



Distributionally robust design for bicycle-sharing closed-loop supply chain network under risk-averse criterion

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ABSTRACT

The requirement of environmental improvement has led to the innovative emergence of shared bicycles. The production and recycling of shared bicycles are a closed logistics network, which can be considered a typical closed-loop supply chain (CLSC) problem. In practice, the CLSC network is influenced by social, economic and environmental factors, which impose high degrees of uncertainty and usually trigger various unanticipated risks, so controlling uncertain parameters becomes a key issue in supply chain decisions. The purpose of this research is to construct a new distributionally robust optimization model for a multi-product, multi-echelon CLSC network, in which the distributions of uncertain transportation cost, demand and the returned product are only partially known in advance. In the proposed model, robust mean-CVaR optimization formulation is employed as the objective function for a trade-off between the expected cost and the risk in the CLSC network. Further, to overcome the obstacle of model solvability resulting from imprecise probability distributions, two kinds of ambiguity sets are used to transform the robust counterpart into its computationally tractable forms. Finally, a case study on a Chinese bicycle-sharing company is addressed to validate the proposed robust optimization model. A comparison study is conducted on the performance between our robust optimization method and the traditional optimization method. In addition, a sensitivity analysis is performed with respect to the risk aversion parameter and the confidence level.

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1. Introduction

The growing awareness of the public for protecting the environment and conserving natural resources has led to the rapid development of a shared economy that has been introduced into many aspects of life, including transportation (e.g., bicycle-sharing (Zhang and Schmocker, 2019) and trucks-sharing (Vahdani et al., 2019)), lodging (e.g., home-sharing (Gyodi, 2019)), and consumption (e.g., food-sharing (Ukolov et al., 2016)). In 2017, the number of shared bicycle users reached 70 million, and Chinese bike-sharing companies served approximately 150 cities (Wang et al., 2019). The large demand necessitates higher requirements for the production and recycling of shared bicycles in a CLSC network.

Some important parameters in the CLSC network, such as demands and costs, are significantly uncertain (Keyvanshokoh et al.,

2016), and addressing the uncertainty has been recognized as a critical research issue in supply chain strategic decision making. Some recent studies estimate the probability distributions of model parameters from historical data and employ stochastic optimization (SO) (Birge and Louveaux, 1997; Ruszczyński and Shapiro, 2003) to model the uncertain CLSC network design problem. In practice, however, finding the stochastic nature of uncertain data and further specifying the exact probability distribution might be difficult, especially in large-scale, real-world applications (Ben-Tal et al., 2009; Bertsimas and Sim, 2003). When the exact probability distributions of model parameters are unknown and only partial distribution information is available, the modeling methodology will lead to the distributionally robust optimization (DRO) approach, which has recently been applied to supply chain management (Xu et al., 2018; Fu et al., 2018; Gao et al., 2019).

On the other hand, the risk associated with distribution uncertainty is a critical issue that needs to be addressed in the design of CLSC networks. Distribution uncertainty originates from two aspects: i) the uncertainty in the realization of uncertain parameters

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produces a large number of scenarios; *ii*) the ambiguity in the imprecise probability means that a realization has many possibilities. Both cases enhance the decision risk in the CLSC network. How should decision makers effectively avoid risk in designing CLSC networks? Introducing an appropriate and effective risk-averse evaluation criterion is a common way to avoid risk. As a popular coherent risk measure, the conditional value-at-risk (CVaR) (Rockafellar and Uryasev, 2000) is widely used to manage risk in various fields of science (Yu et al., 2017; Noyan, 2012). In the literature, earlier studies used CVaR to handle risk under exact probability distribution when designing a CLSC network (Soleimani et al., 2014; Cardoso et al., 2016; Baptista et al., 2019).

In contrast, this paper focuses on the design of a multi-scenario CLSC network under the robust mean-CVaR criterion, in which the imprecise probability occurs in transportation costs, demands and the returned products. More specifically, uncertain parameters in our model are considered by means of stochastic scenarios whose probability distributions are ambiguous. The uncertainty in transportation costs comes from fluctuations in fuel prices, and demands depend on customers' decisions, which are rooted in their survival circumstances, such as policy, cultural and natural environments. In addition, the amount of returned products depends on demand and then contains uncertainty. To model this uncertain CLSC network design problem, a distributionally robust optimization formulation is developed. Furthermore, the robust counterparts of the proposed robust optimization model can be converted to computationally tractable mixed-integer linear programming models. Finally, a case study on a real-world bicycle-sharing company in China is addressed to demonstrate the credibility of the proposed model. The computational results show that the proposed model can effectively balance the expected cost and the CVaR to prevent the inefficiencies of the former and reduce the uncertainty risk of the latter; While the distributionally robust optimization approach can overcome the disadvantage that the exact probability distribution information is unknown.

Compared with the existing literature, the main contributions of this paper include the following three aspects:

- This paper develops a distributionally robust model for the CLSC network design problem. The model possesses two distinct advantages: *i*) The proposed model does not require complete distribution information of uncertain model data. In reality, due to the complexity of the CLSC network, it is usually difficult to estimate and predict the distribution of transportation costs, demands and returned products. Instead, the distribution information of uncertain parameters is only partially known. A distributionally robust CLSC network formulation is designed, where uncertain parameters are characterized by imprecise discrete probability distributions; *ii*) Based on the risk-averse criterion, a mean-CVaR formulation is introduced to find a better trade-off between the average cost and cost overrun. The novelty of our formulation is that it adopts a flexible network configuration strategy, and the trade-off can be adjusted to decision makers' risk preferences.
- The robust counterpart of our distributionally robust formulation is a semi-infinite programming model that belongs to the family of hard optimization problems for general ambiguity sets. To transform the robust counterparts into their computationally tractable forms, two new theoretical results are obtained under the cases of box and polyhedral ambiguity sets. Finally, the optimal solution can be obtained by solving the equivalent mixed-integer linear programming model.
- To demonstrate the effectiveness and practicability of our proposed optimization model, we provide a realistic case study about a bicycle-sharing company in the Jing-Jin-Ji Metropolitan

Region of China. A comparison study between our distributionally robust optimization models and the nominal stochastic model is conducted via a number of numerical experiments. The computational results demonstrate the advantages of the proposed optimization model.

The structure of this paper is arranged as follows. Section 2 provides the related literature review and highlights our new approach. Section 3 describes the studied problem in detail, and develops an optimization model for our CLSC network design problem. Under the cases of box and polyhedral ambiguity sets, Section 4 discusses the equivalent deterministic programming models for the robust counterpart of our distributionally robust mean-CVaR optimization model. Section 5 introduces a real-world bicycle-sharing case in the Jing-Jin-Ji Metropolitan Region to demonstrate the applicability of the proposed model. Finally, Section 6 concludes the paper and suggests several areas for future research. All proofs are provided in the Appendix.

2. Literature review

The research on the CLSC network design begins with Fleischmann et al. (2001), following his pioneering work, numerous extended studies have emerged (Min and Zhou, 2002; Ko and Evans, 2007; Cheraghalipour et al., 2018). This section reviews the related literature in two main areas: uncertain CLSC network design and risk measures in uncertain CLSC network design. Finally, the proposed optimization approach in this paper is highlighted.

2.1. Uncertain CLSC network design

In realistic supply chain network systems, many works have attempted to design and optimize the supply chain network under increasing internal and external uncertainty (Govindan et al., 2017). To the best of our knowledge, a large number of CLSC network researchers have focused on inherent uncertainties caused by several factors, such as the price, cost, supply, returned products and especially demands (Fattahi and Govindan, 2017; Polo et al., 2018; Ahmadi and Amin, 2019). The related literature summarization is illustrated in Table 1 wherein the uncertain parameters have been shown in columns 4 to 11.

To handle these uncertainties, some existing studies use stochastic optimization as a common paradigm for designing the CLSC network with exact probability distribution. For example, Ma and Liu (2017) discussed a stochastic CLSC network model with uncertain transportation costs and demands, where uncertain parameters were characterized by discrete joint probability distributions. Ebrahimi (2018) proposed a multi-objective stochastic optimization model for a location-allocation-routing problem from the perspective of sustainable development.

Considering the reality in the CLSC, however, it may be difficult to obtain the exact probability distribution. In this case, robust optimization approach is an alternative to handle this issue. Several studies employed robust optimization approach to address uncertainty in the CLSC network (Kim et al., 2018; Ghahremani-Nahr et al., 2019; Saedinia et al., 2019). Keyvanshokoh et al. (2016) developed a hybrid robust-stochastic optimization approach to modeling qualitatively uncertainties in the CLSC network design, where transportation costs are random parameters and demands and returns belong to polyhedral uncertainty sets. Additionally, Safaei et al. (2017) considered uncertain demand in a cardboard recycling network model and solved the model with robust optimization approach.

In addition, a few scholars have analyzed the situation of partially known distribution information in supply chain network

Table 1
Gap analysis of the related researches.

Researches	Network type		Uncertain parameter					Distribution information			Risk measure	
	Forward	Reverse	Prices	Costs	Supply	Demands	Returned products	Others	Exact	Partially known	Unknown	
Pishvaei et al. (2009)	*	*	*			*	*	*				Mean
El-Sayed et al. (2010)	*	*				*	*	*				Mean
Soleimani et al. (2014)	*	*	*			*	*	*				MAD, VaR, CVaR
Keyvanshokoh et al. (2016)	*	*	*			*	*	*			*	—
Bai and Liu (2016)	*	*	*			*	*	*		*		VaR
Cardoso et al. (2016)	*	*				*	*	*				Variance, Downside risk, Variability index, CVaR
Fattahi and Govindan (2017)	*	*				*	*	*				Mean
Ma and Liu (2017)	*	*	*			*	*	*				VaR
Yang and Liu (2017)	*	*	*			*	*	*				VaR
Safaei et al. (2017)	*	*				*	*	*				Mean, Variance
Salimi and Vahdani (2018)	*	*				*	*	*		*		Mean
Kim et al. (2018)	*	*				*	*	*			*	—
Polo et al. (2018)	*	*				*	*	*			*	Net Present Value, Internal Rate of Return
Ebrahimi (2018)	*	*				*	*	*				Mean
Cheraghalipour et al. (2018)	*	*				*	*	*				—
Prakash et al. (2018)	*	*			*	*	*	*			*	Supply side risk, Transportation side risk
Ahmadi and Amin (2019)	*	*				*	*	*				—
Gahremani-Nahr et al. (2019)	*	*	*	*		*	*	*				Mean
Saedinia et al. (2019)	*	*	*	*		*	*	*			*	—
Baptista et al. (2019)	*	*	*	*		*	*	*	*	*		Time stochastic dominance
Zhen et al. (2019)	*	*				*	*	*				Mean
Zhao et al. (2018)	*	*				*	*	*				CVaR
This paper	*	*	*	*		*	*	*	*	*		Mean-CVaR

design models. Bai and Liu (2016) presented a robust supply chain network design model by incorporating the uncertainties in variable distributions. Salimi and Vahdani (2018) designed a bio-fuel network considering the variability of the failure probability. Vahdani and Mohammadi (2015) developed an interval-robust design model for the CLSC network problem with imprecise distributions, and a new hybrid solution approach was proposed by combining several approaches and programs. In Table 1, the aforementioned literature is classified and summarized with respect to the different types of uncertain distribution information in columns 12 to 14.

2.2. Risk measures in uncertain CLSC network design

Risk neutrality is a common risk attitude in dealing with risky issues. When decision makers are interested in the average performance of a CLSC network system, the model constructed with the expected cost is appropriate. Many CLSC network design problems have considered the expected profit or cost in the objective function and the constraints. For example, based on the perspective of risk neutrality, El-Sayed et al. (2010) formulated a stochastic, multi-stage decision making model to maximize the total expected profit in the objective function; Zhen et al. (2019) took into account the total expected cost in the objective function by a scenario optimization approach.

In many decision processes, however, risk neutrality is not suitable for the reality of CLSC management. To obtain more realistic strategies, some studies handled the decision risk caused by uncertainty. Some usual risk measures were introduced to the CLSC network design problem, such as the variance (Safaei et al., 2017), VaR (Ma and Liu, 2017) and the CVaR (Zhao et al., 2018). Soleimani et al. (2014) drew risk measures (VaR, CVaR and MAD) from financial optimization problems into the CLSC network design problem and compared three types of models, including three risk measures. Furthermore, Cardoso et al. (2016) used four financial risk measures (the CVaR, variance, downside risk and variability index)

to assess the risk in CLSC design and planning problems. Both of these studies concluded that the CVaR is a well-behaving and predominant risk measure.

The measures used in the aforementioned studies are regarded as risk measure-based functions (Baptista et al., 2019). Recently, some scholars introduced some risk-averse methods to reduce the risk in CLSC networks. Polo et al. (2018) designed a robust CLSC model, which employed the net present value and the internal rate of return to prevent the occurrence of financial risk. According to Prakash et al. (2018), supply side risk and transportation side risk are embedded in designing the CLSC network model to address risk events in the supply chain network. Baptista et al. (2019) mixed the chance-constrained and time stochastic dominance risk-averse strategies to reduce the risk of a multi-period, multi-product CLSC design and operation planning problem. In Table 1, the used risk measures of the related CLSC studies have been sorted out in the last column.

2.3. Our new approach

To identify the research gap of existing studies on CLSC networks and to clarify the position of the present study in the related literature, we classify some of the literature with four terms: network type, uncertain parameters, the distribution information and risk measures in Table 1, and the characteristics of our study have been shown in the last row.

More specifically, we discover that some researchers characterize uncertain parameters with exact probability distributions from the risk-neutral perspective. In practice, however, it is difficult to evaluate the probability distributions in capturing the historical data with high accuracy. In addition, the risk-averse criteria may be more suitable for the reality of CLSC management in many situations, and the CVaR is regarded as an outperforming risk criterion from the comparative analysis of several risk measures. Motivated by these observations, this paper studies a multi-scenario CLSC network design problem with the CVaR to help supply chain

managers measure organizational risk. Since only partial information about the probability distributions of the model parameters is available, a new distributionally robust optimization model is developed. Under box and polyhedral ambiguity sets, the robust counterparts of our proposed distributionally robust mean-CVaR optimization model are translated into their equivalent computationally tractable optimization models, which can be solved by CPLEX optimization software to obtain the optimal strategy.

3. Multi-scenario CLSC network design model

3.1. Problem description and assumptions

In this paper, our CLSC network is a single-period, multi-part, multi-product CLSC network with uncertain demands, returned products and transportation costs. It has been suggested that this kind of CLSC network suits many related industry fields, such as the automobile, electronics and bicycle industries. Taking a bicycle-sharing CLSC network as an example, Fig. 1 illustrates a complete network that consists of the suppliers, manufactories, distribution centers, user areas, recycling/dismantling centers, and waste disposal centers.

In the entire CLSC network, the manufacturers buy components needed for producing products from potential suppliers and then send products to distribution centers. Further, distribution centers transfer them to user areas based on demand. Note that the user areas are supposed to be predetermined in advance. In such a CLSC network, hybrid facilities save on potential costs compared with separate recycling or dismantling centers. Thus, both recycling and dismantling centers are established at the same location, and the returned products are sent there. After dismantling, the useful components of the products are sent to the manufactories, and the useless components are transported to waste disposal centers. In particular, potential suppliers will give different price discounts according to the number of orders for the components that are required by the manufactories. In addition to obtaining components from suppliers, manufactories can also receive components from recycling/dismantling centers. In this paper, based on the mean-CVaR optimization criterion, a multi-scenario mathematical model under uncertainty will be presented. In addition, all assumptions needed in the proposed optimization model are given as follows:

- (A1) Manufactories and user areas are fixed.
- (A2) According to the order quantity, potential suppliers offer different price discounts which are known.
- (A3) Each product is made up of multi-components.
- (A4) The backlogging of the unsatisfied demand is not allowed in the network, recycling/dismantling centers will fully collect all the returned products from user areas.
- (A5) The average disposal fraction is deterministic.

Under the above assumptions, a multi-scenario CLSC network design model will be proposed in the next section.

3.2. Formulation of CLSC network design model

3.2.1. Constraints

The following constraints play an important role in the formulation of our CLSC network design problem.

3.2.1.1. Demand and return satisfaction constraints.

$$\sum_{k \in \mathcal{K}} z_{klp}^s + \omega_{lp}^s \geq d_{lp}^s, \quad \forall l, p, s, \tag{1}$$

$$\sum_{m \in \mathcal{M}} o_{imp}^s = r_{lp}^s, \quad \forall l, p, s. \tag{2}$$

The demands of all user areas are dominated by constraint (1). The returned products from all user areas will be collected as shown in constraint (2).

3.2.1.2. Balance constraints.

$$\sum_{i \in \mathcal{I}} \sum_{h \in \mathcal{H}} x_{ijrh}^s + \sum_{m \in \mathcal{M}} t_{mjr}^s = \sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} y_{jkp}^s \delta_{rp}, \quad \forall j, r, s, \tag{3}$$

$$\sum_{j \in \mathcal{J}} y_{jkp}^s = \sum_{l \in \mathcal{L}} z_{klp}^s, \quad \forall k, p, s, \tag{4}$$

$$\sum_{j \in \mathcal{J}} t_{mjr}^s = (1 - \theta_r) \sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}} o_{lmp}^s \delta_{rp}, \quad \forall r, m, s, \tag{5}$$

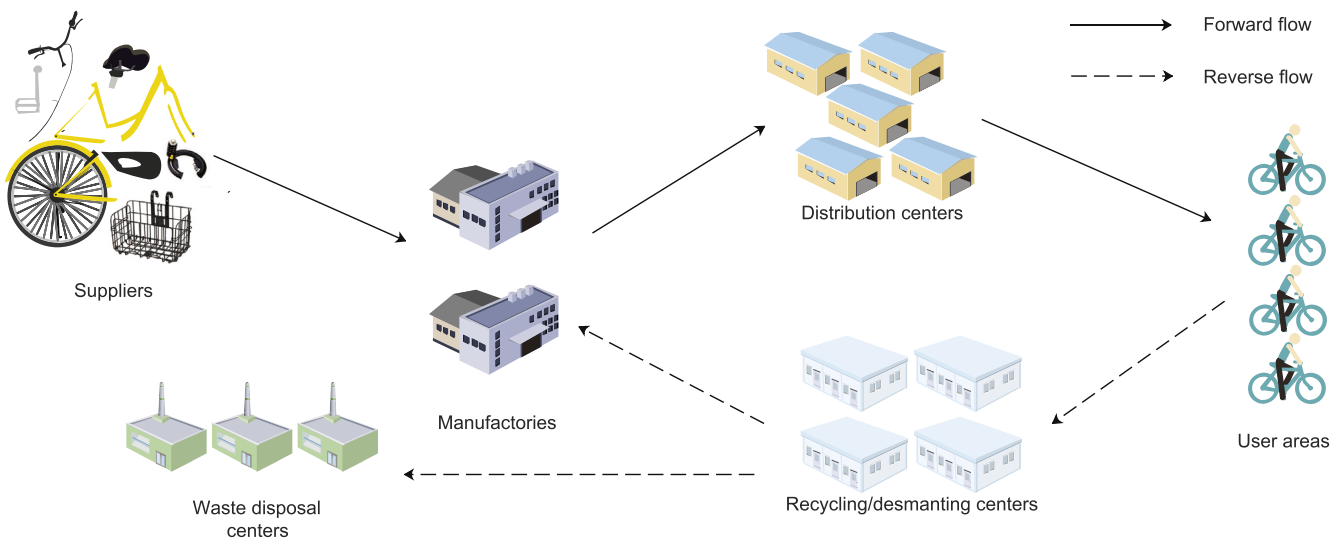


Fig. 1. The structure of a bicycle-sharing CLSC network.

$$\sum_{n \in \mathcal{N}} f_{mnr}^s = \theta_r \sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}} o_{lmp}^s \delta_{rp}, \quad \forall r, m, s. \quad (6)$$

Constraints (3)–(6) are the balance constraints. Constraint (3) guarantees that the components needed for the products produced by manufactories can be satisfied by suppliers and the recycling/dismantling centers. Constraint (4) assures that all products from manufacturers are transported to user areas. Constraints (5)–(6) ensure that the returned products from the user areas are completely disassembled and transported to manufactories or waste disposal centers.

3.2.1.3. Constraints of the quantity discount schemes for suppliers.

$$g_{ijrh} \rho_{irh-1} \leq x_{ijrh}^s \leq g_{ijrh} \rho_{irh}^*, \quad \forall i, j, r, h, s, \quad (7)$$

$$\sum_{h \in \mathcal{H}} g_{ijrh} \leq 1, \quad \forall i, j, r. \quad (8)$$

Let ρ_{irh} be the maximum number of discounts $h \in \mathcal{H}$ for a component $r \in \mathcal{R}$ supplied by supplier $i \in \mathcal{I}$, and $\rho_{irh-1} < \rho_{irh}^* \leq \rho_{irh}$. To ensure that manufacturers are able to buy components reasonably, constraint (7) requires that the number of components bought from a supplier is within the discount interval $[\rho_{irh-1}, \rho_{irh}^*]$, and the supplier will provide a specific price discount. If only one type of component is procured from a supplier, then the corresponding discount level is just one. This criterion is reflected in constraint (8).

3.2.1.4. Capacity constraints.

$$\sum_{j \in \mathcal{J}} \sum_{h \in \mathcal{H}} x_{ijrh}^s \leq ss_{ir} u_i, \quad \forall i, r, s, \quad (9)$$

$$\sum_{k \in \mathcal{K}} y_{jpk}^s \leq sp_{jp}, \quad \forall j, p, s, \quad (10)$$

$$\sum_{j \in \mathcal{J}} y_{jpk}^s \leq v_k sd_{kp}, \quad \forall k, p, s, \quad (11)$$

$$\sum_{l \in \mathcal{L}} z_{klp}^s \leq v_k sd_{kp}, \quad \forall k, p, s, \quad (12)$$

$$\sum_{l \in \mathcal{L}} o_{lmp}^s \leq c_m sc_{mp}, \quad \forall m, p, s, \quad (13)$$

$$\sum_{m \in \mathcal{M}} f_{mnr}^s \leq w_n sp_{nr}, \quad \forall n, r, s, \quad (14)$$

$$\sum_{j \in \mathcal{J}} t_{mjr}^s + \sum_{n \in \mathcal{N}} f_{mnr}^s \leq c_m \sum_{p \in \mathcal{P}} sc_{mp} \delta_{rp}, \quad \forall m, r, s. \quad (15)$$

Constraints (9)–(15) set capacity limits. Constraint (9) is the supply capacity of the parts from suppliers; constraints (10)–(13)

are the storage capacities of the products at the manufactories, the distribution centers, and the recycling/dismantling centers, respectively; constraints (14)–(15) are the storage capacities of the parts at the recycling/dismantling centers and the waste disposal centers, respectively.

3.2.1.5. Binary and non-negativity constraints.

$$g_{ijrh}, u_i, v_k, c_m, w_n \in \{0, 1\}, \quad \forall i, j, r, h, k, m, n, \quad (16)$$

$$x_{ijrh}^s, y_{jpk}^s, z_{klp}^s, p_{lmp}^s, t_{mjr}^s, f_{mnr}^s \geq 0, \quad \forall i, j, k, l, m, n, r, p, h, s. \quad (17)$$

Constraint (16) expresses the binary restrictions of decision variables. Constraint (17) indicates the non-negativity restrictions of the corresponding decision variables.

3.2.2. The objective function

Before establishing the objective function, we first introduce the total cost of the entire CLSC network, which includes four terms under a scenario s .

The first term is the fixed costs of the opening facilities,

$$TFC = \sum_{i \in \mathcal{I}} fs_i u_i + \sum_{k \in \mathcal{K}} fd_k v_k + \sum_{m \in \mathcal{M}} fc_m c_m + \sum_{n \in \mathcal{N}} fp_n w_n.$$

The second term is the processing costs under a particular scenario s :

$$TPC_s = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{h \in \mathcal{H}} k_h cs_{ir} x_{ijrh}^s + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} cm_{jp} y_{jpk}^s + \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} cr_{mr} t_{mjr}^s + \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} \sum_{r \in \mathcal{R}} cd_{nr} f_{mnr}^s.$$

The third term is the transportation costs between the facilities under a scenario s :

$$TTC_s = \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} \sum_{h \in \mathcal{H}} tsp_{ijr}^s x_{ijrh}^s + \sum_{j \in \mathcal{J}} \sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} tpd_{jpk}^s y_{jpk}^s + \sum_{k \in \mathcal{K}} \sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}} tdc_{klp}^s z_{klp}^s + \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} \sum_{p \in \mathcal{P}} tcc_{lmp}^s o_{lmp}^s + \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{J}} \sum_{r \in \mathcal{R}} tcp_{mjr}^s t_{mjr}^s + \sum_{m \in \mathcal{M}} \sum_{n \in \mathcal{N}} \sum_{r \in \mathcal{R}} tcd_{mnr}^s f_{mnr}^s.$$

The fourth term is the penalty costs of the network under a particular scenario s :

$$PC_s = \sum_{l \in \mathcal{L}} \sum_{p \in \mathcal{P}} \pi_{lp} \omega_{lp}^s.$$

Based on the above notations, the total cost of the CLSC under a particular scenario s is obtained as follows:

$$TC_s = TFC + TPC_s + TTC_s + PC_s = \mathbf{C}_s^T \boldsymbol{\tau}_s,$$

where

$$\mathbf{C}_s = (fs_1, \dots, fs_{|\mathcal{I}|}, fd_1, \dots, fd_{|\mathcal{K}|}, fc_1, \dots, fc_{|\mathcal{M}|}, fp_1, \dots, fp_{|\mathcal{N}|}, k_1 cs_{11}, \dots, k_{|\mathcal{K}|} cs_{|\mathcal{J}||\mathcal{P}|} + tsp_{111}^s, \dots, k_{|\mathcal{K}|} cs_{|\mathcal{J}||\mathcal{P}|} + tsp_{|\mathcal{J}||\mathcal{K}||\mathcal{P}|}^s, cm_{11} + tpd_{111}^s, \dots, cm_{|\mathcal{J}||\mathcal{K}||\mathcal{P}|} + tpd_{|\mathcal{J}||\mathcal{K}||\mathcal{P}|}^s, cp_{11} + tdc_{111}^s, \dots, cp_{|\mathcal{J}||\mathcal{K}||\mathcal{P}|} + tdc_{|\mathcal{J}||\mathcal{K}||\mathcal{P}|}^s, cc_{11} + tcc_{111}^s, \dots, cc_{|\mathcal{L}||\mathcal{M}||\mathcal{P}|} + tcc_{|\mathcal{L}||\mathcal{M}||\mathcal{P}|}^s, cr_{11} + tcp_{111}^s, \dots, cr_{|\mathcal{M}||\mathcal{J}||\mathcal{R}|} + tcp_{|\mathcal{M}||\mathcal{J}||\mathcal{R}|}^s, cd_{11} + tcd_{111}^s, \dots, cd_{|\mathcal{M}||\mathcal{N}||\mathcal{R}|} + tcd_{|\mathcal{M}||\mathcal{N}||\mathcal{R}|}^s, \pi_{11}, \dots, \pi_{|\mathcal{L}||\mathcal{P}|}, 0, \dots, 0)^T$$

and assumed to be known, and the optimal supply chain configuration

$$\tau_s = \left(u_1, \dots, u_{|\mathcal{S}|}, v_1, \dots, v_{|\mathcal{S}|}, c_1, \dots, c_{|\mathcal{S}|}, w_1, \dots, w_{|\mathcal{S}|}, x_{1111}^s, \dots, x_{|\mathcal{S}||\mathcal{S}||\mathcal{S}||\mathcal{S}|}^s, y_{111}^s, \dots, y_{|\mathcal{S}||\mathcal{S}||\mathcal{S}|}^s, z_{111}^s, \dots, z_{|\mathcal{S}||\mathcal{S}||\mathcal{S}|}^s, o_{111}^s, \dots, o_{|\mathcal{S}||\mathcal{S}||\mathcal{S}|}^s, t_{111}^s, \dots, t_{|\mathcal{S}||\mathcal{S}||\mathcal{S}|}^s, f_{|\mathcal{S}||\mathcal{S}||\mathcal{S}|}^s, \omega_{11}^s, \dots, \omega_{|\mathcal{S}||\mathcal{S}||\mathcal{S}|}^s, g_{1111}, \dots, g_{|\mathcal{S}||\mathcal{S}||\mathcal{S}||\mathcal{S}|} \right)^T.$$

As a consequence, the expected value of the total cost can be represented by

$$E_{\mathbf{p}}[TC] = \sum_{s \in \mathcal{S}} p_s TC_s, \tag{18}$$

where $\mathbf{p} = (p_1, p_2, \dots, p_{|\mathcal{S}|})^T$, $p_s > 0$ is the probability of scenario s and $\sum_{s \in \mathcal{S}} p_s = 1$.

Noting that the CVaR can quantify the losses that might be encountered in the tail of the probability distribution function, it becomes a common tool in the risk decision-making problem of a supply chain. Hence, this paper employs the CVaR measure to construct the following cost function for our CLSC network design problem:

$$CVaR_{\alpha, \mathbf{p}}[TC] = \min_{\phi \in \mathbf{R}^+} \left\{ \phi + \frac{1}{1 - \alpha} E[\max\{TC - \phi, 0\}] \right\}, \tag{19}$$

where $\alpha \in (0, 1)$ is the confidence level. By introducing the additional variables t_s to represent $\max\{TC_s - \phi, 0\}$ for $s = 1, 2, \dots, |\mathcal{S}|$ and expanding the expected value of $\max\{TC - \phi, 0\}$ for all scenarios, problem (19) can be equivalently represented as the following linear programming model:

$$\begin{aligned} \min_{\phi \in \mathbf{R}^+} \quad & \phi + \frac{1}{1 - \alpha} \sum_{s \in \mathcal{S}} p_s t_s \\ \text{s.t.} \quad & TC_s - \phi \leq t_s, \quad \forall s, \\ & t_s \geq 0, \quad \forall s. \end{aligned}$$

Using the CVaR criterion, only the mean value of the costs above the confidence level is measured in the model, and the part of the costs below the confidence level is ignored. In this paper, the expected value and the CVaR of the total cost are simultaneously used to formulate our objective function. Combining (18) and (19), a multi-scenario mean-CVaR CLSC model is formally built as follows:

$$\begin{aligned} \min_{\tau} \quad & \lambda E_{\mathbf{p}}[TC] + (1 - \lambda) CVaR_{\alpha, \mathbf{p}}[TC] \\ \text{s.t.} \quad & \text{constraints (1) - (17)}, \end{aligned} \tag{20}$$

where $\tau = (\tau_1, \dots, \tau_{|\mathcal{S}|})^T$, and $\lambda \in [0, 1]$ is a risk aversion parameter.

In practice, it might be difficult to reliably specify the probability distributions of random parameters. Thus, probability distributions are assumed to be only partially known and belong to some ambiguity sets in this paper. In the next section, we will address the issue of imprecise discrete probability distributions.

4. Robust CLSC model with probability uncertainty

In the CLSC literature (Pishvae et al., 2009; Soleimani et al., 2014), the exact distributions of random variables are usually

obtained by solving the corresponding stochastic optimization model. In this section, we assume that the information about the probability distributions is partially available, and the discrete probability distribution vector \mathbf{p} belongs to an ambiguity set \wp .

4.1. Ambiguity sets

Due to the assumption of imprecise discrete probability distributions, our robust optimization models can be built with respect to the box and polyhedral ambiguity sets, where the box and polyhedral ambiguity sets are defined as

$$\wp_{\infty} = \left\{ \mathbf{p} = \mathbf{p}_0 + \xi \mid \mathbf{e}^T \xi = 0, \|\xi\|_{\infty} \leq \Psi \right\}, \tag{21}$$

$$\wp_1 = \left\{ \mathbf{p} = \mathbf{p}_0 + \mathbf{P}_1 \xi \mid \mathbf{e}^T \mathbf{P}_1 \xi = 0, \mathbf{p}_0 + \mathbf{P}_1 \xi \geq 0, \|\xi\|_1 \leq 1 \right\}. \tag{22}$$

In Eqs. (21) and (22), \mathbf{p}_0 is the nominal distribution that signifies the most likely probability distribution; \mathbf{e} represents the vector of ones; $\xi = (\xi_1, \xi_2, \dots, \xi_{|\mathcal{S}|})$ denotes the perturbation vector; Ψ is a real value in $[0, 1]$; and \mathbf{P}_1 is a known scaling matrix. The conditions $\mathbf{e}^T \mathbf{P}_1 \xi = 0$ and the nonnegativity constraint $\mathbf{p}_0 + \mathbf{P}_1 \xi \geq 0$ ensure \mathbf{p} meets the nonnegative property of the probability distribution. It is sensible to consider the box and polyhedral ambiguity sets, which are the simplest ambiguity sets to be specified, and the resulting problem can also be formulated in a computationally tractable manner.

4.2. Distributionally robust CLSC optimization model

In this section, the distributionally robust CLSC network design model is presented under the box and polyhedral ambiguity sets. This network design model is a family of stochastic optimization models (20) with probability \mathbf{p} belonging to the ambiguity set \wp , i.e.,

$$\left(\begin{aligned} \min_{\tau} \quad & \lambda E_{\mathbf{p}}[TC] + (1 - \lambda) CVaR_{\alpha, \mathbf{p}}[TC] \\ \text{s.t.} \quad & \text{constraints (1) - (17)} \end{aligned} \right)_{\mathbf{p} \in \wp}.$$

Further, based on “worst-case” oriented rule, the robust counterpart of the proposed distributionally robust model is formulated as

$$\begin{aligned} \min_{\tau} \quad & \lambda \max_{\mathbf{p} \in \wp} E_{\mathbf{p}}[TC] + (1 - \lambda) \max_{\mathbf{p} \in \wp} CVaR_{\alpha, \mathbf{p}}[TC] \\ \text{s.t.} \quad & \text{constraints (1) - (17)}. \end{aligned} \tag{23}$$

Note that $\max_{\mathbf{p} \in \wp} E_{\mathbf{p}}[TC]$ and $\max_{\mathbf{p} \in \wp} CVaR_{\alpha, \mathbf{p}}[TC]$ in problem (23) depend on the structure of the ambiguity set of discrete probability distributions. In addition, the equivalent form of the expected value of the total cost $\max_{\mathbf{p} \in \wp} E_{\mathbf{p}}[TC]$ can be represented by

$$\max_{\mathbf{p} \in \wp} E_{\mathbf{p}}[TC] = \max_{\mathbf{p} \in \wp} \mathbf{TC}^T \mathbf{p}, \tag{24}$$

while the equivalent form of the CVaR of the total cost $\max_{\mathbf{p} \in \wp} \text{CVaR}_{\alpha, \mathbf{p}}[TC]$ can be represented by

$$\begin{aligned} \min_{\phi \in \mathbf{R}^+} \quad & \phi + \frac{1}{1-\alpha} \max_{\mathbf{p} \in \wp} \mathbf{t}^T \mathbf{p} \\ \text{s.t.} \quad & TC_s - \phi \leq t_s, \quad \forall s, \\ & t_s \geq 0, \quad \forall s, \end{aligned} \tag{25}$$

where $\mathbf{TC}^T = (TC_1, \dots, TC_{|\mathcal{S}|}) = (\mathbf{C}_1^T \boldsymbol{\tau}_1, \dots, \mathbf{C}_{|\mathcal{S}|}^T \boldsymbol{\tau}_{|\mathcal{S}|}) = \mathbf{C}^T \boldsymbol{\tau}, \mathbf{C} = (\mathbf{C}_1^T, \dots, \mathbf{C}_{|\mathcal{S}|}^T)$ and $\mathbf{t} = (t_1, t_2, \dots, t_{|\mathcal{S}|})$.

Problem (23) is a semi-infinite programming model, which is usually computationally intractable for general ambiguity sets; finding its solution is a time-consuming process. To transform problem (23) into a tractable optimization model, problems (24) and (25) must be transformed into their computationally tractable models.

Under box and polyhedral ambiguity sets, the following observations are obtained.

Theorem 1. *If probability \mathbf{p} belongs to box ambiguity set (21), then problem (23) can be equivalently turned into the following computationally tractable optimization model with respect to variables $(\mu, \mu', \boldsymbol{\eta}, \boldsymbol{\eta}', \boldsymbol{\gamma}, \boldsymbol{\gamma}') \in \mathbf{R} \times \mathbf{R} \times \mathbf{R}^{|\mathcal{S}|} \times \mathbf{R}^{|\mathcal{S}|} \times \mathbf{R}^{|\mathcal{S}|} \times \mathbf{R}^{|\mathcal{S}|}$:*

$$\begin{aligned} \min_{\tau, \phi, \mu, \eta, \gamma, \mu', \eta', \gamma'} \quad & \lambda (\mathbf{C}^T \boldsymbol{\tau} \mathbf{p}_0 + \boldsymbol{\Psi}^T \boldsymbol{\eta} + \boldsymbol{\Psi}^T \boldsymbol{\gamma}) + (1-\lambda) \\ & \left[\phi + \frac{1}{1-\alpha} (\mathbf{t}^T \mathbf{p}_0 + \boldsymbol{\Psi}^T \boldsymbol{\eta}' + \boldsymbol{\Psi}^T \boldsymbol{\gamma}') \right] \text{ s.t. } TC_s - \phi \\ & \leq t_s, \quad \forall s, \quad \mathbf{e}\mu - \boldsymbol{\eta} + \boldsymbol{\gamma} = \boldsymbol{\tau}^T \mathbf{C}, \quad \mathbf{e}\mu' - \boldsymbol{\eta}' + \boldsymbol{\gamma}' = \mathbf{t}, \quad t_s \geq 0, \quad \forall s, \\ & \phi \geq 0, \quad \gamma \geq 0, \quad \boldsymbol{\eta}' \geq 0, \quad \boldsymbol{\gamma}' \geq 0, \quad \text{constraints (1) - (17)} \end{aligned} \tag{26}$$

Remark 1. In the case $\boldsymbol{\Psi} = 0$, problem (26) reduces to its nominal stochastic CLSC problem (20).

Theorem 2. *If probability \mathbf{p} belongs to polyhedral ambiguity set (22), then problem (23) can be equivalently turned into the following computationally tractable optimization model with respect to variables $(\vartheta, \vartheta', \nu, \nu', \boldsymbol{\zeta}, \boldsymbol{\zeta}') \in \mathbf{R}^{|\mathcal{S}|} \times \mathbf{R}^{|\mathcal{S}|} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R} \times \mathbf{R}$:*

$$\begin{aligned} \min_{\tau, \phi, \vartheta, \nu, \boldsymbol{\zeta}, \vartheta', \nu', \boldsymbol{\zeta}'} \quad & \lambda (\mathbf{C}^T \boldsymbol{\tau} \mathbf{p}_0 + \mathbf{p}_0^T \vartheta + \nu) + (1-\lambda) \\ & \left[\phi + \frac{1}{1-\alpha} \mathbf{p}_0^T \mathbf{t} + \frac{1}{1-\alpha} (\mathbf{p}_0^T \vartheta' + \nu') \right] \text{ s.t. } TC_s - \phi \\ & \leq t_s, \quad \forall s, \quad \left\| \mathbf{P}_1^T \boldsymbol{\tau}^T \mathbf{C} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \mathbf{e}\boldsymbol{\zeta} \right\|_{\infty} \\ & \leq \nu, \quad \left\| \mathbf{P}_1^T \mathbf{t} + \mathbf{P}_1^T \vartheta' + \mathbf{P}_1^T \mathbf{e}\boldsymbol{\zeta}' \right\|_{\infty} \leq \nu', \quad t_s \geq 0, \quad \forall s, \quad \vartheta \geq 0, \nu \geq 0, \vartheta' \\ & \geq 0, \nu' \geq 0, \quad \text{constraints (1) - (17)} \end{aligned} \tag{27}$$

Next, we give the following proposition to ensure that the optimal solution of model (23) with $\wp = \wp_{\vartheta}$ is the optimal solution of model (27).

Proposition 1. *If $(\boldsymbol{\tau}^*, \phi^*, \mathbf{t}^*, \vartheta^*, \nu^*, \boldsymbol{\zeta}^*, \vartheta'^*, \nu'^*, \boldsymbol{\zeta}'^*)$ solves model (27), then $(\boldsymbol{\tau}^*, \phi^*, \mathbf{t}^*)$ solves model (23) with $\wp = \wp_{\vartheta}$; Conversely, if $(\boldsymbol{\tau}^*, \phi^*, \mathbf{t}^*)$ solves model (23) with $\wp = \wp_{\vartheta}$, then $(\boldsymbol{\tau}^*, \phi^*, \mathbf{t}^*, \vartheta^*, \nu^*, \boldsymbol{\zeta}^*, \vartheta'^*, \nu'^*, \boldsymbol{\zeta}'^*)$ solves model (27), i.e. $(\boldsymbol{\tau}^*, \vartheta^*, \nu^*, \boldsymbol{\zeta}^*)$ solves model (32),*

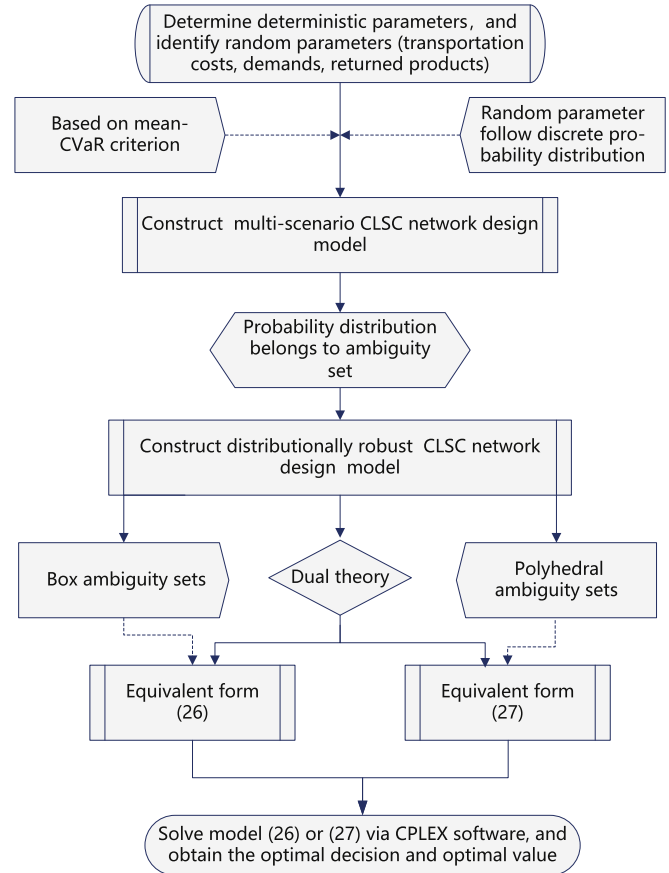


Fig. 2. The procedures for the solution of distributionally robust model.

where $(\tilde{\vartheta}^*, \tilde{\nu}^*, \tilde{\boldsymbol{\zeta}}^*)$ solves (31), and $(\tilde{\boldsymbol{\tau}}^*, \tilde{\phi}^*, \tilde{\mathbf{t}}^*, \tilde{\vartheta}^{**}, \tilde{\nu}^{**}, \tilde{\boldsymbol{\zeta}}^{**})$ solves (33).

Remark 2. In the case $\mathbf{P}_1 = 0$, problem (27) reduces to its nominal stochastic CLSC problem (20).

4.3. Methodology step

To solve the CLSC network problem, we first need to introduce some parameters and then construct a multi-scenario model by stochastic optimization. Furthermore, a distributionally robust model is proposed, and the ambiguity sets are determined. Based on dual theory, the equivalent forms of the distributionally robust model are obtained. Finally, the equivalent models are solved by CPLEX. The methodology steps are visualized, and the detailed description is presented in Fig. 2.

5. Case study about bicycle-sharing in the Jing-Jin-Ji region

Bicycle-sharing is currently a popular mode of travel, which mainly relies on the bicycle-sharing company (Industry analysis report, 2017). Bicycle-sharing not only solves “the last kilometer” traffic problem to promote the efficiency of short-distance urban life but also realizes the low-carbon environment and green travel (DeMaio, 2009). Currently, bicycle-sharing has attracted the attention of many city dwellers as a green, flexible, sustainable mobility model.

5.1. Data description

In this subsection, a real-world case study on ofo, a bicycle-

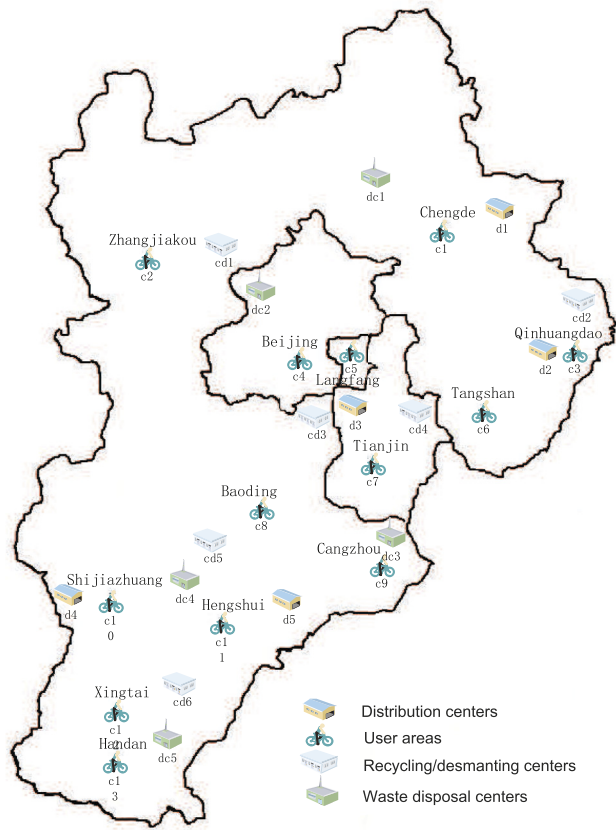


Fig. 3. The locations of all facilities.

Table 2
The value ranges of parameters in case study, originated from Industry analysis report (2017).

Costs	Value (¥)	Parameter	Value
fs_i	20–40 thousand	ss_{ir}	200–300 thousand
fd_k	20–40 thousand	sp_{jp}	400–600 thousand
fc_m	60–80 thousand	sd_{kp}	100–200 thousand
fp_n	40–60 thousand	sc_{mp}	250–300 thousand
cs_{irh}^s	25–35	sp_{nr}	400–500 thousand
cm_{jp}^s	300	ρ_{ir1}	120–150 thousand
cp_{kp}^s	13–20	ρ_{ir2}	180–250 thousand
cc_{mp}^s	0.5–1.5	d^s	6–10
cr_{mr}^s	0.5–1.5	θ_r	0.1
cd_{nr}^s	43–50	tsp^s, tpd^s, tdc^s	
π_{lp}^s	40–60 thousand	tcc^s, tcp^s, tcd^s	[0.7, 3]

sharing company in the Jing-Jin-Ji Metropolitan Region in China, is presented. There are 10 suppliers (s1 to s10) located in Tianjin. There are 3 manufacturers, Feige, Fenghuang and Fushida (p1 to p3, respectively), located in Tianjin. The five distribution centers are numbered from d1 to d5. In total, 13 cities are defined as user areas (c1 to c13). The six recycling/dismantling centers are numbered

Table 3
Problem size for each instance.

Model	Network structure						Variables			Constraints
	SU	MA	DC	UA	RDC	WDC	Binary	Integer	Other	
(20)	10	3	5	13	6	5	746	5877	–	6185
(26)	10	3	5	13	6	5	746	5877	23	6209
(27)	10	3	5	13	6	5	746	5877	37	6215

SU = suppliers, MA = manufacturers, DC = distribution centers, UA = user areas, RDC = recycling/dismantling centers, WDC = waste disposal centers.

from cd1 to cd6, and the waste disposal centers are numbered from dc1 to dc5. To relieve the environmental pressure of first-tier cities, the distribution centers and recycling/dismantling centers are set up in second- or third-tier cities (see Fig. 3). There are 8 types of components with 3 price discounts to 3 manufactories for producing 5 types of bicycles: ofo 1.0, ofo 2.0, ofo 3.0, ofo 3.1 and ofo curve.

In our CLSC network design experiment, the demands, transportation costs and the return amount of the used products are uncertain parameters, which have been predicted to have 3 possible scenarios with unknown occurrence probabilities.

Assume that the demand d_{lp}^s is a linear function of d^s , i.e., $d_{lp}^s = d_r \times d_a \times d^s + d_b$, where d^s is the unit demand per hundred people under scenario s ; d_a is the size of the population in user area l ; d_r is the permeability of the ofo sharing bicycle p in the city and d_b is the extra demand. The amount of the returned products depends on demand, and further assume the returned products r_{lp}^s is a linear function of the demand d_{lp}^s . Concretely, $r_{lp}^s = r \times d_{lp}^s$, where r is the return rate of the used products. In general, the value on the return rate to ofo is in the interval [1.2, 1.4].

In addition, the transportation costs are also uncertain, which are related to the fuel prices and the distances between cities. The unit transportation cost tsp_{ijr}^s depends on the random unit fuel cost tsp^s under three scenarios. Assume tsp_{ijr}^s is a linear function of tsp^s , i.e., $tsp_{ijr}^s = tsp_r \times tsp_{a_{ij}} \times tsp^s + tsp_{b_{ijr}}$. Here, tsp^s is the unit fuel cost per kilometer from suppliers to manufactories under three scenarios, tsp_r is the cost coefficient for part, $tsp_{a_{ij}}$ is the distance between two facilities and $tsp_{b_{ijr}}$ is the extra cost. Similarly, $tpd_{jkr}^s, tdc_{klp}^s, tcc_{lmp}^s, tcp_{mjr}^s$ and tcd_{mnr}^s are characterized by the linear functions of $tsp^s, tpd^s, tdc^s, tcc^s, tcp^s$ and tcd^s , respectively.

In our numerical experiments, the fixed, operating and penalty costs, and other relevant determinate parameters are collected in Table 2. Moreover, the size of this case study is given in Table 3, which includes the CLSC network structure, the variables and the constraints of the mathematical models. In addition, assume that the suppliers will offer a 90% discount when x_{ijr1} is in the range of 1 to ρ_{ir1}^* ; if x_{ijr2} is in the range of ρ_{ir1} to ρ_{ir2}^* , then the suppliers will provide an 85% discount; if x_{ijr3} exceeds ρ_{ir2}^* , then the suppliers will offer a 75% discount.

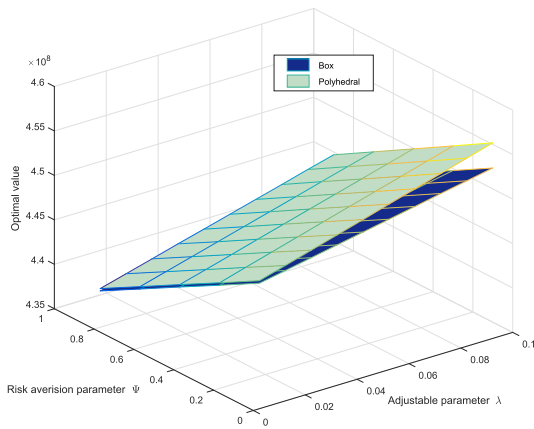
Note that the uncertain demands, returned products and transportation costs in our CLSC network design have been predicted to fall into 3 possible scenarios with unknown occurrence probabilities. However, the nominal probability distribution can be fixed in advance and is denoted as $\mathbf{p}_0 = (0.4, 0.5, 0.1)^T$. The value of the adjustable parameter Ψ in the box uncertainties equals to 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 and 0.1, respectively. The scaling matrix is set in the polyhedral ambiguity set as $\mathbf{P}_1 = \Psi | \mathcal{I}$, where \mathbf{I} is an identity matrix. The software for solving the numerical example is IBM ILOG CPLEX 12.6.3 solver, and the PC's configuration includes a 2.50 GHz Intel(R) Core i5-7200 CPU processor with 8.0 GB of RAM.

5.2. Computational results under the distributionally robust model

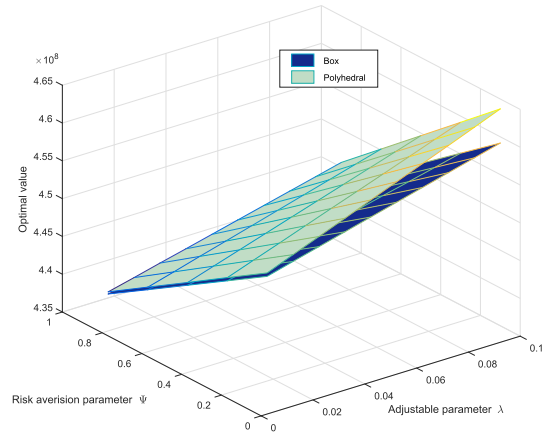
When imprecise probabilistic distributions of uncertain

Table 4
Summary of computational results under imprecise probabilistic distribution.

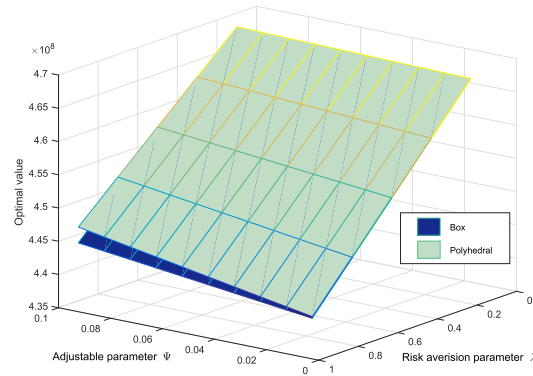
Ambiguity set	Confidence level (α)	Risk aversion parameter (λ)	Adjustable parameter (Ψ)										
			0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	
Box (Ψ)	0.9	0.1	467212337.72	467250069.04	467303036.82	467369674.98	467422121.42	467460626.26	467512325.08	467564848.64	467617257.90	467669724.31	
		0.3	460271183.50	460428582.72	460585981.95	460759840.68	460900801.08	461077145.02	461215776.85	461372978.08	461530377.30	461687776.53	
		0.5	453341686.16	453604076.01	453866465.85	454147436.83	454391245.54	454653635.38	454916355.22	455178735.07	455440804.91	455703194.75	
		0.7	446410561.35	446777907.13	447145252.91	447531125.89	447879944.47	448247290.26	448614636.04	448981981.82	449349327.60	449716673.38	
	0.7	0.9	439479436.54	439951738.26	440424039.97	440896341.69	441368643.41	441840945.13	442313246.85	442785548.57	443257850.28	443730152.00	
		0.1	451028229.24	451947835.56	452851819.79	453755804.02	454659875.75	455563857.48	456467839.21	457371740.94	458275725.16	459178624.79	
		0.3	447691705.89	448511303.26	449330900.55	450150497.88	450971009.53	451789896.57	452625666.97	453428887.21	454248647.80	455085375.76	
		0.5	444355182.53	445090546.24	445826280.05	446561273.68	447296637.40	448032341.12	448767364.99	449519730.24	450238402.29	450990592.13	
	0.5	0.7	441018659.17	441686799.04	442320919.38	442975731.60	443623179.59	444291416.27	444942570.58	445593724.89	446244879.19	446878830.12	
		0.9	437700482.41	438249716.30	438815928.80	439382825.29	439951950.18	440533964.20	441100841.87	441650987.26	442231993.47	442801474.87	
		0.1	447297206.04	447845641.90	448408929.65	448972217.41	449535505.16	450098724.92	450662014.67	451240866.82	451788594.18	452351883.94	
		0.3	444778186.81	445348803.29	445887650.73	446442382.69	447010422.78	447551846.62	448106778.00	448661966.84	449216042.50	449778628.08	
	Polyhedral ($\Gamma = \Psi \mathcal{J} $)	0.9	0.5	442290243.99	442820271.64	443366815.81	443912619.98	444458857.24	445004968.51	445551472.49	446097585.01	446643491.00	447189664.99
			0.7	439770008.14	440307624.51	440845240.89	441383361.26	441920473.64	442458090.02	442995706.39	443533322.77	444070939.15	444608975.66
			0.9	437266620.80	437795661.38	438324035.97	438853094.55	439399413.01	439911211.72	440440270.30	440969328.88	441498387.47	442027446.05
			0.1	467238637.67	467302459.45	467381159.06	467478907.14	467553243.11	467631916.13	467697159.75	467774657.12	467853356.74	467946608.20
	0.7	0.3	460350114.11	460585981.95	460838244.84	461058179.63	461294278.47	461530377.30	461766476.14	462002574.98	462238832.82	462480248.47	
		0.5	453472881.09	453866465.85	454260050.62	454653635.38	455047220.15	455440804.91	455834389.68	456230772.84	456621559.21	457015143.97	
		0.7	446594773.24	447145252.91	447696271.58	448247290.26	448798308.93	449349327.60	449900346.27	450451364.94	451002383.61	451553402.28	
		0.9	439715587.40	440424039.97	441132492.55	441840945.13	442549397.71	443257850.28	443966833.87	444674755.44	445383208.02	446108924.48	
	0.5	0.1	451480144.71	452851819.79	454194917.82	455550380.82	456919830.07	458275725.18	459631701.51	460970425.63	462326116.05	463699253.89	
		0.3	448117925.01	449347197.65	450560296.55	451800770.34	453035881.79	454248484.54	455477880.54	456707276.54	457954127.32	459168482.20	
		0.5	444723324.97	445825909.96	446928955.54	448032001.12	449135780.81	450238402.29	451341137.86	452444183.44	453547229.02	454650524.59	
		0.7	441361221.89	442321438.02	443314684.81	444274309.70	445251004.87	446227700.03	447204808.18	448181090.34	449157785.50	450134830.66	
0.1	0.9	437983014.70	438815928.80	439683647.70	440516618.28	441366963.02	442217307.76	443068183.50	443917997.24	444768341.98	445618686.72		
	0.7	447578896.10	448408855.65	449253790.29	450098792.92	450943724.55	451804324.43	452649317.10	453494387.26	454339523.68	455184527.60		
	0.5	445055552.79	445903311.57	446735501.48	447567691.39	448399745.66	449232303.72	450048141.01	450880238.41	451712336.33	452544679.92		
	0.3	442563828.46	443366445.81	444186062.06	445021276.18	445840585.84	446659895.50	447463047.08	448282276.78	449101902.62	449920535.84		
0.01	0.7	440038816.32	440845240.89	441651665.45	442458090.02	443264969.58	444070939.15	444877363.71	445700626.53	446507007.99	447296638.97		
	0.9	437530448.09	438324035.97	439118262.84	439928445.77	440704799.59	441498387.47	442291975.34	443085563.22	443896191.51	444673198.10		



(a) The optimal values for $\alpha = 0.5$



(b) The optimal values for $\alpha = 0.7$



(c) The optimal values for $\alpha = 0.9$

Fig. 4. The computational results of distributionally robust model.

parameters belong to box and polyhedral ambiguity sets, the equivalent models (26) and (27) of the distributionally robust model are employed to solve the CLSC network design problem. Table 4 provides the computational results with respect to the adjustable parameters, risk aversion parameters and confidence levels. We consider the risk aversion parameter to be 0.1, 0.3, 0.5, 0.7, and 0.9 and the confidence level to be 0.5, 0.7, and 0.9. From the computational results in Table 4, the objective function value decreases when the risk aversion parameter λ increases, and the optimal value increases when the confidence level α increases. In

addition, as the adjustable parameter Ψ increases, the optimal value of the distributionally robust model worsens. As shown in Fig. 4, a comparison of the distributionally robust model under box and polyhedral ambiguity sets also indicates the same trend. Fig. 4 depicts that the increase in the adjustable parameter Ψ and risk aversion parameter λ yields a moderate, linear increase in the optimal value. These phenomena are perfectly consistent with the theoretical facts. For example, it is easy to see that the optimal objective function of the model is linear with respect to the parameters Ψ and λ . Note that the changes in the optimal value when

Table 5
Summary of computational results under nominal probabilistic distribution.

Confidence level (α)	Risk aversion parameter (λ)				
	0.1	0.3	0.5	0.7	0.9
0.9	467145060.22	460114024.27	453079296.32	446043215.57	439007134.82
0.7	450126298.69	446872108.55	443619818.81	440367529.06	437115239.32
0.5	446718986.39	444223454.85	441727923.30	439232391.76	436736860.22

$\alpha = 0.9$ coincided well with those of $\alpha = 0.7$ and $\alpha = 0.5$, and they both maintain obvious increases as the adjustable parameter increases. In addition, when the confidence level increases, the optimal value worsens. Therefore, it is very important for decision makers to determine the confidence level and perturbation intervals of the uncertain parameters in their real CLSC network decision process.

On the other hand, the results show that the total cost under a polyhedral ambiguity set is higher than that under box ambiguity set. The same trend is displayed in Fig. 4. It is confirmed that the “polyhedral” ambiguity set is larger than the “box” ambiguity set; that is, the “polyhedral” ambiguity set completely covers the “box” ambiguity set. Therefore, the ambiguity set is larger and becomes more conservative. Decision makers should select a suitable ambiguity set based on their own economic situation to resist the risk from imprecise probability distribution.

5.3. Computational results under the nominal stochastic model

In this subsection, the numerical experiment is conducted by solving the nominal stochastic model (20), and the computational results under different confidence levels and risk aversion parameters are reported in Table 5. It can be observed that the optimal value increases with respect to the confidence level when the risk aversion parameter is fixed. In addition, when the confidence level is fixed, the objective function value decreases as the risk aversion parameter increases. Fig. 5 depicts the linear variation in the optimal value when the risk aversion parameter λ increases.

5.4. A comparison between the robust and nominal stochastic models

In the following subsection, a comparison study is performed between our distributionally robust optimization model and the nominal stochastic model with exact probabilistic distributions. The selection of the facilities is different under the nominal stochastic model and distributionally robust model. For example, when $\alpha = 0.9$, $\lambda = 0.5$ and $\Psi = 0.01$, facilities s1, s8 and s10 are selected as potential suppliers in the nominal stochastic model. Nevertheless, facilities s2, s3, s10 are selected in the distributionally robust model under box ambiguity set, and facilities s1, s2, s7 are selected in the distributionally robust model under polyhedral

ambiguity set. Based on the above statement, we can know that the location strategies are distinct in the network design of the nominal and robust solution models.

In addition, Fig. 6 compares the computational results between the nominal stochastic model and the distributionally robust model under different λ with $\alpha = 0.9$ and $\Psi = 0.01$. The comparative study indicates that the optimal value under the nominal stochastic model is less than the optimal values under distributionally robust models, and all the models decrease with increasing λ . In particular, the results of the relevant management/operational performance measures such as expected cost, CVaR and total transportation costs under $\Psi = 0.01$ are given in Table 6. From Table 6, it is clearly seen that the expected costs and CVaR of distributionally robust models (26) and (27) are greater than the nominal stochastic model (20). Besides, transportation costs under different scenarios of the nominal stochastic model and distributionally robust models are also distinct, and the nominal stochastic model are not beyond the distributionally robust models. Note that in the case of $\lambda = 1$, the objective function only includes the part of expected costs. Transportation costs have no difference under different α because they are independent of parameter α . In contrast, when $\lambda = 0$, the objective function only includes CVaR, and transportation costs vary greatly with respect to parameter α . Additionally, we find out that parameters λ and α have a less influence on the total supplied volume to customers. The results of the total supplied volume to customers are 3294377, 3303717 and 3282822 under model (20), (26) and (27), respectively. The largest total supplied volume is from distributionally robust model with box ambiguity set. The comparative study shows the differences between distributionally robust models and nominal stochastic model. The dominant advantage of distributionally robust model is it can effectively resist the risk brought by ambiguity of probability distribution.

To analyze the cause of aforementioned phenomenon, the price of distributional robustness (DRP) is given as follows:

$$DRP = DROV - NSOV,$$

where DROV is the optimal value of the distributionally robust model and NSOV is the optimal value of the nominal stochastic model. From Fig. 7, it is easy to see that the DRP increases with the adjustable parameter Ψ , particularly in the polyhedral-DRP. That is, the larger the ambiguity set, the higher the cost the decision

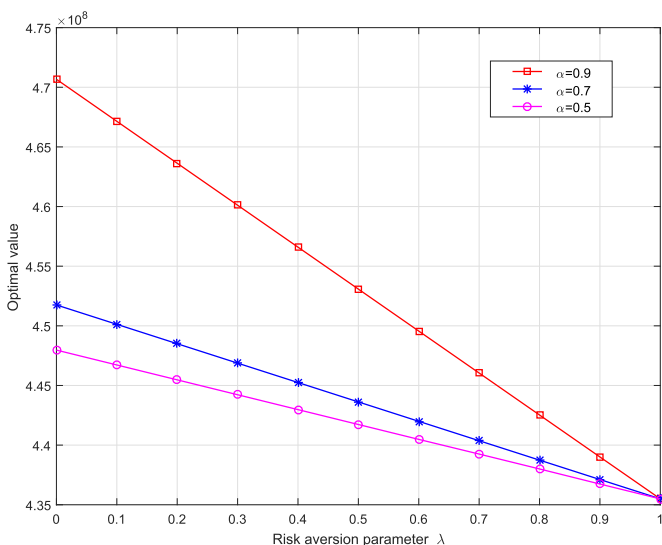


Fig. 5. The computational results of nominal stochastic model.

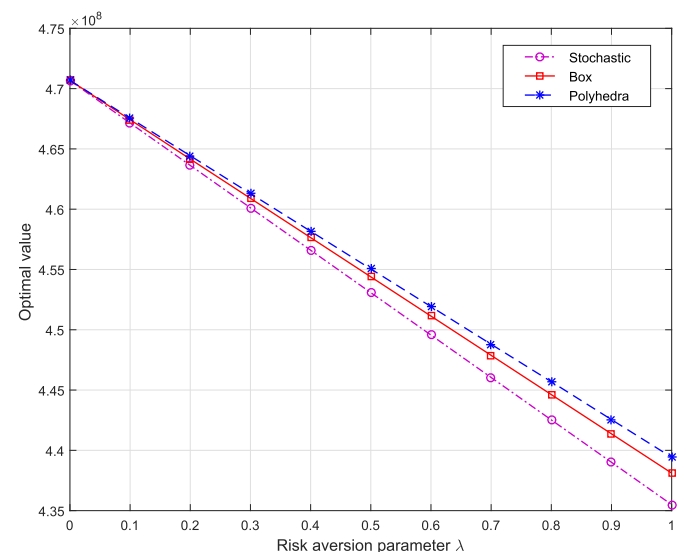


Fig. 6. The comparison of computational results under two types model under $\alpha = 0.9$

Table 6
The computational results of the relevant management/operational measures.

Risk aversion parameter (λ)	Confidence level (α)	Model	Expected cost	CVaR	Transportation costs			
					scenario 1	scenario 2	scenario 3	
0	0.9	(20)	—	470679788.20	23502365.70	34558342.04	50021841.20	
		(26)	—	470679788.35	27730462.64	38456940.68	50021841.20	
		(27)	—	470679788.35	23621849.60	34995875.32	49995801.20	
	0.7	(20)	—	451765763.23	28134808.84	34323970.48	50048797.20	
		(26)	—	452696490.92	21820202.76	34298771.60	50020601.20	
		(27)	—	453169464.80	22836540.00	34298771.60	50020601.20	
	0.5	0.9	(20)	435489094.00	470669499.00	21431132.50	34298771.60	50020601.52
			(26)	436013874.00	470669498.00	21431132.50	34298771.60	50020601.20
			(27)	436276264.00	470669498.00	21433139.82	34298771.60	50020601.20
0.7		(20)	435489094.00	451750543.00	21431132.50	34298771.60	50020601.20	
		(26)	436013874.00	452696491.00	21431132.50	34298771.60	50020601.20	
		(27)	436276264.00	453169465.00	21431132.50	34298771.60	50020601.20	
1		0.9	(20)	435506802.09	—	21440132.50	34323970.48	50048797.20
			(26)	436013874.14	—	21431132.50	34298771.60	50020601.20
			(27)	436276263.98	—	21431132.50	34298771.60	50020601.20
	0.7	(20)	435506802.09	—	21440132.50	34323970.48	50048797.20	
		(26)	436013874.14	—	21431132.50	34298771.60	50020601.20	
		(27)	436276263.98	—	21431132.50	34298771.60	50020601.20	

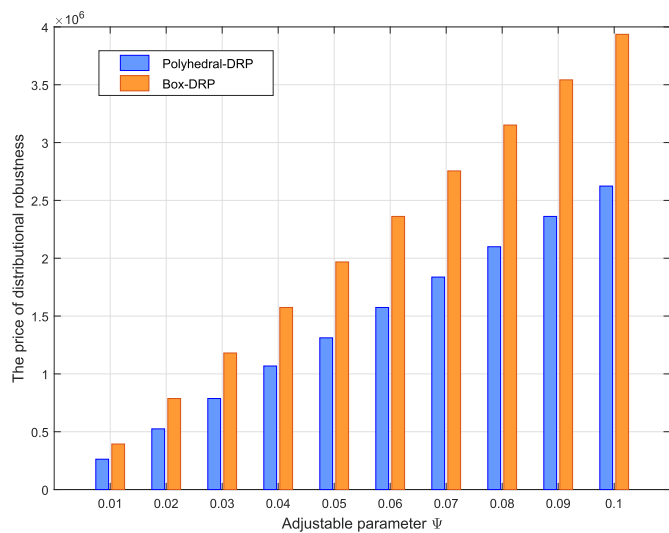


Fig. 7. The price of distributional robustness under two ambiguity sets under $\alpha = 0.9$, $\lambda = 0.5$

makers will have to pay. The additional costs are caused by the ambiguous distribution, which cannot be ignored. In this case, if decision makers still insist on using the optimal solution of the nominal stochastic model, the actual cost increment will greatly increase and even produce infinite costs. Moreover, this situation will lead to decision failure. Therefore, although the total cost of the distributionally robust model is higher than that of the nominal stochastic model, it resists the probability distribution ambiguity better than the stochastic model with exact probabilistic distributions.

5.5. Managerial insights

In this subsection, we analyze the managerial insights of solutions and the optimal values and provide some insights into the distributionally robust optimization approach for engineers.

5.5.1. The insights of the solution results

The computational results show that the obtained solutions of our distributionally robust and nominal stochastic models are

significantly different. In view of this finding, engineers should not ignore the effect of an imprecise probability distribution on the optimal location strategy. Engineers can employ distributionally robust optimization to derive a reasonable location strategy when the distribution information is only partially known.

According to the analysis of the computational results, the type of ambiguity set has a great influence on the optimal value. The larger the ambiguity set is, the higher the cost, and the corresponding model becomes more conservative. If engineers focus on the costs, the conservativeness of the model should be controlled. In addition, the optimal value is also affected by the risk aversion parameter λ and the confidence level α . To obtain a reasonable, optimal scheme, engineers should set the appropriate risk aversion parameter and the confidence level based on the requirements and economic conditions of the firm.

5.5.2. The insights of distributionally robust optimization approach

In many practical CLSC problems, engineers often face diverse types of uncertainty. When the exact probability distribution information can be obtained, engineers can design the CLSC network with stochastic optimization method. In contrast, engineers may only know the support, and any distribution information is out of reach. In this case, they often make decisions by employing the traditional robust method, which is regarded as a method that does not require the distribution information.

In practical situations, however, engineers frequently confront the situation that the distribution information lies somewhere in between these two cases. They can only obtain partial information on the probability distribution. It is easy to see from comparative studies that ignorance to the differences in the distribution information will lead to great decision-making risks and high costs. The proposed distributionally robust method in this study is useful because it should be employed when only partial distribution information is known. Therefore, engineers can use the proposed distributionally robust mean-CVaR model to design a CLSC network and resist the risk caused by imprecise probability distribution information, further ensuring a reasonable decision is made.

6. Conclusions

In this paper, a multi-scenario optimization model is developed to design an optimal CLSC network under distribution ambiguity from a new perspective. The major findings are as follows:

- i) Model: A new distributionally robust mean-CVaR model is proposed for designing the CLSC network. To better reflect the uncertain nature of CLSC decision circumstances, uncertain parameters are characterized by imprecise discrete probability distributions. The objective is to minimize the total cost and risk of the entire supply chain network by an optimal trade-off between the expected value and the CVaR.
- ii) Tractability: The robust counterpart of the proposed model is a hard optimization problem and is usually computationally intractable for general ambiguity sets. Consequently, two types of ambiguity sets are selected and applied to obtain computationally tractable forms of the proposed model. As a result, the optimal solution can be obtained via conventional optimization software such as CPLEX.
- iii) Application: A realistic case study about a bicycle-sharing company in the Jing-Jin-Ji Metropolitan Region of China is implemented with the proposed approach. Actual data from ofo are used to examine the performance of the proposed model. To evaluate the effectiveness, a sensitivity analysis and a comparison study are conducted via a number of numerical experiments, from which some managerial insights are recommended. Decision makers can make informed decisions to design the CLSC configuration under imprecise probability distributions of uncertain parameters.

A limitation of this research is the assumption of discrete probability distributions for uncertain parameters. The intention of this assumption is to construct a multi-scenario optimization model that can be effectively solved. However, in many decision problems, the uncertain parameter is continuous. In this case, constructing a proper model for the bicycle-sharing CLSC network problem and transforming it to a tractable form are two valuable problems to be solved.

Moreover, the lack of consideration on carbon decisions is another limitation, so an extension direction would be to reduce the environmental pressures. A further attempt at other application cases in different countries and regions is also an opportunity for future research.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix C. Supplementary data

Supplementary data to this article can be found online at <https://doi.org/10.1016/j.jclepro.2019.118967>.

Appendix A. The main notations of CLSC network in order to formulate our CLSC network design model, the following notations are necessary:

Related parameters of costs

fs_i	Fixed cost for selecting supplier $i \in \mathcal{I}$
fd_k	Fixed cost for opening distribution center $k \in \mathcal{K}$
fc_m	Fixed cost for opening recycling/dismantling center $m \in \mathcal{M}$
fp_n	Fixed cost for opening waste disposal center $n \in \mathcal{N}$
cs_{ir}	Purchasing cost per unit component $r \in \mathcal{R}$ from supplier $i \in \mathcal{I}$
cm_{jp}	Manufacturing cost per unit product $p \in \mathcal{P}$ for manufactory $j \in \mathcal{J}$
cp_{kp}	Operating cost per unit product $p \in \mathcal{P}$ for distribution center $k \in \mathcal{K}$
cc_{mp}	recycling and dismantling cost per unit returned product $p \in \mathcal{P}$ for recycling/dism-antling center $m \in \mathcal{M}$
cr_{mr}	Recovery cost per unit component $r \in \mathcal{R}$ from recycling/dismantling center $m \in \mathcal{M}$ to manufactory $j \in \mathcal{J}$
cd_{nr}	Disposal cost per unit useless component $r \in \mathcal{R}$ for waste disposal center $n \in \mathcal{N}$
tsp_{ijr}^s	Cost of shipping component $r \in \mathcal{R}$ from supplier $i \in \mathcal{I}$ to manufactory $j \in \mathcal{J}$ under scenario $s \in \mathcal{S}$
tpd_{jkp}^s	Cost of shipping product $p \in \mathcal{P}$ from manufactory $j \in \mathcal{J}$ to distribution center $k \in \mathcal{K}$ under scenario $s \in \mathcal{S}$
tdc_{klp}^s	Cost of shipping product $p \in \mathcal{P}$ from distribution center $k \in \mathcal{K}$ to user area $l \in \mathcal{L}$ under scenario $s \in \mathcal{S}$
tcc_{lmp}^s	Cost of shipping product $p \in \mathcal{P}$ from user area $l \in \mathcal{L}$ to recycling/dismantling center $m \in \mathcal{M}$ under scenario $s \in \mathcal{S}$
tcp_{mjr}^s	Cost of shipping component $r \in \mathcal{R}$ from recycling/dismantling center $m \in \mathcal{M}$ to manufactory $j \in \mathcal{J}$ under scenario $s \in \mathcal{S}$
tcd_{mnr}^s	Cost of shipping component $r \in \mathcal{R}$ from recycling/dismantling center $m \in \mathcal{M}$ to waste disposal center $n \in \mathcal{N}$ under scenario $s \in \mathcal{S}$
π_{lp}	Penalty costs of unit product $p \in \mathcal{P}$ that don't satisfy demand for customer $l \in \mathcal{L}$

Other parameters of CLSC network

ss_{ir}	The capacity of supplier $i \in \mathcal{I}$ to store component $r \in \mathcal{R}$
sp_{jp}	The capacity of manufactory $j \in \mathcal{J}$ to store product $p \in \mathcal{P}$
sd_{kp}	The capacity of distribution center $k \in \mathcal{K}$ to store product $p \in \mathcal{P}$
sc_{mp}	The capacity of recycling/dismantling center $m \in \mathcal{M}$ to store product $p \in \mathcal{P}$
sp_{nr}	The capacity of waste disposal center $n \in \mathcal{N}$ to store component $r \in \mathcal{R}$
d_{lp}^s	Demand for product $p \in \mathcal{P}$ in user area $l \in \mathcal{L}$ under scenario $s \in \mathcal{S}$
r_{lp}^s	Amount of return product $p \in \mathcal{P}$ from the user area $l \in \mathcal{L}$ under scenario $s \in \mathcal{S}$
δ_{rp}	The number of component $r \in \mathcal{R}$ needed to produce one product $p \in \mathcal{P}$
κ_h	The coefficient of discount scheme $h \in \mathcal{H}$
θ_r	Average disposal fraction of component $r \in \mathcal{R}$

Decision variables of CLSC network

x_{ijrh}^s	The number of component $r \in \mathcal{R}$ purchased from supplier $i \in \mathcal{I}$ to manufactory $j \in \mathcal{J}$ with quantity discount $h \in \mathcal{H}$ under scenario $s \in \mathcal{S}$
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y_{jkp}^s	The number of product $p \in \mathcal{P}$ shipped from manufactory $j \in \mathcal{J}$ to distribution center $k \in \mathcal{K}$ under scenario $s \in \mathcal{S}$
z_{klp}^s	The number of product $p \in \mathcal{P}$ shipped from distribution center $k \in \mathcal{K}$ to user area $l \in \mathcal{L}$ under scenario $s \in \mathcal{S}$
o_{imp}^s	The number of returned product $p \in \mathcal{P}$ from user area $l \in \mathcal{L}$ to recycling/dismantling center $m \in \mathcal{M}$ under scenario $s \in \mathcal{S}$
t_{mjr}^s	The number of useful component $r \in \mathcal{R}$ shipped from recycling/dismantling center $m \in \mathcal{M}$ to manufactory $j \in \mathcal{J}$ under scenario $s \in \mathcal{S}$
f_{mnr}^s	The number of useless component $r \in \mathcal{R}$ shipped from recycling/dismantling center $m \in \mathcal{M}$ to waste disposal center $n \in \mathcal{N}$ under scenario $s \in \mathcal{S}$
ω_{lp}^s	The number of product $p \in \mathcal{P}$ that don't meet demand for users area $l \in \mathcal{L}$ under scenario $s \in \mathcal{S}$
u_i	Binary variable indicating whether potential supplier $i \in \mathcal{I}$ is selected or not
v_k	Binary variable indicating whether potential distribution center $k \in \mathcal{K}$ is opened or not
c_m	Binary variable indicating whether potential recycling/dismantling center $m \in \mathcal{M}$ is opened or not
w_n	Binary variable indicating whether potential waste disposal center $n \in \mathcal{N}$ is opened or not
g_{ijrh}	Binary variable indicating whether manufactory $j \in \mathcal{J}$ purchased component $r \in \mathcal{R}$ from supplier $i \in \mathcal{I}$ with discount $h \in \mathcal{H}$ or not

Appendix B. The Proofs of Main Theorems

We first give the proof of Theorem 1.

Suppose probability distribution vector \mathbf{p} belongs to a box ambiguity set, i.e., Eq. (21), problems (24) and (25) are all linear programming problems. To be specific, problem (24) can be represented as follows:

$$\max_{\mathbf{p} \in \wp_{\mathcal{P}}} \mathbf{TC}^T \mathbf{p} = \mathbf{TC}^T \mathbf{p}_0 + \max_{\xi} \left\{ \mathbf{TC}^T \xi \mid \mathbf{e}^T \xi = 0, \|\xi\|_{\infty} \leq \Psi \right\}.$$

where $\xi_{\infty} = \max_{s \in \mathcal{S}} |\xi_s|$.

Thus, on the basis of the strong duality theory of linear programming, the dual form of problem (24) is the following linear programming model:

$$\begin{aligned} \min_{\tau, \mu, \eta, \gamma} \quad & \mathbf{TC}^T \mathbf{p}_0 + \Psi^T \eta + \Psi^T \gamma \\ \text{s.t.} \quad & \mathbf{e}\mu - \eta + \gamma = \mathbf{TC}, \\ & \eta \geq 0, \gamma \geq 0, \end{aligned} \tag{28}$$

while the dual programming of problem (25) is the following linear programming model:

$$\begin{aligned} \min_{\mu', \tau, \eta', \gamma'} \quad & \phi + \frac{1}{1-\alpha} (\mathbf{t}^T \mathbf{p}_0 + \Psi^T \eta' + \Psi^T \gamma') \\ \text{s.t.} \quad & \mathbf{TC} - \phi \leq \mathbf{t}_s, \forall s, \\ & \mathbf{e}\mu' - \eta' + \gamma' = \mathbf{t}, \\ & \mathbf{t}_s \geq 0, \forall s, \\ & \eta' \geq 0, \gamma' \geq 0. \end{aligned} \tag{29}$$

Combining models (28) and (29), under the ambiguity set $\wp = \wp_{\mathcal{P}}$, problem (23) is equivalent to the following linear system:

$$\min_{\tau, \phi, \mu, \eta, \gamma, \mu', \eta', \gamma'} \lambda (\mathbf{TC}^T \mathbf{p}_0 + \Psi^T \eta + \Psi^T \gamma) + (1-\lambda)$$

$$\begin{aligned} & \left[\phi + \frac{1}{1-\alpha} (\mathbf{t}^T \mathbf{p}_0 + \Psi^T \eta' + \Psi^T \gamma') \right] \text{s.t. } \mathbf{TC}_s - \phi \\ & \leq \mathbf{t}_s, \forall s, \quad \mathbf{e}\mu - \eta + \gamma = \mathbf{TC}, \quad \mathbf{e}\mu' - \eta' + \gamma' = \mathbf{t}, \quad \mathbf{t}_s \\ & \geq 0, \forall s, \quad \eta \geq 0, \gamma \geq 0, \eta' \geq 0, \gamma' \\ & \geq 0, \quad \text{constraints (1) - (17)}, \end{aligned}$$

where $\mu, \mu', \eta, \eta', \gamma, \gamma' \in R \times R \times R^{|\mathcal{S}|} \times R^{|\mathcal{S}|} \times R^{|\mathcal{S}|} \times R^{|\mathcal{S}|}$ are the dual variables/vectors.

For the sake of clarity, the above-mentioned linear system can be rewritten as

$$\begin{aligned} & \min_{\tau, \phi, \mu, \eta, \gamma, \mu', \eta', \gamma'} \lambda (\mathbf{TC}^T \mathbf{p}_0 + \Psi^T \eta + \Psi^T \gamma) + (1-\lambda) \\ & \left[\phi + \frac{1}{1-\alpha} (\mathbf{t}^T \mathbf{p}_0 + \Psi^T \eta' + \Psi^T \gamma') \right] \text{s.t. } \mathbf{TC}_s - \phi \\ & \leq \mathbf{t}_s, \forall s, \quad \mathbf{e}\mu - \eta + \gamma = \mathbf{TC}, \quad \mathbf{e}\mu' - \eta' + \gamma' = \mathbf{t}, \quad \mathbf{t}_s \geq 0, \forall s, \quad \eta \\ & \geq 0, \gamma \geq 0, \eta' \geq 0, \gamma' \geq 0, \quad \text{constraints (1) - (17)}. \end{aligned}$$

The proof of theorem is complete.

We next presents the proof of Theorem 2.

Suppose probability distribution vector \mathbf{p} belongs to a polyhedral ambiguity set, i.e., Eq. (22), optimization problem (24) can be represented as

$$\begin{aligned} \max_{\mathbf{p} \in \wp_{\mathcal{P}}} \mathbf{TC}^T \mathbf{p} = \mathbf{TC}^T \mathbf{p}_0 + \max_{\xi} \left\{ \mathbf{TC}^T \mathbf{P}_1 \xi \mid \mathbf{e}^T \mathbf{P}_1 \xi = 0, \mathbf{p}_0 \right. \\ \left. + \mathbf{P}_1 \xi \geq 0, \|\xi\|_1 \leq 1 \right\} = \mathbf{TC}^T \mathbf{p}_0 + Y^*(\mathbf{TC}), \end{aligned}$$

where $\|\xi\|_1 = \sum |\xi_s|$, and $Y^*(\mathbf{TC})$ is the optimal value of the following convex program

$$\max_{\xi} \left\{ \mathbf{TC}^T \mathbf{P}_1 \xi \mid \mathbf{e}^T \mathbf{P}_1 \xi = 0, \mathbf{p}_0 + \mathbf{P}_1 \xi \geq 0, \|\xi\|_1 \leq 1 \right\}. \tag{30}$$

The Lagrange function of problem (30) is

$$\mathcal{L}(\vartheta, \nu, \zeta, \xi) = \mathbf{TC}^T \mathbf{P}_1 \xi + \vartheta^T (\mathbf{p}_0 + \mathbf{P}_1 \xi) + \zeta^T \mathbf{P}_1 \xi + \nu(1 - \|\xi\|_1).$$

Then we obtain the following Lagrange dual function with variables $(\vartheta, \nu, \zeta) \in R^{|\mathcal{S}|} \times R \times R$,

$$\begin{aligned} g(\vartheta, \nu, \zeta) &= \max_{\xi} \mathcal{L}(\vartheta, \nu, \zeta, \xi) \\ &= (\mathbf{p}_0^T \vartheta + \nu) + \max_{\xi} \left\{ (\mathbf{P}_1^T \mathbf{TC} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \zeta) \xi - \nu \|\xi\|_1 \right\} \\ &= (\mathbf{p}_0^T \vartheta + \nu) + f^* (\mathbf{P}_1^T \mathbf{TC} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \zeta), \end{aligned}$$

where

$$f^* (\mathbf{P}_1^T \mathbf{TC} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \zeta) = \begin{cases} 0, & \|\mathbf{P}_1^T \mathbf{TC} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \zeta\|_{\infty} \leq \nu \\ \infty, & \text{otherwise} \end{cases}$$

is the conjugate function of $f(\xi) = \nu \|\xi\|_1$, and $\|\mathbf{P}_1^T \mathbf{TC} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \zeta\|_{\infty} = \max_{s \in \mathcal{S}} |(\mathbf{P}_1^T \mathbf{TC} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \zeta)_s|$. For any $\vartheta \geq 0$ and $\nu \geq 0$, the dual of problem (30) is the following optimization problem

$$\min_{(\vartheta, \nu, \zeta) \in R^{|\mathcal{S}|} \times R \times R} \left\{ \mathbf{p}_0^T \vartheta + \nu \mid \|\mathbf{P}_1^T \mathbf{TC} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \zeta\|_{\infty} \leq \nu, \vartheta \geq 0 \text{ and } \nu \geq 0 \right\}. \tag{31}$$

Thus the equivalent programming of problem (24) with regard to variables $(\vartheta, \nu, \zeta) \in R^{|\mathcal{S}|} \times R \times R$ can be represented as follows:

$$\begin{aligned} \min_{\tau, \vartheta, \nu, \zeta} \quad & \mathbf{TC}^T \mathbf{p}_0 + \mathbf{p}_0^T \vartheta + \nu \\ \text{s. t.} \quad & \left\| \mathbf{P}_1^T \mathbf{TC} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \mathbf{e}_\zeta \right\|_\infty \leq \nu, \\ & \vartheta \geq 0, \nu \geq 0. \end{aligned} \tag{32}$$

Similarly, the equivalent programming of problem (25) can be represented as the following form:

$$\begin{aligned} \min_{\tau, \vartheta, \nu, \zeta'} \quad & \phi + \frac{1}{1-\alpha} \mathbf{p}_0^T \mathbf{t} + \frac{1}{1-\alpha} (\mathbf{p}_0^T \vartheta' + \nu') \\ \text{s. t.} \quad & TC_s - \phi \leq t_s, \forall s, \\ & \left\| \mathbf{P}_1^T \mathbf{t} + \mathbf{P}_1^T \vartheta' + \mathbf{P}_1^T \mathbf{e}_{\zeta'} \right\|_\infty \leq \nu', \\ & t_s \geq 0, \forall s, \\ & \vartheta' \geq 0, \nu' \geq 0. \end{aligned} \tag{33}$$

Combining models (32) and (33), under the ambiguity $\wp = \wp_{\mathcal{S}}$, problem (23) is equivalent to the following solvable system:

$$\begin{aligned} \min_{\tau, \phi, \vartheta, \nu, \zeta, \vartheta', \nu'} \quad & \lambda (\mathbf{TC}^T \mathbf{p}_0 + \mathbf{p}_0^T \vartheta + \nu) + (1-\lambda) \\ & \left[\phi + \frac{1}{1-\alpha} \mathbf{p}_0^T \mathbf{t} + \frac{1}{1-\alpha} (\mathbf{p}_0^T \vartheta' + \nu') \right] \text{s. t. } TC_s - \phi \\ & \leq t_s, \forall s, \left\| \mathbf{P}_1^T \mathbf{TC} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \mathbf{e}_\zeta \right\|_\infty \leq \nu, \left\| \mathbf{P}_1^T \mathbf{t} + \mathbf{P}_1^T \vartheta' + \mathbf{P}_1^T \mathbf{e}_{\zeta'} \right\|_\infty \\ & \leq \nu', t_s \geq 0, \forall s, \vartheta \geq 0, \nu \geq 0, \vartheta' \geq 0, \nu' \\ & \geq 0, \text{constraints (1) - (17)}, \end{aligned}$$

where $\nu, \zeta, \nu', \zeta', \vartheta, \vartheta' \in R \times R \times R \times R \times R^{|\mathcal{S}|} \times R^{|\mathcal{S}'|}$ are dual variables/vectors.

For the sake of clarity, the above-mentioned linear system can be rewritten as

$$\begin{aligned} \min_{\tau, \phi, \vartheta, \nu, \zeta, \vartheta', \nu'} \quad & \lambda (\mathbf{C}^T \tau \mathbf{p}_0 + \mathbf{p}_0^T \vartheta + \nu) + (1-\lambda) \\ & \left[\phi + \frac{1}{1-\alpha} \mathbf{p}_0^T \mathbf{t} + \frac{1}{1-\alpha} (\mathbf{p}_0^T \vartheta' + \nu') \right] \text{s. t. } TC_s - \phi \\ & \leq t_s, \forall s, \left\| \mathbf{P}_1^T \tau \mathbf{TC} + \mathbf{P}_1^T \vartheta + \mathbf{P}_1^T \mathbf{e}_\zeta \right\|_\infty \leq \nu, \left\| \mathbf{P}_1^T \mathbf{t} + \mathbf{P}_1^T \vartheta' + \mathbf{P}_1^T \mathbf{e}_{\zeta'} \right\|_\infty \\ & \leq \nu', t_s \geq 0, \forall s, \vartheta \geq 0, \nu \geq 0, \vartheta' \geq 0, \nu' \\ & \geq 0, \text{constraints (1) - (17)}. \end{aligned}$$

The proof of theorem is complete. ■

Finally, we show the proof of Proposition 1.

If $(\tau^*, \phi^*, \mathbf{t}^*, \vartheta^*, \nu^*, \zeta^*, \vartheta'^*, \nu'^*, \zeta'^*)$ is the optimal solution of model (27) with $\wp = \wp_{\mathcal{S}}$, then $(\tau^*, \vartheta^*, \nu^*, \zeta^*)$ is the optimal solution of model (32). By the lagrange weak duality theorem, we can show that

$$\begin{aligned} \max_{\mathbf{p} \in \wp_{\mathcal{S}}} \quad & \mathbf{C}^T \tau^* \mathbf{p} = \mathbf{C}^T \tau^* \mathbf{p}_0 + \Upsilon^*(\mathbf{TC}) \\ & \leq \mathbf{C}^T \tau^* \mathbf{p}_0 + \mathbf{p}_0^T \vartheta^* + \nu^*. \end{aligned}$$

So $(\tau^*, \phi^*, \mathbf{t}^*)$ is a feasible solution to model (23). We assume that $(\bar{\tau}^*, \bar{\phi}^*, \bar{\mathbf{t}}^*, \bar{\vartheta}^*, \bar{\nu}^*, \bar{\zeta}^*, \bar{\vartheta}'^*, \bar{\nu}'^*, \bar{\zeta}'^*)$ is another solution and it is the optimal solution to model (23), then $(\bar{\tau}^*, \bar{\vartheta}^*, \bar{\nu}^*, \bar{\zeta}^*)$ is the optimal solution of model (32) such that

$$\mathbf{C}^T \bar{\tau}^* \mathbf{p}_0 + \mathbf{p}_0^T \bar{\vartheta}^* + \bar{\nu}^* \leq \mathbf{C}^T \tau^* \mathbf{p}_0 + \mathbf{p}_0^T \vartheta^* + \nu^*, \tag{34}$$

where $(\bar{\vartheta}^*, \bar{\nu}^*, \bar{\zeta}^*)$ solves model (31).

By the strong duality theorem, we have

$$\max_{\mathbf{p} \in \wp_{\mathcal{S}}} \mathbf{C}^T \bar{\tau}^* \mathbf{p} = \mathbf{C}^T \bar{\tau}^* \mathbf{p}_0 + \Upsilon^*(\mathbf{TC}) = \mathbf{C}^T \bar{\tau}^* \mathbf{p}_0 + \mathbf{p}_0^T \bar{\vartheta}^* + \bar{\nu}^*.$$

The CVaR criterion is similar to expected value. Combine with constraints in model (23) and (31), it means that $(\bar{\tau}^*, \bar{\phi}^*, \bar{\mathbf{t}}^*, \bar{\vartheta}^*, \bar{\nu}^*, \bar{\zeta}^*, \bar{\vartheta}'^*, \bar{\nu}'^*, \bar{\zeta}'^*)$ is a feasible solution to model (27), which contradicted the assumption that $(\tau^*, \phi^*, \mathbf{t}^*, \vartheta^*, \nu^*, \zeta^*, \vartheta'^*, \nu'^*, \zeta'^*)$ is an optimal solution to model (27). Therefore, $(\tau^*, \phi^*, \mathbf{t}^*)$ is the optimal solution to model (23).

Conversely, if $(\bar{\tau}^*, \bar{\phi}^*, \bar{\mathbf{t}}^*)$ is a solution to model (23) with $\wp = \wp_{\mathcal{S}}$, then $(\bar{\tau}^*, \bar{\vartheta}^*, \bar{\nu}^*, \bar{\zeta}^*)$ solves model (32), where $(\bar{\vartheta}^*, \bar{\nu}^*, \bar{\zeta}^*)$ solves (31), and $(\bar{\tau}^*, \bar{\phi}^*, \bar{\mathbf{t}}^*, \bar{\vartheta}'^*, \bar{\nu}'^*, \bar{\zeta}'^*)$ solves model (33). So $(\bar{\tau}^*, \bar{\phi}^*, \bar{\mathbf{t}}^*, \bar{\vartheta}^*, \bar{\nu}^*, \bar{\zeta}^*, \bar{\vartheta}'^*, \bar{\nu}'^*, \bar{\zeta}'^*)$ is a feasible solution to model (27). Assume $(\tau^*, \phi^*, \mathbf{t}^*, \vartheta^*, \nu^*, \zeta^*, \vartheta'^*, \nu'^*, \zeta'^*)$ is another solution to model (23), then, for expected value criterion, $(\bar{\tau}^*, \bar{\vartheta}^*, \bar{\nu}^*, \bar{\zeta}^*)$ is the optimal solution of model (32) such that

$$\mathbf{C}^T \bar{\tau}^* \mathbf{p}_0 + \mathbf{p}_0^T \bar{\vartheta}^* + \bar{\nu}^* \leq \mathbf{C}^T \tau^* \mathbf{p}_0 + \mathbf{p}_0^T \vartheta^* + \nu^*, \tag{35}$$

where $(\bar{\vartheta}^*, \bar{\nu}^*, \bar{\zeta}^*)$ solves model (31). According to the discussion in the first part, $(\bar{\tau}^*, \bar{\phi}^*, \bar{\mathbf{t}}^*)$ is an optimal solution to model (23), which contradicts the assumption that $(\tau^*, \phi^*, \mathbf{t}^*)$ solves model (23). Therefore, $(\bar{\tau}^*, \bar{\phi}^*, \bar{\mathbf{t}}^*, \bar{\vartheta}^*, \bar{\nu}^*, \bar{\zeta}^*, \bar{\vartheta}'^*, \bar{\nu}'^*, \bar{\zeta}'^*)$ solves model (27).

References

Ahmadi, S., Amin, S.H., 2019. An integrated chance-constrained stochastic model for a mobile phone closed-loop supply chain network with supplier selection. *J. Clean. Prod.* 226, 998–1003.

Bai, X., Liu, Y., 2016. Robust optimization of supply chain network design in fuzzy decision system. *J. Intell. Manuf.* 27 (6), 1131–1149.

Baptista, S., Barbosa-Povoa, A.P., Escudero, L.F., Gomes, M.I., Pizarro, C., 2019. On risk management of a two-stage stochastic mixed 0-1 model for the closed-loop supply chain design problem. *Eur. J. Oper. Res.* 274 (1), 91–107.

Ben-Tal, A., El Ghaoui, L., Nemirovski, A., 2009. *Robust Optimization*. Princeton University Press.

Bertsimas, D., Sim, M., 2003. Robust discrete optimization and network flows. *Math. Program.* 98 (1–3), 49–71.

Bicycle sharing—mobike and ofo analysis report. Available online. <https://wenku.baidu.com/view/fbd80609ae1ffc4ffe4733687e21af45b207fe13.html>–. (Accessed 1 April 2018).

Birge, J.R., Louveaux, F., 1997. *Introduction to Stochastic Programming*. Springer, New York.

Cardoso, S.R., Barbosa-Povoa, A.P., Relvas, S., 2016. Integrating financial risk measures into the design and planning of closed-loop supply chains. *Comput. Chem. Eng.* 85, 105–123.

Cheraghali, A., Paydar, M.M., Hajiaghahi-Keshteli, M., 2018. A bi-objective optimization for citrus closed-loop supply chain using Pareto-based algorithms. *Appl. Soft Comput.* 69, 33–59.

DeMaio, P., 2009. Bike-sharing: history, impacts, models of provision, and future. *J. Public Transp.* 12 (4), 41–56.

Ebrahimi, S.B., 2018. A stochastic multi-objective location-allocation-routing problem for tire supply chain considering sustainability aspects and quantity discounts. *J. Clean. Prod.* 198, 704–720.

El-Sayed, M., Afia, N., El-Kharbotly, A., 2010. A stochastic model for forward-reverse logistics network design under risk. *Comput. Ind. Eng.* 58 (3), 423–431.

Fattahi, M., Govindan, K., 2017. Integrated forward/reverse logistics network design under uncertainty with pricing for collection of used products. *Ann. Oper. Res.* 253 (1), 193–225.

Fleischmann, M., Beullens, P., Bloemhof-Ruwaard, J.M., Van Wassenhove, L.N., 2001. The impact of product recovery on logistics network design. *Prod. Oper. Manag.* 10 (2), 156–173.

Fu, Q., Sim, C.K., Teo, C.P., 2018. Profit sharing agreements in decentralized supply chains: a distributionally robust approach. *Oper. Res.* 66 (2), 500–513.

Gao, J., Ning, C., You, F., 2019. Data-driven distributionally robust optimization of shale gas supply chains under uncertainty. *AIChE J.* 65 (3), 947–963.

Ghahremani-Nahr, J., Kian, R., Sabet, E., 2019. A robust fuzzy mathematical programming model for the closed-loop supply chain network design and a whale optimization solution algorithm. *Expert Syst. Appl.* 116, 454–471.

Govindan, K., Fattahi, M., Keyvanshokoh, E., 2017. Supply chain network design under uncertainty: a comprehensive review and future research directions. *Eur. J. Oper. Res.* 263 (1), 108–141.

Gyodi, K., 2019. Airbnb in European cities: business as usual or true sharing economy? *J. Clean. Prod.* 221, 536–551.

Keyvanshokoh, E., Ryan, S.M., Kabir, E., 2016. Hybrid robust and stochastic

- optimization for closed-loop supply chain network design using accelerated benders decomposition. *Eur. J. Oper. Res.* 249 (1), 76–92.
- Kim, J., Do Chung, B., Kang, Y., Jeong, B., 2018. Robust optimization model for closed-loop supply chain planning under reverse logistics flow and demand uncertainty. *J. Clean. Prod.* 196, 1314–1328.
- Ko, H.J., Evans, G.W., 2007. A genetic algorithm-based heuristic for the dynamic integrated forward/reverse logistics network for 3PLs. *Comput. Oper. Res.* 34 (2), 346–366.
- Ma, L., Liu, Y., 2017. A stochastic chance constrained closed-loop supply chain network design model with VaR criterion. *J. Uncertain Syst.* 11 (4), 306–320.
- Min, H., Zhou, G., 2002. Supply chain modeling: past, present and future. *Comput. Ind. Eng.* 43 (1–2), 231–249.
- Noyan, N., 2012. Risk-averse two-stage stochastic programming with an application to disaster management. *Comput. Oper. Res.* 39 (3), 541–559.
- Pishvaei, M.S., Jolai, F., Razmi, J., 2009. A stochastic optimization model for integrated forward/reverse logistics network design. *J. Manuf. Syst.* 28 (4), 107–114.
- Polo, A., Pena, N., Munoz, D., Canon, A., Escobar, J.W., 2018. Robust design of a closed-loop supply chain under uncertainty conditions integrating financial criteria. *Omega* 1–23. <https://doi.org/10.1016/j.omega.2018.09.003>.
- Prakash, S., Kumar, S., Soni, G., Jain, V., Rathore, A.P.S., 2018. Closed-loop supply chain network design and modelling under risks and demand uncertainty: an integrated robust optimization approach. *Ann. Oper. Res.* 1–28. <https://doi.org/10.1007/s10479-018-2902-3>.
- Rockafellar, R.T., Uryasev, S., 2000. Optimization of conditional value-at-risk. *J. Risk* 2 (3), 21–42.
- Ruszczynski, A., Shapiro, A. (Eds.), 2003. *Stochastic Programming. Handbook in Operations Research and Management Science*, vol. 10. Elsevier Science, Amsterdam.
- Saedinia, R., Vahdani, B., Etebari, F., Nadjafi, B.A., 2019. Robust gasoline closed loop supply chain design with redistricting, service sharing and intra-district service transfer. *Transport. Res. E Logist. Transport. Rev.* 123, 121–141.
- Safaei, A.S., Roozbeh, A., Paydar, M.M., 2017. A robust optimization model for the design of a cardboard closed-loop supply chain. *J. Clean. Prod.* 166, 1154–1168.
- Salimi, F., Vahdani, B., 2018. Designing a bio-fuel network considering links reliability and risk-pooling effect in bio-refineries. *Reliab. Eng. Syst. Saf.* 174, 96–107.
- Soleimani, H., Seyyed-Esfahani, M., Kannan, G., 2014. Incorporating risk measures in closed-loop supply chain network design. *Int. J. Prod. Res.* 52 (6), 1843–1867.
- Ukolov, V.F., Solomatin, A.V., Solomatin, Y.V., Chernikov, S.U., Ukolov, A.V., 2016. Food-sharing economy pattern comparison in UK and Russian markets. *Int. Bus. Manag.* 10 (18), 4268–4282.
- Vahdani, B., Mohammadi, M., 2015. A bi-objective interval-stochastic robust optimization model for designing closed loop supply chain network with multi-priority queuing system. *Int. J. Prod. Econ.* 170, 67–87.
- Vahdani, B., Mansour, F., Soltani, M., Veysmoradi, D., 2019. Bi-objective optimization for integrating quay crane and internal truck assignment with challenges of trucks sharing. *Knowl. Based Syst.* 163, 675–692.
- Wang, J., Huang, J., Dunford, M., 2019. Rethinking the utility of public bicycles: the development and challenges of station-less bike sharing in China. *Sustainability* 11 (6), 1539.
- Xu, J., Bai, Q., Xu, L., Hu, T., 2018. Effects of emission reduction and partial demand information on operational decisions of a newsvendor problem. *J. Clean. Prod.* 188, 825–839.
- Yang, G., Liu, Y., 2017. Optimizing an equilibrium supply chain network design problem by an improved hybrid biogeography based optimization algorithm. *Appl. Soft Comput.* 58, 657–668.
- Yu, J.R., Chiou, W.J.P., Liu, R.T., 2017. Incorporating transaction costs, weighting management, and floating required return in robust portfolios. *Comput. Ind. Eng.* 109, 48–58.
- Zhang, C., Schmocker, J.D., 2019. A Markovian model of user adaptation with case study of a shared bicycle scheme. *Transportmetrica B* 7 (1), 223–236.
- Zhao, X., Xia, X., Wang, L., Yu, G., 2018. Risk-averse facility location for green closed-loop supply chain networks design under uncertainty. *Sustainability* 10 (11), 4072.
- Zhen, L., Huang, L., Wang, W., 2019. Green and sustainable closed-loop supply chain network design under uncertainty. *J. Clean. Prod.* 227, 1195–1209.