	2100	4550	0	0	1350

5.3.2. Effects of cost uncertainty on decisions

To observe the effects of cost uncertainty on decisions, problems with different cost uncertainty levels are solved: (I) $(C^{pur})^l = [0.55,\ 0.2,\ 0.355,\ 0.355,\ 0.2]$ and $(C^{tran})^l = [0.02,\ 0.02,\ 0.02,\ 0.02,\ 0.02]$ $(\forall\ l \in L)$; (II) $(C^{pur})^l = [0.5,\ 1.2,\ 0.7,\ 0.8,\ 0.14]$ and $(C^{tran})^l = [0.03,\ 0.03,\ 0.03,\ 0.03,\ 0.03]$ $(\forall\ l \in L)$. It is worth noting that $(C^{pur})^l = [(C_1^{pur})^l,\ (C_2^{pur})^l,\ (C_3^{pur})^l,\ (C_4^{pur})^l,\ (C_5^{pur})^l]$ and $(C^{tran})^l = [(C_1^{tran})^l,\ (C_2^{tran})^l,\ (C_3^{tran})^l,\ (C_4^{tran})^l,\ (C_5^{tran})^l]$. The set of experiment is conducted under $\theta = 0.1$, $\epsilon = 0.1$, $\sigma = 0.4126421$ and $(\alpha,\ \beta) = (6000,\ 10000)$. The results of deviations in case (I) are $d_4^- = 275.38$, while the others are 0, and the deviations in the case (II) are $d_1^+ = 252150$, $d_2^+ = 0$, $d_3^- = 178.17$ and $d_4^- = 591.8$. A change in the cost uncertainty evidently impacts the realization of the goals. The results of supplier selection and order allocation for cases (I) and (II) are shown in Fig. 5.

Fig. 5 displays two sets of supplier portfolios and order allocations with respect to two sets of perturbation coefficients (I) and (II). In Fig. 5, the blank area indicates that the supplier is not selected, that is, the order allocation is 0, and the coloured area indicates that the supplier is selected to provide raw materials. Specifically, in the coloured area, the longer the radius is, the greater the order quality the supplier provides. The effect of cost uncertainty on the supplier portfolio and order allocation are intuitively observed. In case (I), suppliers 2, 4 and 5 are selected, and the corresponding order qualities q_2, q_3 and q_5 are 6,700, 900 and 750. Unlike in case (I), when the perturbation coefficients are set as in case (II), suppliers 1 and 5 are contracted, and the resulting order qualities q_1 and q_5 are 2,550 and 5,000, respectively. This result means that different cost uncertainty levels greatly impact on supplier selection and order allocation. It is extremely important for decision makers to determine the uncertainty level of demand in order to formulate effective decisions.

5.3.3. Effects of CO_2 emissions uncertainty on decisions

To illustrate the impact of CO_2 emissions uncertainty on decisions, experiments are carried out with two different set of perturbation coefficients $CO_2^l = [CO_{21}^l,\ CO_{22}^l,\ CO_{23}^l,\ CO_{24}^l,\ CO_{25}^l]$: (i) $CO_2^l = [6,\ 6,\ 6,\ 6,\ 6]$; (ii) $CO_2^l = [8,\ 8,\ 8,\ 8,\ 8]$. This set of experiments is conducted under $\theta = 0.1$, $\epsilon = 0.1$, $\sigma = 0.4126421$ and $(\alpha,\ \beta) = (6000,\ 10000)$.

Based on cases (i), we obtain deviations $d_3^- = 200$ and $d_4^- = 598.7$, while the others are zero. This is different from deviations

Table 6
Sensitivity analysis with $\theta = 0.09$ and $\epsilon = 0.1$ for different σ.

μ	$\bar{d}^+$	σ	d_1^+	d_2^+	d_3^-	d_4^-	q_1	q_2	q_3	q_4	q_5
0	0.05	0.4126421	0	0	204.77	456.99	1600	4900	0	0	1400
	0.025	0.3695402	0	0	183.82	431.97	1600	5000	0	0	1350
0.05	0.05	0.4111048	0	0	204.32	455.89	1650	4850	0	0	1400
	0.025	0.3690423	0	0	183.37	430.88	1650	4950	0	0	1350
0.25	0.05	0.460666	0	0	205.22	429.78	1700	4900	0	0	1350
	0.025	0.4345143	0	0	129.54	428.69	1750	4850	0	0	1350

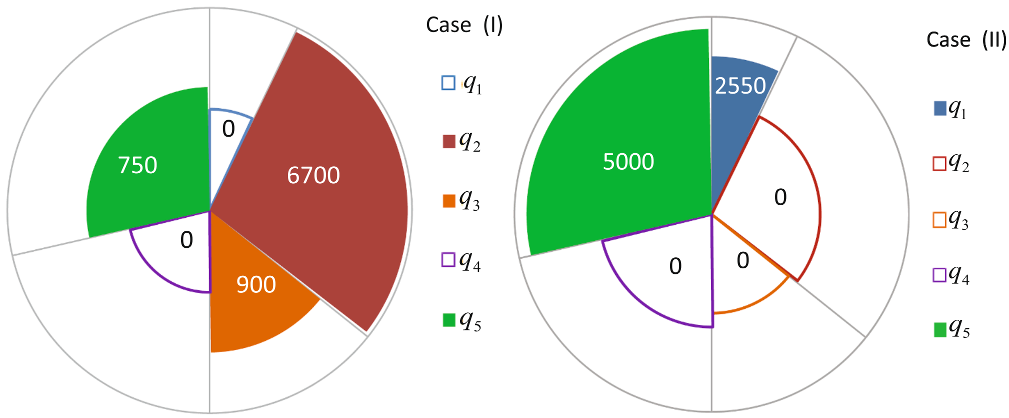


Fig. 5. Optimal result about cost uncertainty.

$d_1^+ = 380070$, $d_2^+ = 0$, $d_3^- = 132.83$ and $d_4^- = 639.5$ corresponding to case (ii). This means that the larger the perturbation coefficients are, the less likely the goals is to be achieved. Therefore, acquiring as much CO_2 emissions information as possible in the process of planning makes a certain positive impact on decision making. Moreover, the results regarding selection and order allocation are shown in Fig. 6.

The impact of uncertain CO_2 emissions levels on the resulting suppliers selection and corresponding order allocation are depicted in Fig. 6. We can intuitively observe from the figure that the solutions corresponding to different levels of CO_2 emissions uncertainty are different. When the perturbation coefficient is set as in case (i), the supplier portfolio includes suppliers 1, 2, 3 and 5, and the order quantities assigned for the contracted suppliers are 1,550, 3,050, 1,700 and 1,350. Different from case (i), the supplier combination in case (ii) is 2 and 5, and the order allocation are 1,200 and 6,350, respectively. As uncertainty levels vary, the optimal supplier portfolio and order allocation change accordingly. Hence, decision makers can make substantial decisions and provide sustainable strategic planning by ascertaining the levels of CO_2 emissions uncertainty.

5.3.4. *Effects of joint cost and* CO_2 *emissions uncertainty on decisions*

To investigate the effects of uncertainty on joint cost and CO_2 emissions on decisions, several experiments have been conducted by simultaneously adjusting cost uncertainty and CO_2 emissions uncertainty. The set of experiments is conducted under $\theta = 0.1$, $\epsilon = 0.1$, $\sigma = 0.4126421$ and $(\alpha,\ \beta) = (6000,\ 10000)$. The resulting order allocations shown in Fig. 7 are obtained with respect to case 1 and case 2. Case 1 represents that the cost perturbation coefficients and CO_2^l perturbations are set as in case (I) and case (i), respectively. Similarly, case 2 represents that the cost perturbation coefficients and CO_2 perturbation coefficients are set as in case (II) and case (ii), respectively.

The realization degrees of the goals with respect to cases 1 and 2 are different. For case 1, the deviations d_3^- and d_4^- are 119.2 and 645.4, while the others are zero. Comparing the deviations in the two cases, the deviations in case 2 are $d_1^+ = 398310$, $d - 2^+ = 0$, $d_3^- = 45.725$ and $d_4^- = 648.5$. The difference in deviations d_1^+ and d_3^- is distinct. Hence, the joint cost and CO_2 emissions uncertainty obviously and significantly impacts the unfulfillment degree of the goals. Moreover, the resulting supplier selection and order allocation for cases 1 and 2 are listed in Fig. 7. Fig. 7 reveals the impacts of joint cost and CO_2 emissions uncertainty on decision making. For case 1, suppliers 2, 3, 4 and 5 are assigned to supply order qualities of 3,750, 1,250, 1,750 and 850, respectively. When the perturbation coefficients are set as in case (ii), the suppliers 3 and 5 are selected to provide order qualities of 1,150 and 6,400, quite different from case 1. This indicates that the corresponding optimal decision will also change when the uncertainty levels of cost and CO_2 emissions vary at the same time.

5.3.5. *Effects of demand uncertainty on decisions*

To evaluate the impact of demand uncertainty on optimal decisions, the proposed model has been run with different perturbation coefficients D^l: $D^l = 200$ and $D^l = 400$. The set of experiments is conducted under $\theta = 0.1$, $\epsilon = 0.1$, $\sigma = 0.4126421$ and $(\alpha,\ \beta) = (6000,\ 10000)$. Under the two different D^l, the deviations differ slightly. The deviations are $d_3^- = 152.84$ and $d_4^- = 394.32$ in the case of $D^l = 200$, and $d_3^- = 157.18$ and $d_4^- = 399$ for $D^l = 400$. This shows that the demand uncertainty of constraints makes a small impact on the realization of the goals. In addition, Fig. 8 summarizes the resulting order allocation for $D^l = 200$ and $D^l = 400$.

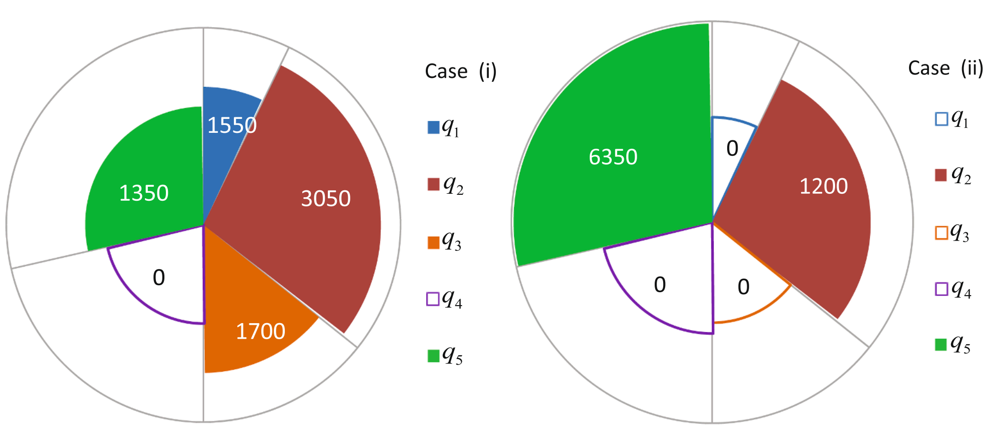


Fig. 6. Optimal result under CO_2 emissions uncertainty.

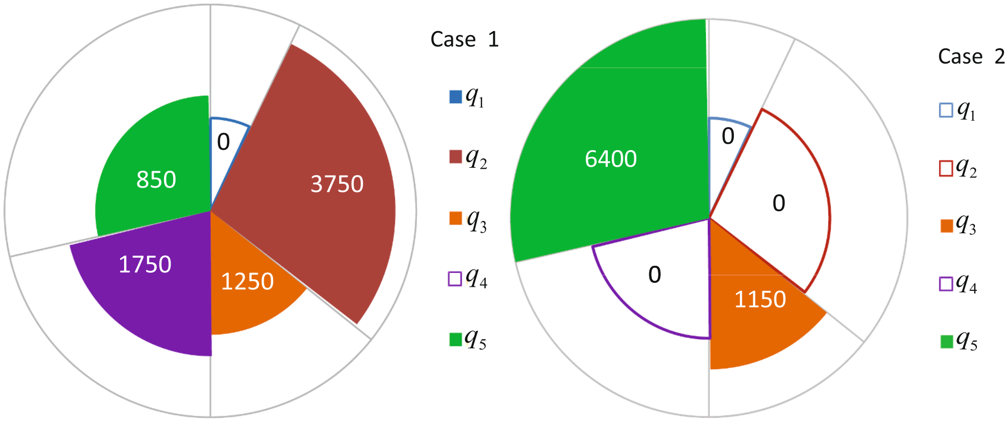


Fig. 7. Optimal result under joint uncertainty on cost and CO_2 emissions.

Fig. 8 provides two sets of optimal supplier portfolios and corresponding order allocations in the cases of different demand perturbation coefficients D^l. When $D^l = 200$, suppliers 1, 2 and 5 are contracted and supply order qualities of 2,150, 4,450 and 1,400, which are different from the order qualities 1,950, 4,700 and 1,350 requested from suppliers 1, 2 and 5 in the case of $D^l = 400$. That is, although different uncertainty levels of demand have no effect on the choice of suppliers, they have some influence on the order allocation. As a result, decision makers can reasonably allocate orders among the contracted suppliers by determining the perturbation coefficient D^l in order to achieve sustainability.

5.3.6. Effects of weights and performance coefficients on decisions

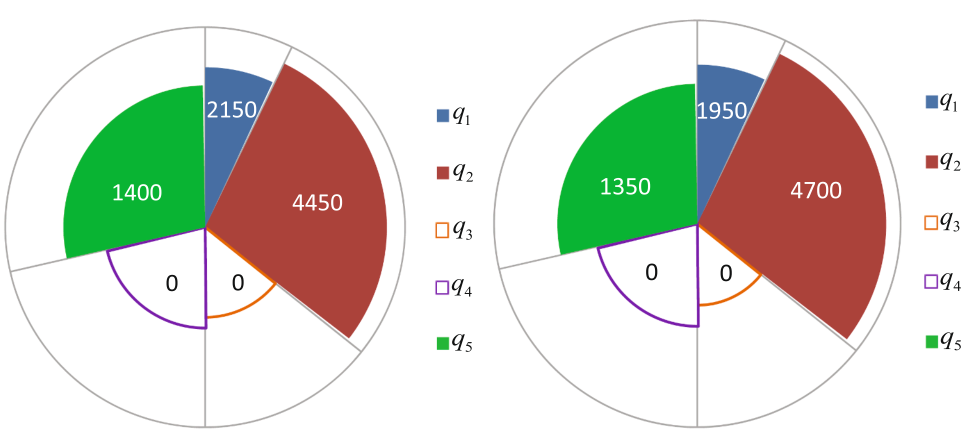
In this subsection, we explore the effects of weights W^{eco}, W^{env} and W^{soc} and performance coefficients w_j^{eco}, w_j^{env} and w_j^{soc} on results, and experiments are conducted under $\theta = 0.09$, $\epsilon = 0.1$, $\sigma = 0.4126421$ and $(\alpha, \beta) = (5500, 10000)$.

We show that the influence of weights on the optimal results is exerted through the following eight groups of weights $[W^{eco}, W^{env}, W^{soc}]$: (i) [0.2934, 0.3108, 0.3958], (ii) [0.2934, 0.4108, 0.2958], (iii) [0.3958, 0.3508, 0.2458], (iv) [0.3534, 0.3108, 0.3458], (v) [0.4934, 0.2108, 0.2958], (vi) [0.5934, 0.2608, 0.1458], (vii) [0.6934, 0.1108, 0.1958], (viii) [0.7934, 0.1108, 0.0958]. The results of supplier selection and order allocation are $q_1 = 1750$, $q_2 = 4200$ and $q_5 = 1650$ under these weights. This invariable supplier selection and order allocation is primarily achieved because weights W^{eco}, W^{env} and W^{soc} are obtained based on the importance evaluation of the three sets of criteria by decision makers. Therefore, a change in the weights will not affect the supplier selection and order allocation. However, it will affect the realization of the comprehensive value. That is, the values of deviation d_4^- under the eight groups of weights are different, which is shown in Fig. 9.

In Fig. 9, the horizontal axis corresponds to the eight groups of weights and the vertical axis corresponds to the deviation d_4^-. From this figure, we can intuitively observe the variation in the unfulfillment degree of the 4th goal for the comprehensive value under the different weights. Specifically, the deviation d_4^- exhibits a decreasing trend with weight from (i) to (viii). From (i) to (viii), the weight of economic criteria is increasing, and the remaining weights of the environmental and social criteria are decreasing. Therefore, this shows that the deviation d_4^- decreases with increasing economic weight and decreasing environmental and social weights. This means that the decision makers must make more effort when they decide to pursue certain positive impacts of non-monetary criteria related to environment and society instead of only pursuing interests.

Furthermore, the performance coefficients w_j^{eco}, w_j^{env} and w_j^{soc} are obtained according to the evaluation of suppliers by decision makers with respect to the three sets of criteria. Hence, changes in these coefficients may produce a certain effect on the order allocation. To demonstrate this conjecture, we conducted experiments based on the two cases shown in Table 8. The solved deviations d_3^- and d_4^- differ between the two cases. The deviations d_3^- and d_4^- in case A are 58.375 and 296.94, respectively, which are different from the values of 362.35 and 615.08 corresponding to case B. Furthermore, the order allocation results with respect to the two cases are provided in Fig. 10.

Fig. 10 displays two sets of different order allocations among assigned suppliers 1, 2 and 5. For case A, suppliers 1 and 2 supply order qualities of 1,950 and 4,700, which are different from the order



(a) q_j under $D^l = 200$ (b) q_j under $D^l = 400$

Fig. 8. Optimal result under demand uncertainty.

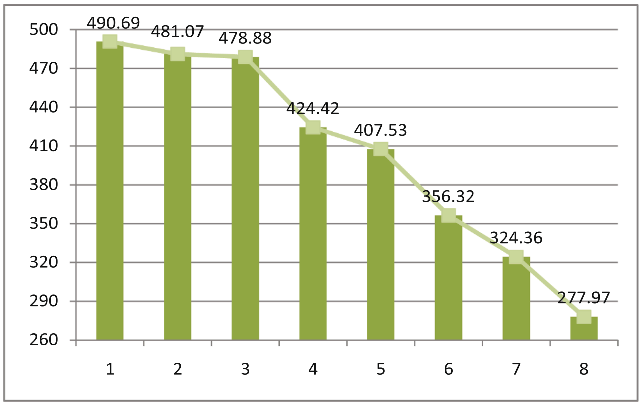


Fig. 9. The deviations d_4^- under different weights.

qualities 2,000 and 4,650 requested from suppliers 1 and 2 in case B. This reveals that the evaluation by decision maker regarding suppliers' performance with respect to the three sets of criteria has a certain impact on the results. Consequently, the above experiments demonstrate that the weights have a certain impact on the order allocation. Furthermore, the decision maker plays a vital role in the determination of the weights and even in the entire optimization process.

5.4. Managerial implications

The case study confirms that the distributionally robust goal programming method is beneficial to the purchasing company with respect to selecting proper suppliers and allocating the orders in pursuit of sustainability. The results illustrate the managerial and practical implications of the proposed method. We summarize these implications as follows.

- It should be noted that in the steel company application, the decision makers are not obligated to obtain accurate and true distributions of the input data. Decision makers can plan the sustainable SS/OA using the proposed method even if they do not have access to the complete input data information.
- According to the experimental results, it is important for decision makers to note that the multiple conflicting objectives are almost impossible to simultaneously achieve. The newly presented method with priority structure can explore the trade-offs among multiple conflicting objectives in an uncertain environment to fulfil as many goals as possible.
- Numerical experiments show that satisfactory results can be obtained by assigning the parameters involved in this model. By adjusting the acceptable waste rate θ and risk level (α, β), the corresponding changes in the satisfactory solution can be observed. The results can be regarded as a guideline for decision makers to make appropriate decisions in order to meet the needs of company development.

Table 8
Two sets of performance coefficients.

Case A	S1	S2	S3	S4	S5
w_j^{eco}	0.508915	0.56469	0.540057	0.437148	0.571704
w_j^{env}	0.52003	0.503307	0.466368	0.46827	0.478947
w_j^{soc}	0.454708	0.49308	0.570297	0.513594	0.546273
Case B	S1	S2	S3	S4	S5
w_j^{eco}	0.562485	0.48402	0.462906	0.534292	0.517256
w_j^{env}	0.57477	0.431406	0.399744	0.57233	0.433333
w_j^{soc}	0.502572	0.42264	0.488826	0.627726	0.494247

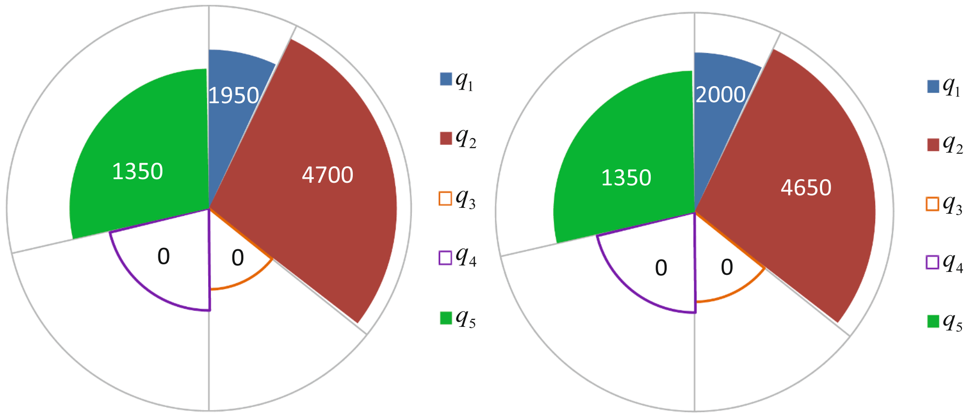
- By adjusting the parameter σ, probability level ϵ and demand uncertainty level D^l, the supplier selection does not change; however, the results of the optimal order allocation among selected suppliers will change accordingly. After the suppliers are selected, decision maker may formulate better decisions on order allocation by adjusting σ, ϵ and D^l.
- Adjusting the uncertainty level of cost and CO_2 emissions not only changes the supplier choice, but also changes the distribution of orders among the contracted suppliers. Decision makers may choose parameters according to personal experience and knowledge in order to make the satisfactory decision.
- The developed method for solving the sustainable SS/OA problem is flexible; it can be extended or modified in lines with the practical needs of the problem. For example, decision makers can provide the proper priority levels among multiple objectives under the requirements of time and company development.

6. Conclusions

This paper proposed a new distributionally robust goal programming model including expected constraints and chance constraints for sustainable SS/OA problems under distribution uncertainty and multiple conflicting objectives. In detail, the exact distributions of the uncertain per unit purchasing cost, per unit transportation cost, per unit CO_2 emissions, demand, supply capacity and minimum order quality are unavailable. We characterized the imprecise distributions by ambiguous distribution sets, and incorporated this type of uncertainty into our model under the idea of the distributionally robust optimization method rather than conventional methods. Moreover, we optimized the multiple conflicting objectives related to cost, CO_2 emissions, society and suppliers' comprehensive value with the priority structure, while incorporating risk measures for cost and CO_2 emissions into our sustainable SS/OA model. Because of the intractability of the distributionally robust sustainable SS/OA goal programming model, we derived robust counterpart forms of the expected constraints and safe approximation systems of the chance constraints under the mean absolute semi-dispersion ambiguity sets and new perturbation sets. Consequently, this paper derived the new tractable robust approximation model.

We applied the proposed new model to a case study involving a steel company to illustrate the effectiveness of the model and conducted a series of comprehensive numerical experiments. The results demonstrated that the new model can serve as a quantitative tool and offer advice for decision makers to solve the sustainable SS/OA problem in order to achieve sustainability of the company under distribution uncertainty. The proposed optimization method is also applicable to different companies that are committed to long-term and sustainable development. These companies actively respond to sustainability. Thus they need to consider multiple conflicting goals that include some non-monetary goals related to CO_2 emissions, society and the suppliers' comprehensive value in addition to the monetary goal. Under the regulatory environment, the proposed model provides a tool for these companies to select multiple suitable suppliers among those suppliers who are willing to cooperate and act in compliance in the case of uncertainty.

In the scope of this research work, the issues of imprecise random distribution of uncertain input data have been investigated and addressed by developing a new distributionally robust goal programming method. In further work, the issues of imprecise possibility distribution of uncertain parameters can also be studied and investigated by the fuzzy optimization method (Bai, Zhang, & Liu, 2018; Liu, Chen, Liu, & Qin, 2013) and parametric credibilistic optimization method (Liu, Bai, & Yang, 2017). Furthermore, the implementation of other ambiguity sets for the uncertain parameters in the sustainable SS/OA problem can be studied in future works.



(a) q_j under case A

(b) q_j under case B

Fig. 10. Optimal result under different performance coefficients.

Declaration of Competing Interest

The authors declare that they have no conflict of interest.

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